Intermediate Algebra

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Mathematics can be best described as a language, and when one learns the foundations of mathematics, one starts the process of becoming literate. The mathematics covered in this textbook are at an intermediate algebra level, building upon literacies covered in Mathematical Fundamental and Elementary Algebra.

This textbook can trace its origins to three distinct sources:

First, it is adapted from an original work by Wallace: Elementary and Introductory Algebra.

Second, it has been modified after many years of observing student preferences in how they learn. My former KPU students all have fingerprints throughout this book.

Third, it is the work of the world, in that mathematics is universal and global, having history in all ages and cultures.

This being said, this textbook is intended to never be sold for profit. Rather, it is meant to be freely used and adapted by anyone who wishes to teach or learn intermediate algebra.

Please feel free to contact me at KPU for insights and additions that can add richness to this document, as this work is intended to be a living document that can grow and help to increase our understanding of this complex world that we live in.

Best regards

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PART I

CHAPTER 1: ALGEBRA REVIEW

Learning Objectives

This chapter covers:

- Integers
- Fractions
- Order of Operations
- Properties of Algebra
- Terms & Definitions
- Word Problems

1. 1.1 Integers

The ability to work comfortably with negative numbers is essential to success in algebra. For this reason, a quick review of adding, subtracting, multiplying, and dividing integers is necessary.[1] Integers[2] are all the positive whole numbers, all the negative whole numbers, and zero. As this is intended to be a review of integers, descriptions and examples will not be as detailed as in a normal lesson.

When adding integers, there are two cases to consider. The first is when the signs match—that is, the two integers are both positive or both negative.

If the signs match, add the numbers together and retain the sign.

Example 1.1.1

Add $-5+(-3)$.

$$\begin{array}{rl} -5+(-3) & \text{Same sign. Add } 5+3. \text{ Keep the negative sign.} \\ -8 & \text{Solution} \end{array}$$

Example 1.1.2

Add $-7+(-5)$.

$$\begin{array}{rl} -7+(-5) & \text{Same sign. Add } 7+5. \text{ Keep the negative sign.} \\ -12 & \text{Solution} \end{array}$$

The second case is when the signs don't match, and there is one positive and one negative number. Subtract the numbers (as if they were all positive), then use the sign from the number with the greatest absolute value. This means that, if the number with the greater absolute value is positive, the answer is positive. If it is negative, the answer is negative.

1. Read about The History of Integers.
2. The word "integer" is derived from the Latin word *integer*, which means "whole." Integers are written without using a fractional component. Examples are 2, 3, 1042, 28, 0, −42, −2. Numbers that are fractional—such as $\frac{1}{4}$, 0.33, and 1.42—are not integers.

Example 1.1.3

Add $-7+2$.

$$\begin{array}{rl} -7+2 & \text{Different signs. Subtract } 7-2\text{. Negative number has greater absolute value.} \\ -5 & \text{Solution} \end{array}$$

Example 1.1.4

Add $-4+6$.

$$\begin{array}{rl} -4+6 & \text{Different signs. Subtract } 6-4\text{. Positive number has greater absolute value.} \\ 2 & \text{Solution} \end{array}$$

Example 1.1.5

Add $4+(-3)$.

$$\begin{array}{rl} 4+(-3) & \text{Different signs. Subtract } 4-3\text{. Positive number has greater absolute value.} \\ 1 & \text{Solution} \end{array}$$

Example 1.1.6

Add $7+(-10)$.

$$\begin{array}{rl} 7+(-10) & \text{Different signs. Subtract } 10-7\text{. Negative number has greater absolute value.} \\ -3 & \text{Solution} \end{array}$$

For subtraction of negatives, change the problem to an addition problem, which is then solved using the above methods.

The way to change a subtraction problem to an addition problem is by adding the opposite of the number after the subtraction sign to the number before the subtraction sign. Often, this method is referred to as "adding the opposite."

Example 1.1.7

Subtract $8-3$.

$$\begin{array}{rl} 8-3 & \text{Add the opposite of 3 to 8.} \\ 8+(-3) & \text{Different signs. Subtract } 8-3\text{. Positive number has greater absolute value.} \\ 5 & \text{Solution} \end{array}$$

Example 1.1.8

Subtract $-4-6$.

$$\begin{array}{rl} -4-6 & \text{Add the opposite of 6 to } -4. \\ -4+(-6) & \text{Same sign. Add } 4+6\text{. Keep the negative sign.} \\ -10 & \text{Solution} \end{array}$$

Example 1.1.9

Subtract $9-(-4)$.

$$\begin{array}{rl} 9-(-4) & \text{Add the opposite of } -4 \text{ to 9.} \\ 9+4 & \text{Same sign. Add } 9+4\text{. Keep the positive sign.} \\ 13 & \text{Solution} \end{array}$$

Example 1.1.10

Subtract $-6-(-2)$.

$$\begin{array}{rl} -6-(-2) & \text{Add the opposite of } -2 \text{ to } -6. \\ -6+2 & \text{Different signs. Subtract } 6-2. \text{ Negative number has greater absolute value.} \\ -4 & \text{Solution} \end{array}$$

Multiplication and division of integers both work in a very similar pattern. The short description of the process is to multiply and divide like normal. If the signs match (numbers are both positive or both negative), the answer is positive. If the signs don't match (one positive and one negative), then the answer is negative.

Example 1.1.11

Multiply $(4)(-6)$.

$$\begin{array}{rl} (4)(-6) & \text{Signs do not match, so the answer is negative.} \\ -24 & \text{Solution} \end{array}$$

Example 1.1.12

Divide $-36 \div -9$.

$$\begin{array}{rl} -36 \div -9 & \text{Signs match, so the answer is positive.} \\ 4 & \text{Solution} \end{array}$$

Example 1.1.13

Multiply $-2(-6)$.

$$\begin{array}{rl} -2(-6) & \text{Signs match, so the answer is positive.} \\ 12 & \text{Solution} \end{array}$$

Example 1.1.14

Divide $15 \div -3.$

$$\begin{array}{rl} 15 \div -3 & \text{Signs do not match, so the answer is negative.} \\ -5 & \text{Solution} \end{array}$$

Key Takeaways: A few things to be careful of when working with integers.

Be sure not to confuse a problem like −3 − 8 with −3(−8).

- The −3 − 8 problem is subtraction because the subtraction sign separates the −3 from what comes after it.
- The −3(−8) is a multiplication problem because there is nothing between the −3 and the parenthesis. If there is no operation written in between the parts, then you assume that you are multiplying.

Be careful not to mix the pattern for adding and subtracting integers with the pattern for multiplying and dividing integers. They can look very similar. For example:

- If the two numbers in an addition problem are negative, then keep the negative sign, such as in −3 + (−7) = −10.
- If the signs of the two numbers in a multiplication problem match, the answer is positive, such as in (−3)(−7) = 21.

Questions

For questions 1 to 30, find the sum and/or difference.

1. $1-3$
2. $4-(-1)$
3. $(-6)-(-8)$
4. $(-6)+8$
5. $(-3)-3$
6. $(-8)-(-3)$
7. $3-(-5)$
8. $7-7$
9. $(-7)-(-5)$
10. $(-4)+(-1)$
11. $3-(-1)$
12. $(-1)+(-6)$
13. $6-3$

14. $(-8) + (-1)$
15. $(-5) + 3$
16. $(-1) - 8$
17. $2 - 3$
18. $5 - 7$
19. $(-8) - (-5)$
20. $(-5) + 7$
21. $(-2) + (-5)$
22. $1 + (-1)$
23. $5 - (-6)$
24. $8 - (-1)$
25. $(-6) + 3$
26. $(-3) + (-1)$
27. $4 - 7$
28. $7 - 3$
29. $(-7) + 7$
30. $(-3) + (-5)$

For questions 31 to 44, find each product.

31. $(4)(-1)$
32. $(7)(-5)$
33. $(10)(-8)$
34. $(-7)(-2)$
35. $(-4)(-2)$
36. $(-6)(-1)$
37. $(-7)(8)$
38. $(6)(-1)$
39. $(9)(-4)$
40. $(-9)(-7)$
41. $(-5)(2)$
42. $(-2)(-2)$
43. $(-5)(4)$
44. $(-3)(-9)$

For questions 45 to 58, find each quotient.

45. $30 \div -10$
46. $-49 \div -7$
47. $-12 \div -4$
48. $-2 \div -1$
49. $30 \div 6$
50. $20 \div 10$
51. $27 \div 3$
52. $-35 \div -5$
53. $80 \div -8$
54. $8 \div -2$
55. $50 \div 5$

56. $-16 \div 2$
57. $48 \div 8$
58. $60 \div -10$

Answer Key 1.1

2. 1.2 Fractions (Review)

Working with fractions is a very important foundational skill in algebra. This section will briefly review reducing, multiplying, dividing, adding, and subtracting fractions. As this is a review, concepts will not be explained in as much detail as they are in other lessons. Final answers of questions working with fractions tend to always be reduced. Reducing fractions is simply done by dividing both the numerator and denominator by the same number.

Example 1.2.1

Reduce $\frac{36}{84}$.

$$\frac{36}{84} \quad \text{Both the numerator and the denominator are divisible by 4.}$$

$$\frac{36 \div 4}{84 \div 4} = \frac{9}{21} \quad \text{Both the numerator and the denominator are divisible by 3.}$$

$$\frac{9 \div 3}{21 \div 3} = \frac{3}{7} \quad \text{Solution}$$

The previous example could have been done in one step by dividing both the numerator and the denominator by 12. Another solution could have been to divide by 2 twice and then by 3 once (in any order). It is not important which method is used as long as the fraction is reduced as much as possible.

The easiest operation to complete with fractions is multiplication. Fractions can be multiplied straight across, meaning all numerators and all denominators are multiplied together.

Example 1.2.2

Multiply $\frac{6}{7} \cdot \frac{3}{5}$.

$$\frac{6}{7} \cdot \frac{3}{5} \quad \text{Multiply numerators and denominators, respectively.}$$

$$\frac{18}{35} \quad \text{Solution}$$

Before multiplying, fractions can be reduced. It is possible to reduce vertically within a single fraction, or diagonally within several fractions, as long as one number from the numerator and one number from the denominator are used.

Example 1.2.3

Multiply $\dfrac{25}{24}\cdot\dfrac{32}{55}$.

$$\frac{25\ 5}{24\ 3}\cdot\frac{32\ 4}{55\ 11} \quad \text{Reduce 25 and 55 by dividing by 5, and reduce 32 and 24 by dividing by 8.}$$

$$\frac{5\cdot 4}{3\cdot 11} \quad \text{Multiply numerators and denominators across.}$$

$$\frac{20}{33} \quad \text{Solution}$$

Dividing fractions is very similar to multiplying, with one extra step. Dividing fractions necessitates first taking the reciprocal of the second fraction. Once this is done, multiply the fractions together. This multiplication problem solves just like the previous problem.

Example 1.2.4

Divide $\dfrac{21}{16}\div\dfrac{28}{6}$.

$$\frac{21}{16}\div\frac{28}{6} \quad \text{Take the reciprocal of the second fraction and multiply it by the first.}$$

$$\frac{21\ 3}{16\ 8}\cdot\frac{6\ 3}{28\ 4} \quad \text{Reduce 21 and 28 by dividing by 7, and reduce 6 and 16 by dividing by 2.}$$

$$\frac{3\cdot 3}{8\cdot 4} \quad \text{Multiply numerators and denominators across.}$$

$$\frac{9}{32} \quad \text{Solution}$$

To add and subtract fractions, it is necessary to first find the least common denominator (LCD). There are several ways to find the LCD. One way is to break the denominators into primes, write out the primes that make up the first denominator, and only add primes that are needed to make the other denominators.

Example 1.2.5

Find the LCD of 8 and 12.

Break 8 and 12 into primes:

$$\begin{aligned} 8 &= 2 \times 2 \times 2 \\ 12 &= 2 \times 2 \times 3 \end{aligned}$$

The LCD will contain all the primes needed to make each number above.

$$\mathrm{LCD} = \rlap{$\overbrace{}^{8}$}2 \times \underbrace{2 \times 2 \times 3}_{12} = 4$$

Adding and subtracting fractions is identical in process. If both fractions already have a common denominator, simply add or subtract the numerators and keep the denominator.

Example 1.2.6

Add $\dfrac{7}{8} + \dfrac{3}{8}$.

$$\begin{aligned} &\frac{7}{8} + \frac{3}{8} && \text{Same denominator, so add } 7 + 3. \\ &\frac{10}{8} && \text{Reduce answer by dividing the numerator and denominator by } 2. \\ &\frac{5}{4} && \text{Solution} \end{aligned}$$

While $\dfrac{5}{4}$ can be written as the mixed number $1\dfrac{1}{4}$, algebra almost never uses mixed numbers. For this reason, always use the improper fraction, not the mixed number.

Example 1.2.7

Subtract $\frac{13}{6} - \frac{9}{6}$.

$$\begin{array}{rl} \frac{13}{6} - \frac{9}{6} & \text{Same denominator, so subtract } 13 - 9. \\ \frac{4}{6} & \text{Reduce answer by dividing by 2.} \\ \frac{2}{3} & \text{Solution} \end{array}$$

If the denominators do not match, it is necessary to first identify the LCD and build up each fraction by multiplying the numerator and denominator by the same number so each denominator is built up to the LCD.

Example 1.2.8

Add $\frac{5}{6} + \frac{4}{9}$.

$$\begin{array}{rl} \overbrace{2 \times 3}^{6} \times 3 \text{ with } 2 \times \underbrace{3 \times 3}_{9} & \text{LCD is 18.} \\ \frac{3 \cdot 5}{3 \cdot 6} + \frac{4 \cdot 2}{9 \cdot 2} & \text{Multiply the first fraction by 3 and the second by 2.} \\ \frac{15}{18} + \frac{8}{18} & \text{Same denominator, so add } 15 + 8. \\ \frac{23}{18} & \text{Solution} \end{array}$$

Example 1.2.9

Subtract $\frac{2}{3} - \frac{1}{6}$.

$$\begin{array}{rl} \dfrac{2}{3} - \dfrac{1}{6} & \text{LCD is 6.} \\[2ex] \dfrac{2\cdot 2}{2\cdot 3} - \dfrac{1}{6} & \text{Multiply the first fraction by 2.} \\[2ex] \dfrac{4}{6} - \dfrac{1}{6} & \text{Same denominator, so subtract } 4-1. \\[2ex] \dfrac{3}{6} & \text{Reduce answer by dividing by 3.} \\[2ex] \dfrac{1}{2} & \text{Solution} \end{array}$$

Questions

For questions 1 to 18, simplify each fraction. Leave your answer as an improper fraction.

1. $\frac{42}{12}$
2. $\frac{25}{20}$
3. $\frac{35}{25}$
4. $\frac{24}{8}$
5. $\frac{54}{36}$
6. $\frac{30}{24}$
7. $\frac{45}{36}$
8. $\frac{36}{27}$

9. $\frac{27}{18}$
10. $\frac{48}{18}$
11. $\frac{40}{16}$
12. $\frac{48}{42}$
13. $\frac{63}{18}$
14. $\frac{16}{12}$
15. $\frac{80}{60}$
16. $\frac{72}{48}$
17. $\frac{72}{60}$
18. $\frac{126}{108}$

For questions 19 to 36, find each product. Leave your answer as an improper fraction.

19. $(9)\left(\frac{8}{9}\right)$
20. $(-2)\left(-\frac{5}{6}\right)$
21. $(2)\left(-\frac{2}{9}\right)$
22. $(-2)\left(\frac{1}{3}\right)$
23. $(-2)\left(\frac{13}{8}\right)$
24. $\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)$
25. $\left(-\frac{6}{5}\right)\left(-\frac{11}{8}\right)$
26. $\left(-\frac{3}{7}\right)\left(-\frac{11}{8}\right)$
27. $(8)\left(\frac{1}{2}\right)$
28. $(-2)\left(-\frac{9}{7}\right)$
29. $\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)$
30. $\left(-\frac{17}{9}\right)\left(-\frac{3}{5}\right)$

31. $(2)\left(\frac{3}{2}\right)$
32. $\left(\frac{17}{9}\right)\left(-\frac{3}{5}\right)$
33. $\left(\frac{1}{2}\right)\left(-\frac{7}{5}\right)$
34. $\left(\frac{1}{2}\right)\left(\frac{5}{7}\right)$
35. $\left(\frac{5}{2}\right)\left(-\frac{0}{5}\right)$
36. $\left(\frac{6}{0}\right)\left(\frac{6}{7}\right)$

For questions 37 to 52, find each quotient. Leave your answer as an improper fraction.

37. $-2 \div \frac{7}{4}$
38. $-\frac{12}{7} \div -\frac{9}{5}$
39. $-\frac{1}{9} \div -\frac{1}{2}$
40. $-2 \div -\frac{3}{2}$
41. $-\frac{3}{2} \div \frac{13}{7}$
42. $\frac{5}{3} \div \frac{7}{5}$
43. $-1 \div \frac{2}{3}$
44. $\frac{10}{9} \div -6$
45. $\frac{8}{9} \div \frac{1}{5}$
46. $\frac{1}{6} \div -\frac{5}{3}$
47. $-\frac{9}{7} \div \frac{1}{5}$
48. $-\frac{13}{8} \div -\frac{15}{8}$
49. $-\frac{2}{9} \div -\frac{3}{2}$
50. $-\frac{4}{5} \div -\frac{13}{8}$
51. $\frac{1}{10} \div \frac{3}{2}$
52. $\frac{5}{3} \div \frac{5}{3}$

For questions 53 to 70, evaluate each expression. Leave your answer as an improper fraction.

53. $\frac{1}{3}+\left(-\frac{4}{3}\right)$
54. $\frac{1}{7}+\left(-\frac{11}{7}\right)$
55. $\frac{3}{7}-\frac{1}{7}$
56. $\frac{1}{3}+\frac{5}{3}$
57. $\frac{11}{6}+\frac{7}{6}$
58. $(-2)+\left(-\frac{15}{8}\right)$
59. $\frac{3}{5}+\frac{5}{4}$
60. $(-1)-\frac{2}{3}$
61. $\frac{2}{5}+\frac{5}{4}$
62. $\frac{12}{7}-\frac{9}{7}$
63. $\frac{9}{8}+\left(-\frac{2}{7}\right)$
64. $(-2)+\frac{5}{6}$
65. $1+\left(-\frac{1}{3}\right)$
66. $\frac{1}{2}-\frac{11}{6}$
67. $\left(-\frac{1}{2}\right)+\frac{3}{2}$
68. $\frac{11}{8}-\frac{1}{22}$
69. $\frac{1}{5}+\frac{3}{4}$
70. $\frac{6}{5}-\frac{8}{5}$

Answer Key 1.2

3. 1.3 Order of Operations (Review)

When simplifying expressions, it is important to do so in the correct order. Consider the problem $2 + 5 \cdot 3$ done two different ways:

Method 1: Add first	Method 2: Multiply first
Add: $2 + 5 \cdot 3$	Multiply: $2 + 5 \cdot 3$
Multiply: $7 \cdot 3$	Add: $2 + 15$
Solution: 21	Solution: 17

The previous example illustrates that if the same problem is done two different ways, it will result in two different solutions. However, only one method can be correct. It turns out the second method is the correct one. The order of operations ends with the most basic of operations, addition (or subtraction). Before addition is completed, do all repeated addition, also known as multiplication (or division). Before multiplication is completed, do all repeated multiplication, also known as exponents. When something is supposed to be done out of order, to make it come first, put it in parentheses (or grouping symbols). This list, then, is the order of operations used to simplify expressions.

Key Takeaways: Order of Operations

1st Brackets (Grouping)

2nd Exponents

3rd Multiplication and Division (Left to Right)

4th Addition and Subtraction (Left to Right)

Multiplication and division are on the same level because they are the same operation (division is just multiplying by the reciprocal). This means multiplication and division must be performed from left to right. Therefore, division will come first in some problems, and multiplication will come first in others. The same is true for adding and subtracting (subtracting is just adding the opposite).

Often, students use the word BEMDAS to remember the order of operations, as the first letter of each operation creates the word (written as B E MD AS). Remember BEMDAS to ensure that multiplication and division are done from left to right (same with addition and subtraction).

Example 1.3.1

Evaluate $2 + 3(9 - 4)^2$ using the order of operations.

$$\begin{aligned} 2+3(9-4)^2 &\quad \text{Parentheses first} \\ 2+3(5)^2 &\quad \text{Exponents} \\ 2+3(25) &\quad \text{Multiply} \\ 2+75 &\quad \text{Add} \\ 77 &\quad \text{Solution} \end{aligned}$$

It is very important to remember to multiply and divide from left to right!

Example 1.3.2

Evaluate $30 \div 3 \cdot 2$ using the order of operations.

$$\begin{aligned} 30 \div 3 \cdot 2 &\quad \text{Divide first (left to right)} \\ 10 \cdot 2 &\quad \text{Multiply} \\ 20 &\quad \text{Solution} \end{aligned}$$

If there are several sets of parentheses in a problem, start with the innermost set and work outward. Inside each set of parentheses, simplify using the order of operations. To make it easier to know which left parenthesis goes with which right parenthesis, different types of grouping symbols will be used, such as braces { }, brackets [ ], and parentheses ( ). These all do the same thing: they are grouping symbols and must be evaluated first.

Example 1.3.3

Evaluate $2\{8^2 - 7\left[32 - 4(3^2+1)\right](-1)\}$ using the order of operations.

$$\begin{aligned}
2\{8^2 - 7\left[32 - 4(3^2+1)\right](-1)\} &\quad \text{Innermost parentheses, exponents first} \\
2\{8^2 - 7[32 - 4(9+1)](-1)\} &\quad \text{Add inside those parentheses} \\
2\{8^2 - 7[32 - 4(10)](-1)\} &\quad \text{Multiply inside innermost parentheses} \\
2\{8^2 - 7[32 - 40](-1)\} &\quad \text{Subtract inside those parentheses} \\
2\{8^2 - 7[-8](-1)\} &\quad \text{Exponents next} \\
2\{64 - 7[-8](-1)\} &\quad \text{Multiply left to right} \\
2\{64 + 56(-1)\} &\quad \text{Finish multiplying inside the parentheses} \\
2\{64 - 56\} &\quad \text{Subtract inside parentheses} \\
2\{8\} &\quad \text{Multiply} \\
16 &\quad \text{Solution}
\end{aligned}$$

As Example 1.3.3 illustrates, it can take several steps to complete a problem. The key to successfully solving order of operations problems is to take the time to show your work and do one step at a time. This will reduce the chance of making a mistake along the way.

There are several types of grouping symbols that can be used besides parentheses, brackets, and braces. One such symbol is a fraction bar. The entire numerator and the entire denominator of a fraction must be evaluated before reducing. Once the fraction is reduced, the numerator and denominator can be simplified at the same time.

Example 1.3.4

Evaluate $\dfrac{2^4 - (-8)\cdot 3}{15 \div 5 - 1}$ using the order of operations.

$$\frac{2^4-(-8)\cdot 3}{15\div 5-1} \quad \text{Evaluate the exponent in the numerator and divide in the denominator}$$

$$\frac{16-(-8)\cdot 3}{3-1} \quad \text{Multiply in the numerator, subtract in the denominator}$$

$$\frac{16-(-24)}{2} \quad \text{Add in the numerator}$$

$$\frac{40}{2} \quad \text{Divide}$$

$$20 \quad \text{Solution}$$

Another type of grouping symbol is the absolute value. Everything inside a set of absolute value brackets must be evaluated, just as if it were a normal set of parentheses. Then, once the inside is completed, take the absolute value—or distance from zero—to make the number positive.

Example 1.3.5

Evaluate $1+3|-4^2-(-8)|+2|3+(-5)^2|$ using the order of operations.

$$1+3|-4^2-(-8)|+2|3+(-5)^2| \quad \text{Evaluate the exponents}$$

$$1+3|-16-(-8)|+2|3+25| \quad \text{Add inside the absolute values}$$

$$1+3|-8|+2|28| \quad \text{Evaluate absolute values}$$

$$1+3(8)+2(28) \quad \text{Multiply left to right}$$

$$1+24+56 \quad \text{Add left to right}$$

$$81 \quad \text{Solution}$$

Key Takeaways: Exponents

The above example also illustrates an important point about exponents:

- Exponents are only considered to be on the number they are attached to.
- This means that, in the expression -4^2, only the 4 is squared, giving us $-(4^2)$ or -16.
- But when the negative is in parentheses, such as in $(-5)^2$, the negative is part of the number and is also squared, giving a positive solution of 25.

Questions

For questions 1 to 24, reduce and solve the following expressions.

1. $-6 \cdot 4(-1)$
2. $(-6 \div 6)^3$
3. $3 + (8) \div |4|$
4. $5(-5+6) \cdot 6^2$
5. $8 \div 4 \cdot 2$
6. $7 - 5 + 6$
7. $[-9 - (2-5)] \div (-6)$
8. $(-2 \cdot 2^3 \cdot 2) \div (-4)$
9. $-6 + (-3-3)^2 \div |3|$
10. $(-7-5) \div [-2 - 2 - (-6)]$
11. $4 - 2|3^2 - 16|$
12. $(-10-6) \div (-2)^2 - 5$
13. $[-1 - (-5)]|3+2|$
14. $-3 - \{3 - [-3(2+4) - (-2)]\}$
15. $[2 + 4|7 + 2^2|] \div [4 \cdot 2 + 5 \cdot 3]$
16. $-4 - [2 + 4(-6) - 4 - 22 - 5 \cdot 2]$
17. $[6 \cdot 2 + 2 - (-6)](-5 + |(-18 \div 6)|)$
18. $2 \cdot (-3) + 3 - 6[-2 - (-1-3)]$
19. $\dfrac{-13-2}{2 - (-1)^3 + (-6) - [-1 - (-3)]}$
20. $\dfrac{5^2 + (-5)^2}{|4^2 - 2^5| - 2 \cdot 3}$
21. $\dfrac{6 \cdot -8 - 4 + (-4) - [-4 - (-3)]}{(4^2 + 3^2) \div 5}$
22. $\dfrac{-9 \cdot 2 - (3-6)}{1 - (-2+1) - (-3)}$
23. $\dfrac{2^3 + 4}{-18 - 6 + (-4) - [-5(-1)(-5)]}$

24. $$\frac{13+(-3)^2+4(-3)+1-[-10-(-6)]}{\{[4+5]\div[4^2-3^2(4-3)-8]\}+12}$$

Answer Key 1.3

4. 1.4 Properties of Algebra (Review)

When doing algebra, it is common not to know the value of the variables. In this case, simplify where possible and leave any unknown variables in the final solution. One way to simplify expressions is to combine like terms.

Like terms are terms whose variables match exactly, exponents included. Examples of like terms would be $3xy$ and $-7xy$, $3a^2b$ and $8a^2b$, or −3 and 5. To combine like terms, add (or subtract) the numbers in front of the variables and keep the variables the same.

Example 1.4.1

Simplify $5x-2y-8x+7y$.

$$\begin{array}{rl} 5x-8x \text{ and } -2y+7y & \text{Combine like terms} \\ \\ -3x+5y & \text{Solution} \end{array}$$

Example 1.4.2

Simplify $8x^2-3x+7-2x^2+4x-3$.

$$\begin{array}{rl} 8x^2-2x^2, -3x+4x, \text{ and } 7-3 & \text{Combine like terms} \\ \\ 6x^2+x+4 & \text{Solution} \end{array}$$

When combining like terms, subtraction signs must be interpreted as part of the terms they precede. This means that the term following a subtraction sign should be treated like a negative term. The sign always stays with the term.

Another method to simplify is known as distributing. Sometimes, when working with problems, there will be a set of parentheses that makes solving a problem difficult, if not impossible. To get rid of these unwanted parentheses, use the distributive property and multiply the number in front of the parentheses by each term inside.

$$\text{Distributive Property: } a(b+c)=ab+ac$$

Several examples of using the distributive property are given below.

Example 1.4.3

Simplify $4(2x-7)$.

$$\begin{array}{rl} 4(2x-7) & \text{Multiply each term by 4.} \\ 8x-28 & \text{Solution} \end{array}$$

Example 1.4.4

Simplify $-7(5x-6)$.

$$\begin{array}{rl} -7(5x-6) & \text{Multiply each term by } -7. \\ -35x+42 & \text{Solution} \end{array}$$

In the previous example, it is necessary to again use the fact that the sign goes with the number. This means −6 is treated as a negative number, which gives (−7)(−6) = 42, a positive number. The most common error in distributing is a sign error. Be very careful with signs! It is possible to distribute just a negative throughout parentheses. If there is a negative sign in front of parentheses, think of it like a −1 in front and distribute it throughout.

Example 1.4.5

Simplify $-(4x-5y+6)$.

$$\begin{aligned} -(4x-5y+6) &\quad \text{Negative can be thought of as } -1. \\ -1(4x-5y+6) &\quad \text{Multiply each term by } -1. \\ -4x+5y-6 &\quad \text{Solution} \end{aligned}$$

Distributing throughout parentheses and combining like terms can be combined into one problem. Order of operations says to multiply (distribute) first, then add or subtract (combine like terms). Thus, do each problem in two steps: distribute, then combine.

Example 1.4.6

Simplify $3x-2(4x-5)$.

$$\begin{aligned} 3x-2(4x-5) &\quad \text{Distribute } -2, \text{ multiplying each term.} \\ 3x-8x+10 &\quad \text{Combine like terms } 3x-8x. \\ -5x+10 &\quad \text{Solution} \end{aligned}$$

Example 1.4.7

Simplify $5+3(2x-4)$.

$$5 + 3(2x - 4) \quad \text{Distribute 3, multiplying each term.}$$

$$5 + 6x - 12 \quad \text{Combine like terms } 5 - 12.$$

$$-7 + 6x \quad \text{Solution}$$

In Example 1.4.6, −2 is distributed, not just 2. This is because a number being subtracted must always be treated like it has a negative sign attached to it. This makes a big difference, for in that example, when the −5 inside the parentheses is multiplied by −2, the result is a positive number. More involved examples of distributing and combining like terms follow.

Example 1.4.8

Simplify $2(5x - 8) - 6(4x + 3)$.

$$2(5x - 8) - 6(4x + 3) \quad \text{Distribute 2 into the first set of parentheses and } -6 \text{ into the second.}$$

$$10x - 16 - 24x - 18 \quad \text{Combine like terms } 10x - 24x \text{ and } -16 - 18.$$

$$-14x - 34 \quad \text{Solution}$$

Example 1.4.9

Simplify $4(3x - 8) - (2x - 7)$.

$$4(3x - 8) - (2x - 7) \quad \text{The negative sign in the middle can be thought of as } -1.$$

$$4(3x - 8) - (2x - 7) \quad \text{Distribute 4 into the first set of parentheses and } -1 \text{ into the second.}$$

$$12x - 32 - 2x + 7 \quad \text{Combine like terms } 12x - 2x \text{ and } -32 + 7.$$

$$10x - 25 \quad \text{Solution}$$

Questions

For questions 1 to 28, reduce and combine like terms.

1. $r - 9 + 10$
2. $-4x + 2 - 4$
3. $n + n$
4. $4b + 6 + 1 + 7b$
5. $8v + 7v$
6. $-x + 8x$
7. $-7x - 2x$
8. $-7a - 6 + 5$
9. $k - 2 + 7$
10. $-8p + 5p$
11. $x - 10 - 6x + 1$
12. $1 - 10n - 10$
13. $m - 2m$
14. $1 - r - 6$
15. $-8(x - 4)$
16. $3(8v + 9)$
17. $8n(n + 9)$
18. $-(-5 + 9a)$
19. $7k(-k + 6)$
20. $10x(1 + 2x)$
21. $-6(1 + 6x)$
22. $-2(n + 1)$
23. $8m(5 - m)$
24. $-2p(9p - 1)$
25. $-9x(4 - x)$
26. $4(8n - 2)$
27. $-9b(b - 10)$
28. $-4(1 + 7r)$

For questions 29 to 58, simplify each expression.

29. $9(b + 10) + 5b$
30. $4v - 7(1 - 8v)$
31. $-3x(1 - 4x) - 4x^2$
32. $-8x + 9(-9x + 9)$
33. $-4k^2 - 8k(8k + 1)$
34. $-9 - 10(1 + 9a)$
35. $1 - 7(5 + 7p)$
36. $-10(x - 2) - 3$
37. $-10 - 4(n - 5)$
38. $-6(5 - m) + 3m$
39. $4(x + 7) + 8(x + 4)$
40. $-2r(1 + 4r) + 8r(-r + 4)$

41. $-8(n+6)-8n(n+8)$
42. $9(6b+5)-4b(b+3)$
43. $7(7+3v)+10(3-10v)$
44. $-7(4x-6)+2(10x-10)$
45. $2n(-10n+5)-7(6-10n)$
46. $-3(4+a)+6a(9a+10)$
47. $5(1-6k)+10(k-8)$
48. $-7(4x+3)-10(10x+10)$
49. $(8n^2-3n)-(5+4n^2)$
50. $(7x^2-3)-(5x^2+6x)$
51. $(5p-6)+(1-p)$
52. $(3x^2-x)-(7-8x)$
53. $(2-4v^2)+(3v^2+2v)$
54. $(2b-8)+(b-7b^2)$
55. $(4-2k^2)+(8-2k^2)$
56. $(7a^2+7a)-(6a^2+4a)$
57. $(x^2-8)+(2x^2-7)$
58. $(3-7n^2)+(6n^2+3)$

Answer Key 1.4

5. 1.5 Terms and Definitions

Digits can be defined as the alphabet of the Hindu–Arabic numeral system that is in common usage today. This alphabet is: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Written in set-builder notation, digits are expressed as:

$$\text{Set of digits is } \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

Natural numbers are often called counting numbers and are usually denoted by $\mathbb{N}$. These numbers start at 1 and carry on to infinity, which is denoted by the symbol ∞. Writing the set of natural numbers in set-builder notation gives:

$$\text{Set of natural numbers } (\mathbb{N}) \text{ is } \{1, 2, 3, 4, 5, \ldots \infty\}$$

Whole numbers include the set of natural numbers and zero. Whole numbers are generally designated by $\mathbb{W}$. In set-builder notation, the set of whole numbers is denoted by:

$$\text{Set of whole numbers } (\mathbb{W}) \text{ is } \{0, 1, 2, 3, 4, 5, \ldots \infty\}$$

Integers include the set of all whole numbers and their negatives. This means the set of integers is composed of positive whole numbers, negative whole numbers, and zero (fractions and decimals are not integers). Common symbols used to represent integers are $\mathbb{Z}$ and $\mathbb{J}$. For this textbook, the symbol $\mathbb{Z}$ will be used to represent integers.

$$\text{Set of integers } (\mathbb{Z}) \text{ is } \{-\infty, \ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots \infty\}$$

Rational numbers include all integers and all fractions, terminating decimals, and repeating decimals. Every rational number can be written as a fraction $\frac{a}{b}$, where a and b are integers. Rational numbers are denoted by the symbol $\mathbb{Q}$. In set-builder notation, the set of rational numbers $\mathbb{Q}$ can be informally written as:

$$\text{Set of rational numbers } (\mathbb{Q}) \text{ is } \{\text{all numbers defined by } \frac{a}{b}, \text{ where } a \text{ and } b \text{ are integers}\}$$

Irrational numbers include any number that cannot be defined by the fraction $\frac{a}{b}$, where a and b are integers. These are numbers that are non-repeating or non-terminating. Classic examples of irrational numbers are pi (π) and the square roots of 2 and 3. The symbol for irrational numbers is commonly given as $\mathbb{I}$ or $\mathbb{H}$. For this textbook, the symbol $\mathbb{I}$ will be used. In set-builder notation, the set of irrational numbers $\mathbb{I}$ can be informally written as:

$$\text{Set of irrational numbers } (\mathbb{I}) \text{ is } \{\text{all non-repeating or non-terminal numbers}\}$$

Real numbers include the set of all rational numbers and irrational numbers. The symbol for real numbers is commonly given as $\mathbb{R}$. In set-builder notation, the set of real numbers $\mathbb{R}$ can be informally written as:

$$\text{Set of real numbers } (\mathbb{R}) \text{ is } \{\text{all rational and irrational numbers}\}$$

Numbers that may not yet have been encountered are **imaginary numbers** (commonly i, sometimes j) and **complex numbers** $(\mathbb{C})$. These numbers will be properly defined later in the textbook.

Imaginary numbers (i) include any real number multiplied by the square root of −1.

Complex numbers $(\mathbb{C})$ are combinations of any real number, imaginary number, or a sum and difference of them.

Consecutive integers are integers that follow each other sequentially. Examples are:

$$1, 2, 3, 4, \ldots$$
$$89, 90, 91, 92, \ldots$$
$$-45, -44, -43, -42, \ldots$$

Consecutive even or odd integers are numbers that skip the odd/even sequence to just show odd, odd, odd, or even, even, even. Examples are:

$$\begin{array}{lllll} \text{Consecutive odds:} & 1, 3, 5, 7, \ldots & \text{or} & -5, -3, -1, 1, \ldots \\ \text{Consecutive evens:} & 4, 6, 8, 10, \ldots & \text{or} & -4, -2, 0, 2, \ldots \end{array}$$

Prime numbers are numbers that cannot be divided by any integer other than 1 and itself. The following is a list of all the prime numbers that are found between 0 and 1000. (Note: 1 is not considered prime.)

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491, 499, 503, 509, 521, 523, 541, 547, 557, 563, 569, 571, 577, 587, 593, 599, 601, 607, 613, 617, 619, 631, 641, 643, 647, 653, 659, 661, 673, 677, 683, 691, 701, 709, 719, 727, 733, 739, 743, 751, 757, 761, 769, 773, 787, 797, 809, 811, 821, 823, 827, 829, 839, 853, 857, 859, 863, 877, 881, 883, 887, 907, 911, 919, 929, 937, 941, 947, 953, 967, 971, 977, 983, 991, 997$$

Squares are numbers multiplied by themselves. A number that is being squared is shown as having a superscript 2 attached to it. For example, 5 squared is written as 5^2, which equals 5 × 5 or 25.

Perfect squares are squares of whole numbers, such as 1, 4, 9, 16, 25, 36, and 49. They are found by squaring natural numbers. The following is the list of perfect squares using numbers up to 20:

$1^2 = 1$	$6^2 = 36$	$11^2 = 121$	$16^2 = 256$
$2^2 = 4$	$7^2 = 49$	$12^2 = 144$	$17^2 = 289$
$3^2 = 9$	$8^2 = 64$	$13^2 = 169$	$18^2 = 324$
$4^2 = 16$	$9^2 = 81$	$14^2 = 196$	$19^2 = 361$
$5^2 = 25$	$10^2 = 100$	$15^2 = 225$	$20^2 = 400$

Cubes are numbers multiplied by themselves three times. A number that is being cubed is shown as having a superscript 3 attached to it. For example, 5 cubed is written as 5^3, which equals 5 × 5 × 5 or 125.

Perfect cubes are cubes of whole numbers, such as 1, 8, 27, 64, 125, 216, and 343. They are found by cubing natural numbers. The following is the list of perfect cubes using numbers up to 20:

$1^3 = 1$	$6^3 = 216$	$11^3 = 1331$	$16^3 = 4096$
$2^3 = 8$	$7^3 = 343$	$12^3 = 1728$	$17^3 = 4913$
$3^3 = 27$	$8^3 = 512$	$13^3 = 2197$	$18^3 = 5832$
$4^3 = 64$	$9^3 = 729$	$14^3 = 2744$	$19^3 = 6859$
$5^3 = 125$	$10^3 = 1000$	$15^3 = 3375$	$20^3 = 8000$

Percentage means parts per hundred. A percentage can be thought of as a fraction $\frac{a}{b}$, where a, the numerator, is the number to the left of the % sign, and b, the denominator, is 100. For example: $42\% = \frac{42}{100} = 0.42$.

Absolute values. The absolute value of an expression a, denoted $|a|$, is the distance from zero of the number or operation that occurs between the absolute value signs. For example:

$$|-4| = 4 \text{ or } |-9| = 9$$

Examples of absolute values of simple operations are:

$$|-8+6| = 2 \quad \text{since} \quad -8+6 = -2 \quad \text{and} \quad |-2| = 2$$

$$\text{or}$$

$$|-8 \times 5| = 40 \quad \text{since} \quad -8 \times 5 = -40 \quad \text{and} \quad |-40| = 40$$

Set-builder notation follows standard patterns and is as follows:

- Begin the set with a left brace {
- A vertical bar | means "such that"
- End the set with a right brace }

So to say X is an integer, write this as:

$$\{X|X \text{ is an integer}\}$$

This means "the set of X, such that X is an integer."

Another way of writing this is to use the symbols that mean "element of" and "not an element of."

"Element of" is shown by the symbol ∈, and "not an element of" is shown by the element symbol with a line drawn through it, ∉.

In simplest terms, if something is an element of something else, it means that it belongs to or is part of it. For example, a set of numbers called A can only be made up of any natural number $(\mathbb{N})$, like 4, 6, 9, and 15. This can be stated as $\{A|A \in \mathbb{N}\}$, which reads as "the set of A, such that A is an element of the natural number system."

"Not an element of" can be used to state that the set cannot contain excluded values. For example, say there is a set C of all numbers $\mathbb{R}$ except counting numbers $\mathbb{N}$. This can be written as:

$$\{C | C \in \mathbb{R}, \text{ but } C \notin \mathbb{N}\}$$

This can be read as "the set of C, such that C is an element of the set of all real numbers, excluding those numbers that are natural numbers."

Sets of numbers giving excluded values can be seen throughout this textbook. The standard example is to exclude values that would result in a denominator of zero. This exclusion avoids division by zero and getting an undefinable answer.

The empty set. Sometimes, a set contains no elements. This set is termed the "empty set" or the "null set." To represent this, write either { } or Ø.

Names of Large Numbers

10^3	Thousand	10^{108}	Quinquatrigintillion
10^6	Million	10^{111}	Sestrigintillion
10^9	Billion	10^{114}	Septentrigintillion
10^{12}	Trillion	10^{117}	Octotrigintillion
10^{15}	Quadrillion	10^{120}	Noventrigintillion
10^{18}	Quintillion	10^{123}	Quadragintillion
10^{21}	Sextillion	10^{153}	Quinquagintillion
10^{24}	Septillion	10^{183}	Sexagintillion
10^{27}	Octillion	10^{213}	Septuagintillion
10^{30}	Nonillion	10^{243}	Octogintillion
10^{33}	Decillion	10^{273}	Nonagintillion
10^{36}	Undecillion	10^{303}	Centillion
10^{39}	Duodecillion	10^{306}	Uncentillion
10^{42}	Tredecillion	10^{309}	Duocentillion
10^{45}	Quattuordecillion	10^{312}	Trescentillion
10^{48}	Quinquadecillion	10^{333}	Decicentillion

10^{51}	Sedecillion	10^{336}	Undecicentillion
10^{54}	Septendecillion	10^{363}	Viginticentillion
10^{57}	Octodecillion	10^{366}	Unviginticentillion
10^{60}	Novendecillion	10^{393}	Trigintacentillion
10^{63}	Vigintillion	10^{423}	Quadragintacentillion
10^{66}	Unvigintillion	10^{453}	Quinquagintacentillion
10^{69}	Duovigintillion	10^{483}	Sexagintacentillion
10^{72}	Tresvigintillion	10^{513}	Septuagintacentillion
10^{75}	Quattuorvigintillion	10^{543}	Octogintacentillion
10^{78}	Quinquavigintillion	10^{573}	Nonagintacentillion
10^{81}	Sesvigintillion	10^{603}	Ducentillion
10^{84}	Septemvigintillion	10^{903}	Trecentillion
10^{87}	Octovigintillion	10^{1203}	Quadringentillion
10^{90}	Novemvigintillion	10^{1503}	Quingentillion
10^{93}	Trigintillion	10^{1803}	Sescentillion
10^{96}	Untrigintillion	10^{2103}	Septingentillion
10^{99}	Duotrigintillion	10^{2403}	Octingentillion
10^{102}	Trestrigintillion	10^{2703}	Nongentillion
10^{105}	Quattuortrigintillion	10^{3003}	Millinillion

Names of Small Numbers

10^{-3}	Thousandth	10^{-108}	Quinquatrigintillionth
10^{-6}	Millionth	10^{-111}	Sestrigintillionth
10^{-9}	Billionth	10^{-114}	Septentrigintillionth
10^{-12}	Trillionth	10^{-117}	Octotrigintillionth
10^{-15}	Quadrillionth	10^{-120}	Noventrigintillionth
10^{-21}	Sextillionth	10^{-123}	Quadragintillionth
10^{-24}	Septillionth	10^{-153}	Quinquagintillionth
10^{-27}	Octillionth	10^{-213}	Septuagintillionth
10^{-30}	Nonillionth	10^{-243}	Octogintillionth
10^{-33}	Decillionth	10^{-273}	Nonagintillionth
10^{-36}	Undecillionth	10^{-303}	Centillionth
10^{-39}	Duodecillionth	10^{-306}	Uncentillionth
10^{-42}	Tredecillionth	10^{-309}	Duocentillionth
10^{-45}	Quattuordecillionth	10^{-312}	Trescentillionth
10^{-48}	Quinquadecillionth	10^{-333}	Decicentillionth
10^{-51}	Sedecillionth	10^{-336}	Undecicentillionth

10^{-54}	Septendecillionth	10^{-363}	Viginticentillionth
10^{-57}	Octodecillionth	10^{-366}	Unviginticentillionth
10^{-60}	Novendecillionth	10^{-393}	Trigintacentillionth
10^{-63}	Vigintillionth	10^{-423}	Quadragintacentillionth
10^{-66}	Unvigintillionth	10^{-453}	Quinquagintacentillionth
10^{-69}	Duovigintillionth	10^{-483}	Sexagintacentillionth
10^{-72}	Tresvigintillionth	10^{-513}	Septuagintacentillionth
10^{-75}	Quattuorvigintillionth	10^{-543}	Octogintacentillionth
10^{-78}	Quinquavigintillionth	10^{-573}	Nonagintacentillionth
10^{-81}	Sesvigintillionth	10^{-603}	Ducentillionth
10^{-84}	Septemvigintillionth	10^{-903}	Trecentillionth
10^{-87}	Octovigintillionth	10^{-1203}	Quadringentillionth
10^{-90}	Novemvigintillionth	10^{-1503}	Quingentillionth
10^{-93}	Trigintillionth	10^{-1803}	Sescentillionth
10^{-96}	Untrigintillionth	10^{-2103}	Septingentillionth
10^{-99}	Duotrigintillionth	10^{-2403}	Octingentillionth
10^{-102}	Trestrigintillionth	10^{-2703}	Nongentillionth
10^{-105}	Quattuortrigintillionth	10^{-3003}	Millinillionth

6. 1.6 Unit Conversion Word Problems

One application of rational expressions deals with converting units. Units of measure can be converted by multiplying several fractions together in a process known as dimensional analysis.

The trick is to decide what fractions to multiply. If an expression is multiplied by 1, its value does not change. The number 1 can be written as a fraction in many different ways, so long as the numerator and denominator are identical in value. Note that the numerator and denominator need not be identical in appearance, but rather only identical in value. Below are several fractions, each equal to 1, where the numerator and the denominator are identical in value. This is why, when doing dimensional analysis, it is very important to use units in the setup of the problem, so as to ensure that the conversion factor is set up correctly.

Example 1.6.1

If 1 pound = 16 ounces, how many pounds are in 435 ounces?

$$\begin{aligned} 435\text{ oz} &= 435\text{ oz} \times \frac{1\text{ lb}}{16\text{ oz}} && \text{This operation cancels the oz and leaves the lbs} \\ &= \frac{435\text{ lb}}{16} && \text{Which reduces to} \\ &= 27\frac{3}{16}\text{ lb} && \text{Solution} \end{aligned}$$

The same process can be used to convert problems with several units in them. Consider the following example.

Example 1.6.2

A student averaged 45 miles per hour on a trip. What was the student's speed in feet per second?

$$\begin{aligned} 45\text{ mi/h} &= \frac{45\text{ mi}}{\text{hr}} \times \frac{5280\text{ ft}}{1\text{ mi}} \times \frac{1\text{ hr}}{3600\text{ s}} && \text{This will cancel the miles and hours} \\ &= 45 \times \frac{5280}{1} \times \frac{1}{3600}\text{ ft/s} && \text{This reduces to} \\ &= 66\text{ ft/s} && \text{Solution} \end{aligned}$$

Example 1.6.3

Convert 8 ft^3 to yd^3.

$$\begin{aligned} 8\text{ ft}^3 &= 8\text{ ft}^3 \times \frac{(1\text{ yd})^3}{(3\text{ ft})^3} && \text{Cube the parentheses} \\ &= 8\text{ ft}^3 \times \frac{1\text{ yd}^3}{27\text{ ft}^3} && \text{This will cancel the ft}^3\text{ and replace them with yd}^3 \\ &= 8 \times \frac{1\text{ yd}^3}{27} && \text{Which reduces to} \\ &= \frac{8}{27}\text{ yd}^3\text{ or }0.296\text{ yd}^3 && \text{Solution} \end{aligned}$$

Example 1.6.4

A room is 10 ft by 12 ft. How many square yards are in the room? The area of the room is 120 ft^2 (area = length × width).

Converting the area yields:

$$\begin{aligned} 120\text{ ft}^2 &= 120\text{ ft}^2 \times \frac{(1\text{ yd})^2}{(3\text{ ft})^2} && \text{Cancel ft}^2\text{ and replace with yd}^2 \\ &= \frac{120\text{ yd}^2}{9} && \text{This reduces to} \\ &= 13\frac{1}{3}\text{ yd}^2 && \text{Solution} \end{aligned}$$

The process of dimensional analysis can be used to convert other types of units as well. Once relationships that represent the same value have been identified, a conversion factor can be determined.

Example 1.6.5

A child is prescribed a dosage of 12 mg of a certain drug per day and is allowed to refill his prescription twice. If there are 60 tablets in a prescription, and each tablet has 4 mg, how many doses are in the 3 prescriptions (original + 2 refills)?

$$\begin{aligned} \text{3 prescriptions} &= \text{3 pres.} \times \frac{\text{60 tablets}}{\text{1 pres.}} \times \frac{\text{4 mg}}{\text{1 tablet}} \times \frac{\text{1 dosage}}{\text{12 mg}} && \text{This cancels all unwanted units} \\ &= \frac{3 \times 60 \times 4 \times 1}{1 \times 1 \times 12} \text{ or } \frac{720}{12} \text{ dosages} && \text{Which reduces to} \\ &= \text{60 daily dosages} && \text{Solution} \end{aligned}$$

Metric and Imperial (U.S.) Conversions

Distance

$$\begin{aligned} 12 \text{ in} &= 1 \text{ ft} \\ 3 \text{ ft} &= 1 \text{ yd} \\ 1760 \text{ yds} &= 1 \text{ mi} \\ 5280 \text{ ft} &= 1 \text{ mi} \end{aligned} \qquad \begin{aligned} 10 \text{ mm} &= 1 \text{ cm} \\ 100 \text{ cm} &= 1 \text{ m} \\ 1000 \text{ m} &= 1 \text{ km} \end{aligned}$$

Imperial to metric conversions:

$$\begin{aligned} 1 \text{ inch} &= 2.54 \text{ cm} \\ 1 \text{ ft} &= 0.3048 \text{ m} \\ 1 \text{ mile} &= 1.61 \text{ km} \end{aligned}$$

Area

$$\begin{aligned} 144 \text{ in}^2 &= 1 \text{ ft}^2 \\ 43,560 \text{ ft}^2 &= 1 \text{ acre} \\ 640 \text{ acres} &= 1 \text{ mi}^2 \end{aligned} \qquad \begin{aligned} 10,000 \text{ cm}^2 &= 1 \text{ m}^2 \\ 10,000 \text{ m}^2 &= 1 \text{ hectare} \\ 100 \text{ hectares} &= 1 \text{ km}^2 \end{aligned}$$

Imperial to metric conversions:

$$\begin{aligned} 1 \text{ in}^2 &= 6.45 \text{ cm}^2 \\ 1 \text{ ft}^2 &= 0.092903 \text{ m}^2 \\ 1 \text{ mi}^2 &= 2.59 \text{ km}^2 \end{aligned}$$

Volume

$$\begin{aligned} 57.75 \text{ in}^3 &= 1 \text{ qt} \\ 4 \text{ qt} &= 1 \text{ gal} \\ 42 \text{ gal (petroleum)} &= 1 \text{ barrel} \end{aligned} \qquad \begin{aligned} 1 \text{ cm}^3 &= 1 \text{ ml} \\ 1000 \text{ ml} &= 1 \text{ litre} \\ 1000 \text{ litres} &= 1 \text{ m}^3 \end{aligned}$$

Imperial to metric conversions:

$$\begin{aligned} 16.39 \text{ cm}^3 &= 1 \text{ in}^3 \\ 1 \text{ ft}^3 &= 0.0283168 \text{ m}^3 \\ 3.79 \text{ litres} &= 1 \text{ gal} \end{aligned}$$

Mass

$$\begin{aligned} 437.5 \text{ grains} &= 1 \text{ oz} \\ 16 \text{ oz} &= 1 \text{ lb} \\ 2000 \text{ lb} &= 1 \text{ short ton} \end{aligned} \qquad \begin{aligned} 1000 \text{ mg} &= 1 \text{ g} \\ 1000 \text{ g} &= 1 \text{ kg} \\ 1000 \text{ kg} &= 1 \text{ metric ton} \end{aligned}$$

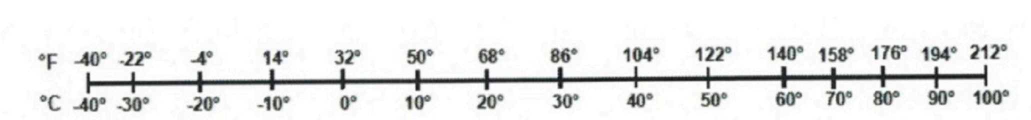
Imperial to metric conversions:

$$\begin{aligned} 453 \text{ g} &= 1 \text{ lb} \\ 2.2 \text{ lb} &= 1 \text{ kg} \end{aligned}$$

Temperature

Fahrenheit to Celsius conversions:

$$\begin{aligned} {}^\circ\text{C} &= \frac{5}{9}({}^\circ\text{F} - 32) \\ {}^\circ\text{F} &= \frac{9}{5}({}^\circ\text{C} + 32) \end{aligned}$$



Celsius to Fahrenheit conversion scale. [Long Description]

Questions

For questions 1 to 18, use dimensional analysis to perform the indicated conversions.

1. 7 miles to yards
2. 234 oz to tons
3. 11.2 mg to grams
4. 1.35 km to centimetres
5. 9,800,000 mm to miles
6. 4.5 ft^2 to square yards
7. 435,000 m^2 to square kilometres
8. 8 km^2 to square feet
9. 0.0065 km^3 to cubic metres
10. 14.62 in^3 to square centimetres
11. 5500 cm^3 to cubic yards
12. 3.5 mph (miles per hour) to feet per second
13. 185 yd per min. to miles per hour
14. 153 ft/s (feet per second) to miles per hour
15. 248 mph to metres per second
16. 186,000 mph to kilometres per year
17. 7.50 tons/yd^2 to pounds per square inch
18. 16 ft/s^2 to kilometres per hour squared

For questions 19 to 27, solve each conversion word problem.

19. On a recent trip, Jan travelled 260 miles using 8 gallons of gas. What was the car's miles per gallon for this trip? Kilometres per litre?
20. A certain laser printer can print 12 pages per minute. Determine this printer's output in pages per day.
21. An average human heart beats 60 times per minute. If the average person lives to the age of 86, how many times does the average heart beat in a lifetime?
22. Blood sugar levels are measured in milligrams of glucose per decilitre of blood volume. If a person's blood sugar level measured 128 mg/dL, what is this in grams per litre?
23. You are buying carpet to cover a room that measures 38 ft by 40 ft. The carpet cost $18 per square yard. How much will the carpet cost?
24. A cargo container is 50 ft long, 10 ft wide, and 8 ft tall. Find its volume in cubic yards and cubic metres.
25. A local zoning ordinance says that a house's "footprint" (area of its ground floor) cannot occupy more than ¼ of the lot it is built on. Suppose you own a $\frac{1}{3}$-acre lot (1 acre = 43,560 ft^2). What is the maximum allowed footprint for your house in square feet? In square metres?
26. A car travels 23 km in 15 minutes. How fast is it going in kilometres per hour? In metres per second?
27. The largest single rough diamond ever found, the Cullinan Diamond, weighed 3106 carats. One carat is equivalent to the mass of 0.20 grams. What is the mass of this diamond in milligrams? Weight in pounds?

Answer Key 1.6

Long Descriptions

Celsius to Fahrenheit conversion scale long description: Scale showing conversions between Celsius and Fahrenheit. The following table summarizes the data:

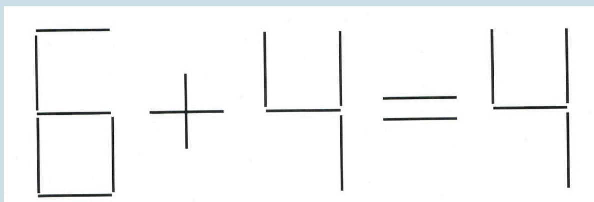
Celsius	Fahrenheit
−40°C	−40°F
−30°C	−22°F
−20°C	−4°F
−10°C	14°F
0°C	32°F
10°C	50°F
20°C	68°F
30°C	86°F
40°C	104°F
50°C	122°F
60°C	140°F
70°C	158°F
80°C	176°F
90°C	194°F
100°C	212°F

[Return to Celsius to Fahrenheit conversion scale]

7. 1.7 Puzzles for Homework

Exercise 1.7.1

There are four known solutions to the following math puzzle, in which you can move just one line to fix the equation. How many solutions can you find?



Exercise 1.7.2

Are the following statements true?

- Letters A, B, C and D do not appear anywhere in the spellings of 1 to 99
- Letter D appears for the first time in “hundred”
- Letters A, B and C do not appear anywhere in the spellings of 1 to 999
- Letter A appears for the first time in “thousand”
- Letters B and C do not appear anywhere in the spellings of 1 to 999,999,999
- Letter B appears for the first time in “billion”
- Letter C does not appear anywhere in any word used to count in English

PART II

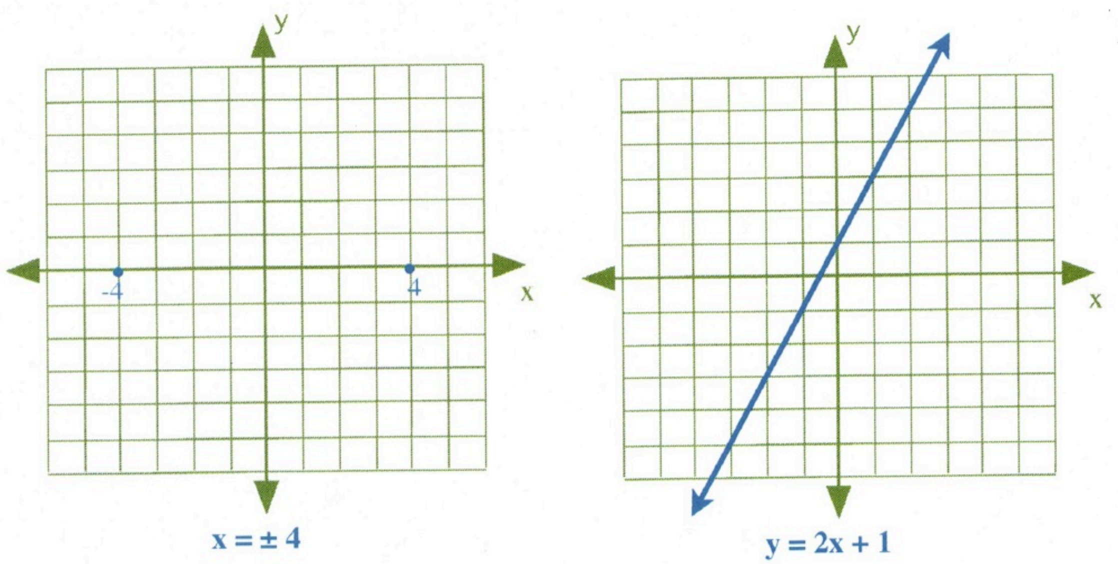
CHAPTER 2: LINEAR EQUATIONS

The study of linear equations is the study of the foundations of algebra that lead into multiple applications and more advanced mathematics, physics, and engineering fields. Linear equations are used quite frequently in these fields in part because the solutions to complex, non-linear systems can often be well approximated using a linear equation.

Linear equations are equations that can define multiple dimensions. Consider, for instance, the following two equations:

(i) $x = \pm 4$ and (ii) $y = 2x + 1$

Here they are in graphed form:



Linear equations can also be shown in three dimensions, but that would require time-spaced snapshots to show how a three-dimensional line would change if the fourth dimension of time were to be added.

Fundamental to all linear equations is that the variables being worked with have no powers attached to them. This means that a four-dimensional space-time linear equation could look like $1000 = x + 2y + 4z + 10t$, but it could not carry any powers, like in the equation $1000 = x^2 + 2y^3 + 4z^2 + 10t^4$. Remember:

Linear equations have no powers on any variables being used.

8. 2.1 Elementary Linear Equations

Solving linear equations is an important and fundamental skill in algebra. In algebra, there are often problems in which the answer is known, but the variable part of the problem is missing. To find this missing variable, it is necessary to follow a series of steps that result in the variable equalling some solution.

Addition and Subtraction Problems

To solve equations, the general rule is to do the opposite of the order of operations. Consider the following.

Example 2.1.1

Solve for x.

1. $x - 7 = 5$

$$\begin{array}{rrrrr} x & - & 7 & = & -5 \\ & + & 7 & & +7 \\ & & x & = & 2 \end{array}$$

2. $4 + x = 8$

$$\begin{array}{rrrrr} 4 & + & x & = & 8 \\ -4 & & & & -4 \\ & & x & = & 4 \end{array}$$

3. $7 = x - 9$

$$\begin{array}{rrrrr} 7 & = & x & - & 9 \\ +9 & & & + & 9 \\ 16 & = & x & & \end{array}$$

4. $5 = 8 + x$

$$\begin{array}{rrrrr} 5 & = & 8 & + & x \\ -8 & & -8 & & \\ -3 & = & x & & \end{array}$$

Multiplication Problems

In a multiplication problem, get rid of the coefficient in front of the variable by dividing both sides of the equation by that number. Consider the following examples.

Example 2.1.2

Solve for x.

1. $$\begin{aligned} 4x &= 20 \\ \frac{4x}{4} &= \frac{20}{4} \\ x &= 5 \end{aligned}$$
2. $$\begin{aligned} 8x &= -24 \\ \frac{8x}{8} &= \frac{-24}{8} \\ x &= -3 \end{aligned}$$
3. $$\begin{aligned} -4x &= -20 \\ \frac{-4x}{-4} &= \frac{-20}{-4} \\ x &= 5 \end{aligned}$$

Division Problems

In division problems, remove the denominator by multiplying both sides by it. Consider the following examples.

Example 2.1.3

Solve for x.

1.
$$\begin{aligned}\frac{x}{-7} &= -2\\ -7\left(\frac{x}{-7}\right) &= (-2)-7\\ x &= 14\end{aligned}$$

2.
$$\begin{aligned}\frac{x}{8} &= 5\\ 8\left(\frac{x}{8}\right) &= (5)8\\ x &= 40\end{aligned}$$

3.
$$\begin{aligned}\frac{x}{-4} &= 9\\ -4\left(\frac{x}{-4}\right) &= (9)-4\\ x &= -36\end{aligned}$$

Questions

For questions 1 to 28, solve each linear equation.

1. $v+9=16$
2. $14=b+3$
3. $x-11=-16$
4. $-14=x-18$
5. $30=a+20$
6. $-1+k=5$
7. $x-7=-26$
8. $-13+p=-19$
9. $13=n-5$
10. $22=16+m$
11. $340=-17x$

12. $4r = -28$
13. $-9 = \frac{n}{12}$
14. $27 = 9b$
15. $20v = -160$
16. $-20x = -80$
17. $340 = 20n$
18. $12 = 8a$
19. $16x = 320$
20. $8k = -16$
21. $-16 + n = -13$
22. $-21 = x - 5$
23. $p - 8 = -21$
24. $m - 4 = -13$
25. $\frac{r}{14} = \frac{5}{14}$
26. $\frac{n}{8} = 40$
27. $20b = -200$
28. $-\frac{1}{3} = \frac{x}{12}$

Answer Key 2.1

Extra Reading and Instructional Videos

Article to read: New theory finds 'traffic jams' in jet stream cause abnormal weather patterns.

The abstract reads:

> A study offers an explanation for a mysterious and sometimes deadly weather pattern in which the jet stream, the global air currents that circle the Earth, stalls out over a region. Much like highways, the jet stream has a capacity, researchers said, and when it's exceeded, blockages form that are remarkably similar to traffic jams — and climate forecasters can use the same math to model them both.

9. 2.2 Solving Linear Equations

When working with questions that require two or more steps to solve, do the reverse of the order of operations to solve for the variable.

Example 2.2.1

Solve $4x + 16 = -4$ for x.

$$\begin{array}{rrrrrl} 4x & + & 16 & = & -4 & \\ & & -16 & & -16 & \text{Subtract 16 from each side} \\ & & \dfrac{4x}{4} & = & \dfrac{-20}{4} & \text{Divide each side by 4} \\ \\ & & x & = & -5 & \text{Solution} \end{array}$$

In solving the above equation, notice that the general pattern followed was to do the opposite of the equation. $4x$ was added to 16, so 16 was then subtracted from both sides. The variable x was multiplied by 4, so both sides were divided by 4.

Example 2.2.2

1. $$\begin{array}{rrrrr} 5x & + & 7 & = & 7 \\ & - & 7 & & -7 \\ & & \dfrac{5x}{5} & = & \dfrac{0}{5} \\ \\ & & x & = & 0 \end{array}$$
2. $$\begin{array}{rrrrr} 4 & - & 2x & = & 10 \\ -4 & & & & -4 \\ & & \dfrac{-2x}{-2} & = & \dfrac{6}{-2} \\ \\ & & x & = & -3 \end{array}$$

3.
$$\begin{array}{rrrrr} -3x & - & 7 & = & 8 \\ & + & 7 & = & +7 \\ & & \dfrac{-3x}{-3} & = & \dfrac{15}{-3} \\ \\ & & x & = & -5 \end{array}$$

Questions

For questions 1 to 20, solve each linear equation.

1. $5+\dfrac{n}{4}=4$
2. $-2=-2m+12$
3. $102=-7r+4$
4. $27=21-3x$
5. $-8n+3=-77$
6. $-4-b=8$
7. $0=-6v$
8. $-2+\dfrac{x}{2}=4$
9. $-8=\dfrac{x}{5}-6$
10. $-5=\dfrac{a}{4}-1$
11. $0=-7+\dfrac{k}{2}$
12. $-6=15+3p$
13. $-12+3x=0$
14. $-5m+2=27$
15. $\dfrac{b}{3}+7=10$
16. $\dfrac{x}{1}-8=-8$
17. $152=8n+64$
18. $-11=-8+\dfrac{v}{2}$
19. $-16=8a+64$
20. $-2x-3=-29$

Answer Key 2.2

10. 2.3 Intermediate Linear Equations

When working with linear equations with parentheses, the first objective is to isolate the parentheses. Once isolated, the parentheses can be removed and then the variable solved.

Example 2.3.1

Solve for x in the equation $4(2x-6)=16$.

$$\begin{array}{rcrl}
\dfrac{4(2x-6)}{4}&=&\dfrac{16}{4}&\text{Divide both sides by 4}\\ \\
(2x-6)&=&4&\text{Remove the parentheses}\\
2x-6&=&4&\text{Add 6 to both sides to remove }-6\\
+6&&+6&\\
\dfrac{2x}{2}&=&\dfrac{10}{2}&\text{Divide both sides by 2}\\ \\
x&=&5&\text{Solution}
\end{array}$$

Example 2.3.2

Solve for x in the equation $3(2x-4)+9=15$.

$$\begin{array}{rcrl}
3(2x-4)+9&=&15&\text{Subtract 9 from both sides}\\
-9&&-9&\\
\dfrac{3(2x-4)}{3}&=&\dfrac{6}{3}&\text{Divide both sides by 3 and remove parentheses}\\ \\
2x-4&=&2&\text{Add 4 to both sides}\\
+4&&+4&\\
\dfrac{2x}{2}&=&\dfrac{6}{2}&\text{Divide both sides by 2}\\ \\
x&=&3&\text{Solution}
\end{array}$$

For some problems, it is too difficult to isolate the parentheses. In these problems, it is necessary to multiply or divide throughout the parentheses by whatever coefficient is in front of it.

Example 2.3.3

Solve for x in the equation $3(4x-5)-4(2x+1)=5.$

$$\begin{array}{rcrl} 3(4x-5)-4(2x+1) & = & 5 & \text{Distribute} \\ 12x-15-8x-4 & = & 5 & \text{Combine similar terms} \\ 4x-19 & = & 5 & \text{Add 19 to both sides} \\ +19 & & +19 & \\ \dfrac{4x}{4} & = & \dfrac{24}{4} & \text{Divide both sides by 4} \\ & & & \\ x & = & 6 & \end{array}$$

Questions

For questions 1 to 26, solve each linear equation.

1. $2-(-3a-8)=1$
2. $2(-3n+8)=-20$
3. $-5(-4+2v)=-50$
4. $2-8(-4+3x)=34$
5. $66=6(6+5x)$
6. $32=2-5(-4n+6)$
7. $-2+2(8x-9)=-16$
8. $-(3-5n)=12$
9. $-1-7m=-8m+7$
10. $56p-48=6p+2$
11. $1-12r=29-8r$
12. $4+3x=-12x+4$
13. $20-7b=-12b+30$
14. $-16n+12=39-7n$
15. $-2-5(2-4m)=33+5m$
16. $-25-7x=6(2x-1)$
17. $-4n+11=2(1-8n)+3n$
18. $-7(1+b)=-5-5b$
19. $-6v-29=-4v-5(v+1)$
20. $-8(8r-2)=3r+16$
21. $2(4x-4)=-20-4x$

22. $-8n - 19 = -2(8n - 3) + 3n$
23. $-2(m - 2) + 7(m - 8) = -67$
24. $7 = 4(n - 7) + 5(7n + 7)$
25. $50 = 8(7 + 7r) - (4r + 6)$
26. $-8(6 + 6x) + 4(-3 + 6x) = -12$

Answer Key 2.3

11. 2.4 Fractional Linear Equations

When working with fractions built into linear equations, it is often easiest to remove the fraction in the very first step. This generally means finding the LCD of the fraction and then multiplying every term in the entire equation by the LCD.

Example 2.4.1

Solve for x in the equation $\dfrac{3}{4}x-\dfrac{7}{2}=\dfrac{5}{6}$.

For this equation, the LCD is 12, so every term in this equation will be multiplied by 12.

$$\frac{3}{4}x(12)-\frac{7}{2}(12)=\frac{5}{6}(12)$$

Cancelling out the denominator yields:

$$3x(3)-7(6)=5(2)$$

Multiplying results in:

$$\begin{array}{rrrrr} 9x & - & 42 & = & 10 \\ & + & 42 & & +42 \\ & & \dfrac{9x}{9} & = & \dfrac{52}{9} \\ \\ & & x & = & \dfrac{52}{9} \end{array}$$

Example 2.4.2

Solve for x in the equation $\dfrac{3\left(\dfrac{5}{9}x+\dfrac{4}{27}\right)}{2}=3$.

First, remove the outside denominator 2 by multiplying both sides by 2:

$$(2)\,\frac{3\left(\frac{5}{9}x+\frac{4}{27}\right)}{2}=3(2)$$

$$3\left(\frac{5}{9}x+\frac{4}{27}\right)=6$$

Now divide both sides by 3, which leaves:

$$\frac{5}{9}x+\frac{4}{27}=2$$

To remove the 9 and 27, multiply both sides by the LCD, 27:

$$\frac{5}{9}x\,(27)+\frac{4}{27}\,(27)=2(27)$$

This leaves:

$$\begin{array}{rrrcl} 5x(3) & + & 4 & = & 54 \\ & - & 4 & & -4 \\ & & 15x & = & 50 \\ \\ & & x & = & \dfrac{50}{15} \text{ or } \dfrac{10}{3} \end{array}$$

Questions

For questions 1 to 18, solve each linear equation.

1. $\dfrac{3}{5}(1+p)=\dfrac{21}{20}$
2. $-\dfrac{1}{2}=\dfrac{3k}{2}+\dfrac{3}{2}$
3. $0=-\dfrac{5}{4}\left(x-\dfrac{6}{5}\right)$
4. $\dfrac{3}{2}n-8=-\dfrac{29}{12}$
5. $\dfrac{3}{4}-\dfrac{5}{4}m=\dfrac{108}{24}$
6. $\dfrac{11}{4}+\dfrac{3}{4}r=\dfrac{160}{32}$
7. $2b+\dfrac{9}{5}=-\dfrac{11}{5}$
8. $\dfrac{3}{2}-\dfrac{7}{4}v=-\dfrac{9}{8}$
9. $\dfrac{3}{2}\left(\dfrac{7}{3}n+1\right)=\dfrac{3}{2}$
10. $\dfrac{41}{9}=\dfrac{5}{2}\left(x+\dfrac{2}{3}\right)-\dfrac{1}{3}x$
11. $-a-\dfrac{5}{4}\left(-\dfrac{8}{3}a+1\right)=-\dfrac{19}{4}$
12. $\dfrac{1}{3}\left(-\dfrac{7}{4}k+1\right)-\dfrac{10}{3}k=-\dfrac{13}{8}$
13. $\dfrac{55}{6}=-\dfrac{5}{2}\left(\dfrac{3}{2}p-\dfrac{5}{3}\right)$

14. $-\frac{1}{2}\left(\frac{2}{3}x-\frac{3}{4}\right)-\frac{7}{2}x=-\frac{83}{24}$

15. $-\frac{5}{8}=\frac{5}{4}\left(r-\frac{3}{2}\right)$

16. $\frac{1}{12}=\frac{4}{3}x+\frac{5}{3}\left(x-\frac{7}{4}\right)$

17. $-\frac{11}{3}+\frac{3}{2}b=\frac{5}{2}\left(b-\frac{5}{3}\right)$

18. $\frac{7}{6}-\frac{4}{3}n=-\frac{3}{2}n+2\left(n+\frac{3}{2}\right)$

Answer Key 2.4

12. 2.5 Absolute Value Equations

When solving equations with absolute values, there can be more than one possible answer. This is because the variable whose absolute value is being taken can be either negative or positive, and both possibilities must be accounted for when solving equations.

Example 2.5.1

Solve $|x| = 7$.

$$x = +7 \text{ and } -7, \text{ or } \pm 7$$

When there are absolute values in a problem, it is important to first isolate the absolute value, then remove the absolute value by considering both the positive and negative solutions. Notice that, in the next two examples, all the numbers outside of the absolute value are moved to one side first before the absolute value bars are removed and both positive and negative solutions are considered.

Example 2.5.2

Solve $5 + |x| = 8$.

$$\begin{array}{rcrcr} 5 & + & |x| & = & 8 \\ -5 & & & & -5 \\ & & |x| & = & 3 \\ & & x & = & \pm 3 \end{array}$$

Example 2.5.3

Solve $-4|x| = -20.$

$$\begin{array}{rcl}\dfrac{-4}{-4}|x| &=& \dfrac{-20}{-4} \\ \\ |x| &=& 5 \\ x &=& \pm 5\end{array}$$

Note: the objective in solving for absolute values is to isolate the absolute value to yield a solution.

Example 2.5.4

Solve $5|x| - 4 = 26.$

$$\begin{array}{rcrcr} 5|x| & - & 4 & = & 26 \\ & + & 4 & & +4 \\ & & \dfrac{5}{5}|x| & = & \dfrac{30}{5} \\ \\ & & |x| & = & 6 \\ & & x & = & \pm 6 \end{array}$$

Often, the absolute value of more than just a variable is being taken. In these cases, it is necessary to solve the resulting equations before considering positive and negative possibilities. This is shown in the next example.

Example 2.5.5

Solve $|2x-1|=7$.

Since absolute value can be positive or negative, this means that there are two equations to solve.

$$\begin{array}{rrrrr} 2x & - & 1 & = & 7 \\ & + & 1 & & +1 \\ & & \dfrac{2x}{2} & = & \dfrac{8}{2} \\ \\ & & x & = & 4 \end{array} \qquad \text{and} \qquad \begin{array}{rrrrr} 2x & - & 1 & = & -7 \\ & + & 1 & & +1 \\ & & \dfrac{2x}{2} & = & \dfrac{-6}{2} \\ \\ & & x & = & -3 \end{array}$$

Remember: the absolute value must be isolated first before solving.

Example 2.5.6

Solve $2-4|2x+3|=-18$.

$$\begin{array}{rrrrrrr} 2 & - & 4|2x & + & 3| & = & -18 \\ -2 & & & & & & -2 \\ & & \dfrac{-4}{-4}|2x & + & 3| & = & \dfrac{-20}{-4} \\ \\ & & |2x & + & 3| & = & 5 \end{array}$$

Now, solve two equations to get the positive and negative solutions:

$$\begin{array}{rcrcr} 2x & + & 3 & = & 5 \\ & - & 3 & & -3 \\ & & \dfrac{2x}{2} & = & \dfrac{2}{2} \\ & & x & = & 1 \end{array} \quad \text{and} \quad \begin{array}{rcrcr} 2x & + & 3 & = & -5 \\ & - & 3 & & -3 \\ & & \dfrac{2x}{2} & = & \dfrac{-8}{2} \\ & & x & = & -4 \end{array}$$

There exist two other possible results from solving an absolute value besides what has been shown in the above six examples.

Example 2.5.7

Consider the equation $|2x-1|=7$ from Example 2.5.5.

What happens if, instead, the equation to solve is $|2x-1|=0$ or $|2x-1|=-5$?

For $|2x-1|=0$, there is no ± 0, so there will be just one solution instead of two. Solving this equation yields:

$$\begin{array}{rcrcr} 2x & - & 1 & = & 0 \\ & + & 1 & & +1 \\ & & \dfrac{2x}{2} & = & \dfrac{1}{2} \\ & & x & = & \dfrac{1}{2} \end{array}$$

For $|2x-1|=-5$, the result will be “no solution” or $\emptyset$, since an absolute value is never negative.

One final type of absolute value problem covered in this chapter is when two absolute values are equal to each other. There will still be both a positive and a negative result—the difference is that a negative must be distributed into the second absolute value for the negative possibility.

Example 2.5.8

Solve $|2x-7|=|4x+6|$.

Solving this means solving:

$$|2x-7|=|4x+6| \text{ and } |2x-7|=-|4x+6|$$

Removing the absolute value signs leaves:

$$\begin{array}{rrrcrrr} 2x & - & 7 & = & 4x & + & 6 \\ -4x & + & 7 & & -4x & + & 7 \\ & & \dfrac{-2x}{-2} & = & \dfrac{13}{-2} & & \\ & & x & = & -\dfrac{13}{2} & & \end{array} \quad \text{and} \quad \begin{array}{rrrcrrr} 2x & - & 7 & = & -4x & - & 6 \\ +4x & + & 7 & & +4x & + & 7 \\ & & \dfrac{6x}{6} & = & \dfrac{1}{6} & & \\ & & x & = & \dfrac{1}{6} & & \end{array}$$

Questions

For questions 1 to 24, solve each absolute value equation.

1. $|x|=8$
2. $|n|=7$
3. $|b|=1$
4. $|x|=2$
5. $|5+8a|=53$
6. $|9n+8|=46$
7. $|3k+8|=2$
8. $|3-x|=6$
9. $-7|-3-3r|=-21$

10. $|2+2b|+1=3$
11. $7|-7x-3|=21$
12. $|-4-3n|=2$
13. $8|5p+8|-5=11$
14. $3-|6n+7|=-40$
15. $5|3+7m|+1=51$
16. $4|r+7|+3=59$
17. $-7+8|-7x-3|=73$
18. $8|3-3n|-5=91$
19. $|5x+3|=|2x-1|$
20. $|2+3x|=|4-2x|$
21. $|3x-4|=|2x+3|$
22. $|2x-5|=|3x+4|$
23. $|4x-2|=|6x+3|$
24. $|3x+2|=|2x-3|$

Answer Key 2.5

13. 2.6 Working With Formulas

In algebra, expressions often need to be simplified to make them easier to use. There are three basic forms of simplifying, which will be reviewed here. The first form of simplifying expressions is used when the value of each variable in an expression is known. In this case, each variable can be replaced with the equivalent number, and the rest of the expression can be simplified using the order of operations.

Example 2.6.1

Evaluate $p(q+6)$ when $p=3$ and $q=5$.

$$\begin{array}{rl}(3)((5)+(6)) & \text{Replace } p \text{ with 3 and } q \text{ with 5 and evaluate parentheses} \\ (3)(11) & \text{Multiply} \\ 33 & \text{Solution}\end{array}$$

Whenever a variable is replaced with something, the new number is written inside a set of parentheses. Notice the values of 3 and 5 in the previous example are in parentheses. This is to preserve operations that are sometimes lost in a simple substitution. Sometimes, the parentheses won't make a difference, but it is a good habit to always use them to prevent problems later.

Example 2.6.2

Evaluate $x+zx(3-z)\left(\dfrac{x}{3}\right)$ when $x=-6$ and $z=-2$.

$$
\begin{array}{rl}
(-6)+(-2)(-6)\left[(3)-(-2)\right]\left(\dfrac{-6}{3}\right) & \text{Evaluate parentheses} \\
-6+(-2)(-6)(5)(-2) & \text{Multiply left to right} \\
-6+12(5)(-2) & \text{Multiply left to right} \\
-6+60(-2) & \text{Multiply} \\
-6-120 & \text{Subtract} \\
-126 & \text{Solution}
\end{array}
$$

Isolating variables in formulas is similar to solving general linear equations. The only difference is, with a formula, there will be several variables in the problem, and the goal is to solve for one specific variable. For example, consider solving a formula such as $A=\pi r^2+\pi rs$ (the formula for the surface area of a right circular cone) for the variable s. This means isolating the s so the equation has s on one side. So a solution might look like $s=\dfrac{A-\pi r^2}{\pi r}$. This second equation gives the same information as the first; they are algebraically equivalent. However, one is solved for the area A, while the other is solved for the slant height of the cone s.

When solving a formula for a variable, focus on the one variable that is being solved for; all the others are treated just like numbers. This is shown in the following example. Two parallel problems are shown: the first is a normal one-step equation, and the second is a formula that you are solving for x.

Example 2.6.3

Isolate the variable x in the following equations.

$$
\begin{aligned}
3x &= 12 \\
\frac{3x}{3} &= \frac{12}{3} \\
x &= 4
\end{aligned}
\qquad\qquad
\begin{aligned}
wx &= z \\
\frac{wx}{w} &= \frac{z}{w} \\
x &= \frac{z}{w}
\end{aligned}
$$

The same process is used to isolate x in $3x = 12$ as in $wx = z$. Because x is being solved for, treat all other variables as numbers. For these two equations, both sides were divided by 3 and w, respectively. A similar idea is seen in the following example.

Example 2.6.4

Isolate the variable n in the equation $m + n = p$.

To isolate n, the variable m must be removed, which is done by subtracting m from both sides:

$$\begin{array}{rcrcl} m & + & n & = & p \\ -m & & & & -m \\ & & n & = & p - m \end{array}$$

Since p and m are not like terms, they cannot be combined. For this reason, leave the expression as $p - m$.

Example 2.6.5

Isolate the variable a in the equation $a(x - y) = b$.

This means that $(x - y)$ must be isolated from the variable a.

$$\frac{a(x-y)}{(x-y)} = \frac{b}{(x-y)} \qquad \Rightarrow \qquad a = \frac{b}{(x-y)}$$

If no individual term inside parentheses is being solved for, keep the terms inside them together and divide by them as a unit. However, if an individual term inside parentheses is being solved for, it is necessary to distribute. The following example is the same formula as in Example 2.6.5, but this time, x is being solved for.

Example 2.6.6

Isolate the variable x in the equation $a(x-y)=b$.

First, distribute a throughout $(x-y)$:

$$\begin{array}{rcrcl} a(x & - & y) & = & b \\ ax & - & ay & = & b \end{array}$$

Remove the term ay from both sides:

$$\begin{array}{rcrcr} ax & - & ay & = & b \\ & + & ay & & +ay \\ & & ax & = & b+ay \end{array}$$

ax is then divided by a:

$$\frac{ax}{a}=\frac{b+ay}{a}$$

The solution is $x=\dfrac{b+ay}{a}$, which can also be shown as $x=\dfrac{b}{a}+y$.

Be very careful when isolating x not to try and cancel the a on the top and the bottom of the fraction. This is not allowed if there is any adding or subtracting in the fraction. There is no reducing possible in this problem, so the final reduced answer remains $x=\dfrac{b+ay}{a}$. The next example is another two-step problem.

Example 2.6.7

Isolate the variable m in the equation $y = mx + b$.

First, subtract b from both sides:

$$\begin{array}{rcrcl} y & = & mx & + & b \\ -\,b & & & - & b \\ y - b & = & mx & & \end{array}$$

Now divide both sides by x:

$$\frac{y-b}{x} = \frac{mx}{x}$$

Therefore, the solution is $m = \dfrac{y-b}{x}$.

It is important to note that a problem is complete when the variable being solved for is isolated or alone on one side of the equation and it does not appear anywhere on the other side of the equation.

The next example is also a two-step equation. It is a problem from earlier in the lesson.

Example 2.6.8

Isolate the variable s in the equation $A = \pi r^2 + \pi rs$.

Subtract πr^2 from both sides:

$$\begin{array}{rcrcl} A & = & \pi r^2 & + & \pi r s \\ -\pi r^2 & & -\pi r^2 & & \\ A - \pi r^2 & = & \pi r s & & \end{array}$$

Divide both sides by πr:

$$\frac{A - \pi r^2}{\pi r} = \frac{\pi r s}{\pi r}$$

The solution is:

$$s = \frac{A - \pi r^2}{\pi r}$$

Formulas often have fractions in them and can be solved in much the same way as any fraction. First, identify the LCD, and then multiply each term by the LCD. After reducing, there will be no more fractions in the problem.

Example 2.6.9

Isolate the variable m in the equation $h = \dfrac{2m}{n}$.

To clear the fraction, multiply both sides by n:

$$(n)h = \frac{2m}{n}(n)$$

This leaves:

$$nh = 2m$$

Divide both sides by 2:

$$\frac{nh}{2} = \frac{2m}{2}$$

Which reduces to:

$$m = \frac{nh}{2}$$

Example 2.6.10

Isolate the variable b in the equation $A = \dfrac{a}{2-b}$.

To clear the fraction, multiply both sides by $(2-b)$:

$$(2-b)A = \frac{a}{2-b}(2-b)$$

Which reduces to:

$$A(2-b)=a$$

Distribute A throughout $(2-b)$, then isolate:

$$\begin{array}{rrrrl} 2A & - & Ab & = & a \\ -2A & & & & -2A \\ & & -Ab & = & a-2A \end{array}$$

Finally, divide both sides by $-A$:

$$\dfrac{-Ab}{-A}=\dfrac{a-2A}{-A}$$

Solution:

$$b=\dfrac{a-2A}{-A}\text{ or }b=\dfrac{2A-a}{A}$$

Questions

For questions 1 to 10, evaluate each expression using the values given.

1. $p+1+q-m\ (m=1,p=3,q=4)$
2. $y^2+y-z\ (y=5,z=1)$
3. $p-[pq\div 6]\ \ (p=6,q=5)$
4. $[6+z-y]\div 3\ (y=1,z=4)$
5. $c^2-(a-1)\ (a=3,c=5)$
6. $x+6z-4y\ (x=6,y=4,z=4)$

7. $5j + kh \div 2 \ (h = 5, j = 4, k = 2)$
8. $5(b + a) + 1 + c \ (a = 2, b = 6, c = 5)$
9. $[4 - (p - m)] \div 2 + q \ (m = 4, p = 6, q = 6)$
10. $z + x - (1^2)^3 \ (x = 5, z = 4)$

For questions 11 to 34, isolate the indicated variable from the equation.

11. $b \text{ in } ab = c$
12. $h \text{ in } g = \dfrac{h}{i}$
13. $x \text{ in } \left(\dfrac{f}{g}\right)x = b$
14. $y \text{ in } p = \dfrac{3y}{q}$
15. $x \text{ in } 3x = \dfrac{a}{b}$
16. $y \text{ in } \dfrac{ym}{b} = \dfrac{c}{d}$
17. $\pi \text{ in } V = \dfrac{4}{3}\pi r^3$
18. $m \text{ in } E = mv^2$
19. $y \text{ in } c = \dfrac{4y}{m + n}$
20. $r \text{ in } \dfrac{rs}{a - 3} = k$
21. $D \text{ in } V = \dfrac{\pi Dn}{12}$
22. $R \text{ in } F = k(R - L)$
23. $c \text{ in } P = n(p - c)$
24. $L \text{ in } S = L + 2B$
25. $D \text{ in } T = \dfrac{D - d}{L}$
26. $E_a \text{ in } I = \dfrac{E_a - E_q}{R}$
27. $L_o \text{ in } L = L_o(1 + at)$
28. $m \text{ in } 2m + p = 4m + q$
29. $k \text{ in } \dfrac{k - m}{r} = q$
30. $T \text{ in } R = aT + b$
31. $Q_2 \text{ in } Q_1 = P(Q_2 - Q_1)$
32. $r_1 \text{ in } L = \pi(r_1 + r_2) + 2d$
33. $T_1 \text{ in } R = \dfrac{kA(T + T_1)}{d}$
34. $V_2 \text{ in } P = \dfrac{V_1(V_2 - V_1)}{g}$

Answer Key 2.6

14. 2.7 Variation Word Problems

Direct Variation Problems

There are many mathematical relations that occur in life. For instance, a flat commission salaried salesperson earns a percentage of their sales, where the more they sell equates to the wage they earn. An example of this would be an employee whose wage is 5% of the sales they make. This is a direct or a linear variation, which, in an equation, would look like:

$$\text{Wage }(x) = 5\% \text{ Commission }(k) \text{ of Sales Completed }(y)$$

$$\text{or}$$

$$x = ky$$

$$\text{(The constant } k \text{ comes from the German word for constant, which is } \textit{konstant}\text{)}$$

A historical example of direct variation can be found in the changing measurement of pi, which has been symbolized using the Greek letter π since the mid 18th century. Variations of historical π calculations are Babylonian $\left(\dfrac{25}{8}\right)$, Egyptian $\left(\dfrac{16}{9}\right)^2$, and Indian $\left(\dfrac{339}{108} \text{ and } 10^{\frac{1}{2}}\right)$. In the 5th century, Chinese mathematician Zu Chongzhi calculated the value of π to seven decimal places (3.1415926), representing the most accurate value of π for over 1000 years.

Pi is found by taking any circle and dividing the circumference of the circle by the diameter, which will always give the same value: 3.14159265358979323846264338327950288419716… (42 decimal places). Using an infinite-series exact equation has allowed computers to calculate π to 10^{13} decimals.

$$\text{Circumference }(c) = \pi \text{ times the diameter }(d)$$

$$\text{or}$$

$$c = \pi d$$

All direct variation relationships are verbalized in written problems as a direct variation or as directly proportional and take the form of straight line relationships. Examples of direct variation or directly proportional equations are:

- $x = ky$
  - x varies directly as y
  - x varies as y
  - x varies directly proportional to y
  - x is proportional to y
- $x = ky^2$

- x varies directly as the square of y
- x varies as y squared
- x is proportional to the square of y

- $x = ky^3$
  - x varies directly as the cube of y
  - x varies as y cubed
  - x is proportional to the cube of y
- $x = ky^{\frac{1}{2}}$
  - x varies directly as the square root of y
  - x varies as the root of y
  - x is proportional to the square root of y

Example 2.7.1

Find the variation equation described as follows:

The surface area of a square surface (A) is directly proportional to the square of either side (x).

Solution:

$$\text{Area } (A) = \text{ constant } (k) \text{ times side}^2 \ (x^2)$$

$$\text{or}$$

$$A = kx^2$$

Example 2.7.2

When looking at two buildings at the same time, the length of the buildings' shadows (s) varies directly as their height (h). If a 5-story building has a 20 m long shadow, how many stories high would a building that has a 32 m long shadow be?

The equation that describes this variation is:

$$h = kx$$

Breaking the data up into the first and second parts gives:

$$\begin{aligned} &\textbf{1st Data} \\ s &= 20\text{ m} \\ h &= 5\text{ stories} \\ k &= \text{find 1st} \\ \\ &\text{Find } k\text{:} \\ h &= kx \\ 5\text{ stories} &= k\ (20\text{ m}) \\ k &= 5\text{ stories}/20\text{ m} \\ k &= 0.25\text{ story/m} \end{aligned} \qquad \begin{aligned} &\textbf{2nd Data} \\ s &= 32\text{ m} \\ h &= \text{find 2nd} \\ k &= 0.25\text{ story/m} \\ \\ &\text{Find } h\text{:} \\ h &= kx \\ h &= (0.25\text{ story/m})(32\text{ m}) \\ h &= 8\text{ stories} \end{aligned}$$

Inverse Variation Problems

Inverse variation problems are reciprocal relationships. In these types of problems, the product of two or more variables is equal to a constant. An example of this comes from the relationship of the pressure (P) and the volume (V) of a gas, called Boyle's Law (1662). This law is written as:

$$\text{Pressure } (P) \text{ times Volume } (V) = \text{ constant}$$

$$\text{or}$$

$$PV = k$$

Written as an inverse variation problem, it can be said that the pressure of an ideal gas varies as the inverse of the volume or varies inversely as the volume. Expressed this way, the equation can be written as:

$$P = \frac{k}{V}$$

Another example is the historically famous inverse square laws. Examples of this are the force of gravity (F_{g}), electrostatic force (F_{el}), and the intensity of light (I). In all of these measures of force and light intensity, as you move away from the source, the intensity or strength decreases as the square of the distance.

In equation form, these look like:

$$F_{\text{g}} = \frac{k}{d^2} \qquad F_{\text{el}} = \frac{k}{d^2} \qquad I = \frac{k}{d^2}$$

These equations would be verbalized as:

- The force of gravity (F_{g}) varies inversely as the square of the distance.
- Electrostatic force (F_{el}) varies inversely as the square of the distance.
- The intensity of a light source (I) varies inversely as the square of the distance.

All inverse variation relationship are verbalized in written problems as inverse variations or as inversely proportional. Examples of inverse variation or inversely proportional equations are:

- $x = \frac{k}{y}$
  - x varies inversely as y
  - x varies as the inverse of y
  - x varies inversely proportional to y
  - x is inversely proportional to y
- $x = \frac{k}{y^2}$
  - x varies inversely as the square of y
  - x varies inversely as y squared
  - x is inversely proportional to the square of y
- $x = \frac{k}{y^3}$
  - x varies inversely as the cube of y
  - x varies inversely as y cubed
  - x is inversely proportional to the cube of y
- $x = \frac{k}{y^{\frac{1}{2}}}$
  - x varies inversely as the square root of y
  - x varies as the inverse root of y
  - x is inversely proportional to the square root of y

Example 2.7.3

Find the variation equation described as follows:

The force experienced by a magnetic field (F_{b}) is inversely proportional to the square of the distance from the source (d_{s}).

Solution:

$$F_{\mathrm{b}} = \frac{k}{{d_{\mathrm{s}}}^2}$$

Example 2.7.4

The time (t) it takes to travel from North Vancouver to Hope varies inversely as the speed (v) at which one travels. If it takes 1.5 hours to travel this distance at an average speed of 120 km/h, find the constant k and the amount of time it would take to drive back if you were only able to travel at 60 km/h due to an engine problem.

The equation that describes this variation is:

$$t = \frac{k}{v}$$

Breaking the data up into the first and second parts gives:

$$\begin{array}{rcl} & & \textbf{1st Data} \\ v & = & 120 \text{ km/h} \\ t & = & 1.5 \text{ h} \\ k & = & \text{find 1st} \\ & & \\ & & \text{Find } k\text{:} \\ k & = & tv \\ k & = & (1.5 \text{ h})(120 \text{ km/h}) \\ k & = & 180 \text{ km} \end{array}$$

$$\begin{array}{rcl} & & \textbf{2nd Data} \\ v & = & 60 \text{ km/h} \\ t & = & \text{find 2nd} \\ k & = & 180 \text{ km} \\ & & \\ & & \text{Find } t\text{:} \\ t & = & \dfrac{k}{v} \\ & & \\ t & = & \dfrac{180 \text{ km}}{60 \text{ km/h}} \\ & & \\ t & = & 3 \text{ h} \end{array}$$

Joint or Combined Variation Problems

In real life, variation problems are not restricted to single variables. Instead, functions are generally a combination of multiple factors. For instance, the physics equation quantifying the gravitational force of attraction between two bodies is:

$$F_{\text{g}} = \frac{Gm_1m_2}{d^2}$$

where:

- F_{g} stands for the gravitational force of attraction
- G is Newton's constant, which would be represented by k in a standard variation problem
- m_1 and m_2 are the masses of the two bodies
- d^2 is the distance between the centres of both bodies

To write this out as a variation problem, first state that the force of gravitational attraction (F_{g}) between two bodies is directly proportional to the product of the two masses (m_1, m_2) and inversely proportional to the square of the distance (d) separating the two masses. From this information, the necessary equation can be derived. All joint variation relationships are verbalized in written problems as a combination of direct and inverse variation relationships, and care must be taken to correctly identify which variables are related in what relationship.

Example 2.7.5

Find the variation equation described as follows:

The force of electrical attraction (F_{el}) between two statically charged bodies is directly proportional to the product of the charges on each of the two objects (q_1, q_2) and inversely proportional to the square of the distance (d) separating these two charged bodies.

Solution:

$$F_{\text{el}} = \frac{kq_1q_2}{d^2}$$

Solving these combined or joint variation problems is the same as solving simpler variation problems.

First, decide what equation the variation represents. Second, break up the data into the first data given—which is used to find k—and then the second data, which is used to solve the problem given. Consider the following joint variation problem.

Example 2.7.6

y varies jointly with m and n and inversely with the square of d. If $y = 12$ when $m = 3, n = 8$, and $d = 2$, find the constant k, then use k to find y when $m = -3, n = 18$, and $d = 3$.

The equation that describes this variation is:

$$y = \frac{kmn}{d^2}$$

Breaking the data up into the first and second parts gives:

$$
\begin{array}{rclcrcl}
& & \textbf{1st Data} & \quad & & & \textbf{2nd Data} \\
y & = & 12 & & y & = & \text{find 2nd} \\
m & = & 3 & & m & = & -3 \\
n & = & 8 & & n & = & 18 \\
d & = & 2 & & d & = & 3 \\
k & = & \text{find 1st} & & k & = & 2
\end{array}
$$

$$
\begin{array}{rclcrcl}
& & \text{Find } k\text{:} & \quad & & & \text{Find } y\text{:} \\
y & = & \dfrac{kmn}{d^2} & & y & = & \dfrac{kmn}{d^2} \\
12 & = & \dfrac{k(3)(8)}{(2)^2} & & y & = & \dfrac{(2)(-3)(18)}{(3)^2} \\
k & = & \dfrac{12(2)^2}{(3)(8)} & & y & = & 12 \\
k & = & 2 & & & &
\end{array}
$$

Questions

For questions 1 to 12, write the formula defining the variation, including the constant of variation (k).

1. x varies directly as y
2. x is jointly proportional to y and z
3. x varies inversely as y
4. x varies directly as the square of y
5. x varies jointly as z and y
6. x is inversely proportional to the cube of y
7. x is jointly proportional with the square of y and the square root of z
8. x is inversely proportional to y to the sixth power
9. x is jointly proportional with the cube of y and inversely to the square root of z
10. x is inversely proportional with the square of y and the square root of z
11. x varies jointly as z and y and is inversely proportional to the cube of p
12. x is inversely proportional to the cube of y and square of z

For questions 13 to 22, find the formula defining the variation and the constant of variation (k).

13. If A varies directly as B, find k when $A = 15$ and $B = 5$.
14. If P is jointly proportional to Q and R, find k when $P = 12, Q = 8$ and $R = 3$.
15. If A varies inversely as B, find k when $A = 7$ and $B = 4$.

16. If A varies directly as the square of B, find k when $A = 6$ and $B = 3$.
17. If C varies jointly as A and B, find k when $C = 24$, $A = 3$, and $B = 2$.
18. If Y is inversely proportional to the cube of X, find k when $Y = 54$ and $X = 3$.
19. If X is directly proportional to Y, find k when $X = 12$ and $Y = 8$.
20. If A is jointly proportional with the square of B and the square root of C, find k when $A = 25$, $B = 5$ and $C = 9$.
21. If y varies jointly with m and the square of n and inversely with d, find k when $y = 10$, $m = 4$, $n = 5$, and $d = 6$.
22. If P varies directly as T and inversely as V, find k when $P = 10$, $T = 250$, and $V = 400$.

For questions 23 to 37, solve each variation word problem.

23. The electrical current I (in amperes, A) varies directly as the voltage (V) in a simple circuit. If the current is 5 A when the source voltage is 15 V, what is the current when the source voltage is 25 V?
24. The current I in an electrical conductor varies inversely as the resistance R (in ohms, Ω) of the conductor. If the current is 12 A when the resistance is 240 Ω, what is the current when the resistance is 540 Ω?
25. Hooke's law states that the distance (d_s) that a spring is stretched supporting a suspended object varies directly as the mass of the object (m). If the distance stretched is 18 cm when the suspended mass is 3 kg, what is the distance when the suspended mass is 5 kg?
26. The volume (V) of an ideal gas at a constant temperature varies inversely as the pressure (P) exerted on it. If the volume of a gas is 200 cm^3 under a pressure of 32 kg/cm^2, what will be its volume under a pressure of 40 kg/cm^2?
27. The number of aluminum cans (c) used each year varies directly as the number of people (p) using the cans. If 250 people use 60,000 cans in one year, how many cans are used each year in a city that has a population of 1,000,000?
28. The time (t) required to do a masonry job varies inversely as the number of bricklayers (b). If it takes 5 hours for 7 bricklayers to build a park wall, how much time should it take 10 bricklayers to complete the same job?
29. The wavelength of a radio signal (λ) varies inversely as its frequency (f). A wave with a frequency of 1200 kilohertz has a length of 250 metres. What is the wavelength of a radio signal having a frequency of 60 kilohertz?
30. The number of kilograms of water (w) in a human body is proportional to the mass of the body (m). If a 96 kg person contains 64 kg of water, how many kilograms of water are in a 60 kg person?
31. The time (t) required to drive a fixed distance (d) varies inversely as the speed (v). If it takes 5 hours at a speed of 80 km/h to drive a fixed distance, what speed is required to do the same trip in 4.2 hours?
32. The volume (V) of a cone varies jointly as its height (h) and the square of its radius (r). If a cone with a height of 8 centimetres and a radius of 2 centimetres has a volume of 33.5 cm^3, what is the volume of a cone with a height of 6 centimetres and a radius of 4 centimetres?
33. The centripetal force (F_c) acting on an object varies as the square of the speed (v) and inversely to the radius (r) of its path. If the centripetal force is 100 N when the object is travelling at 10 m/s in a path or radius of 0.5 m, what is the centripetal force when the object's speed increases to 25 m/s and the path is now 1.0 m?
34. The maximum load (L_{max}) that a cylindrical column with a circular cross section can hold varies directly as the fourth power of the diameter (d) and inversely as the square of the height (h). If an 8.0 m column that is 2.0 m in diameter will support 64 tonnes, how many tonnes can be supported by a column 12.0 m high and 3.0 m in diameter?
35. The volume (V) of gas varies directly as the temperature (T) and inversely as the pressure (P). If the volume is 225 cc when the temperature is 300 K and the pressure is 100 N/cm^2, what is the volume when the temperature drops to 270 K and the pressure is 150 N/cm^2?
36. The electrical resistance (R) of a wire varies directly as its length (l) and inversely as the square of its diameter (d). A wire with a length of 5.0 m and a diameter of 0.25 cm has a resistance of 20 Ω. Find the electrical resistance

in a 10.0 m long wire having twice the diameter.

37. The volume of wood in a tree (V) varies directly as the height (h) and the diameter (d). If the volume of a tree is 377 m^3 when the height is 30 m and the diameter is 2.0 m, what is the height of a tree having a volume of 225 m^3 and a diameter of 1.75 m?

Answer Key 2.7

15. 2.8 The Mystery X Puzzle

The centre number of each square is found by using the order of operations on the numbers that surround it. The challenge is to solve for the variable X.

5	3	12
21	**42**	**64**
3 6	11 9	5 X

Can you solve for X? Can you find any other possible solution?

Answer Key 2.8

PART III

CHAPTER 3: GRAPHING

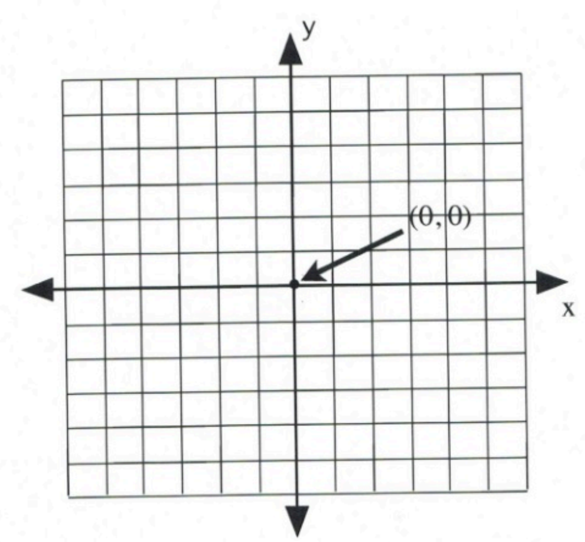
Learning Objectives

This chapter covers:

- Points & Coordinates
- Midpoint & Distance Between Two Points
- Slopes & Their Graphs
- Graphing Linear Equations
- Constructing Linear Equations
- Perpendicular & Parallel Lines
- Numeric Word Problems

16. 3.1 Points and Coordinates

Often, to get an idea of the behaviour of an equation or some function, a visual representation that displays the solutions to the equation or function in the form of a graph will be made. Before exploring this, it is necessary to review the foundations of a graph. The following is an example of what is called the coordinate plane of a graph.

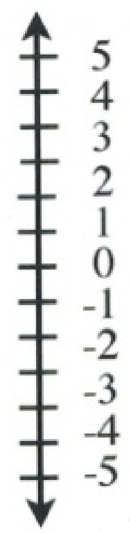


The plane is divided into four sections by a horizontal number line (x-axis) and a vertical number line (y-axis). Where the two lines meet in the centre is called the origin. This centre origin is where $x = 0$ and $y = 0$ and is represented by the ordered pair $(0, 0)$.

Graph 1 arrow

x-axis

For the x-axis, moving to the right from the centre 0, the numbers count up, and $x = 1, 2, 3, 4, 5.$ To the left of the centre 0, the numbers count down, and $x = -1, -2, -3, -4, -5.$

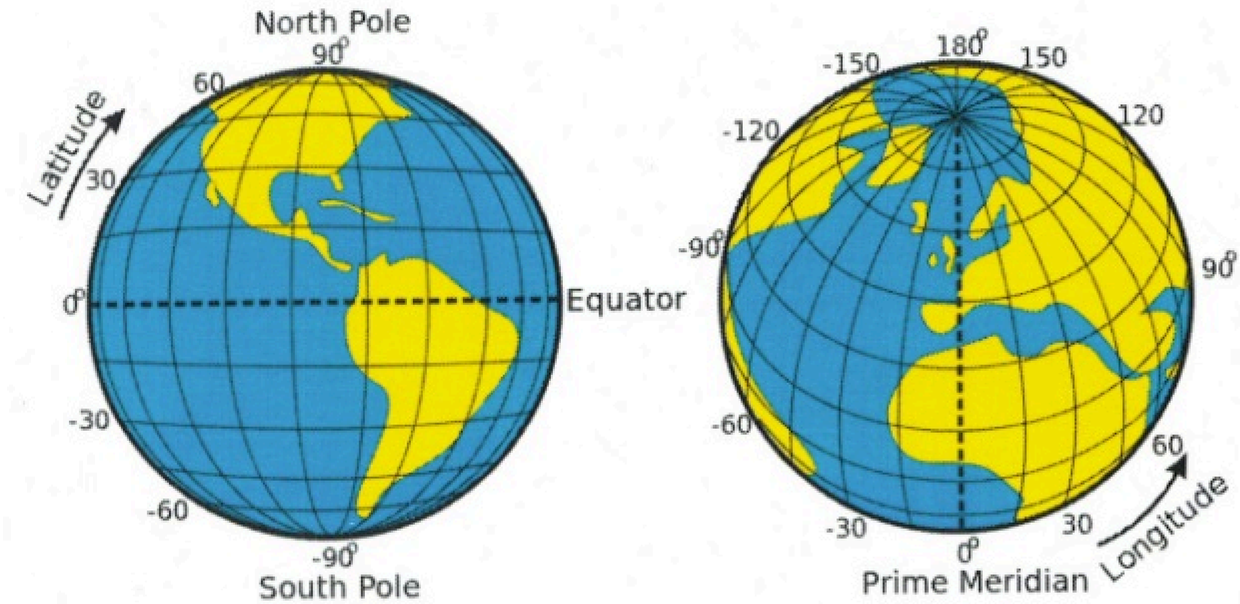


y-axis

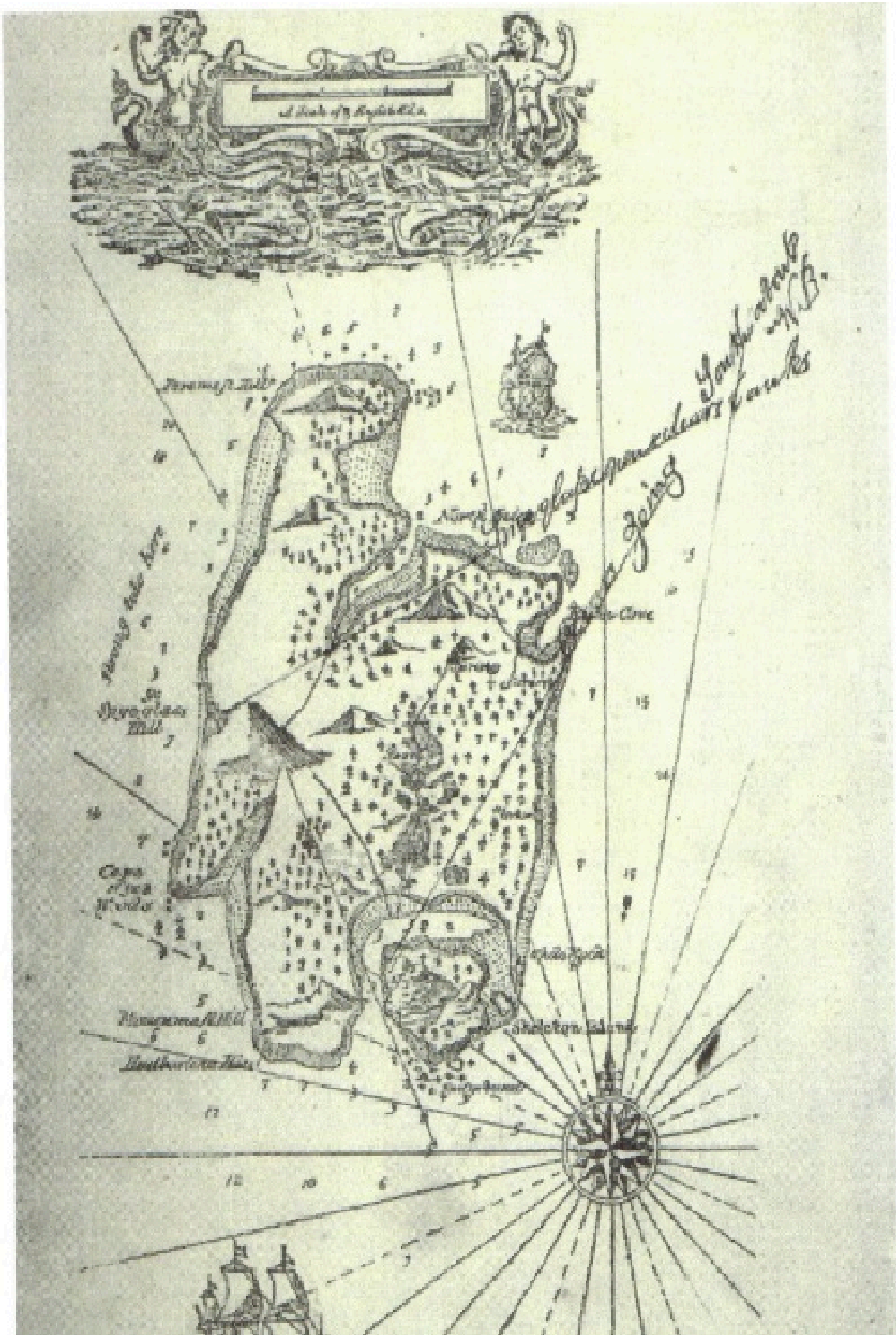
Similarly, for the y-axis, moving up from the centre 0, the numbers count up, and $y = 1, 2, 3, 4, 5.$ Moving down from the centre 0, the numbers count down, and $y = -1, -2, -3, -4, -5.$

When identifying points on a graph, a dot is generally used with a set of parentheses following that gives the x-value followed by the y-value. This will look like $(x\text{-value}, y\text{-value})$ or (x, y) and is given the formal name of an ordered pair.

This coordinate system is universally used, with the simplest example being the kind of treasure map that is usually encountered in childhood, or the longitude and latitude system used to identify any position on the Earth. For this system, the x-axis (which represents latitude) is the equator and the y-axis (which represents longitude) or the prime meridian is the line that passes though Greenwich, England. The origin of the Earth's latitude and longitude (0°, 0°) is a fictional island called "Null Island."



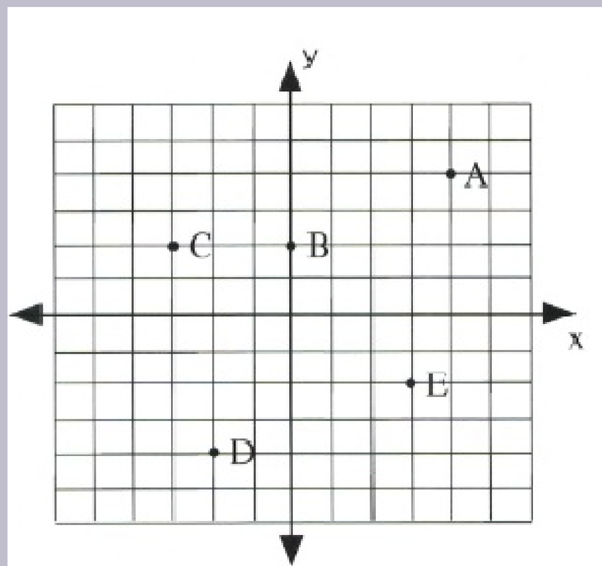
Latitude and longitude. [Long Description]



The treasure map of Robert Louis Stevenson made popular by his work Treasure Island. From Cordingly, David (1995). Under the Black Flag: The romance and the reality of life among the pirates. Times Warner, 1996.

Example 3.1.1

Identify the coordinates of the following data points.



A. For the x-coordinate, move 4 to the right from the origin. For the y-coordinate, move 4 up. This gives the final coordinates of (4, 4).

B. For the x-coordinate, stay at the origin. For the y-coordinate, move 2 up. This gives the final coordinates of (0, 2).

C. For the x-coordinate, move 3 to the left from the origin. For the y-coordinate, move 2 up. This gives the final coordinates of (−3, 2).

D. For the x-coordinate, move 2 to the left from the origin. For the y-coordinate, move 4 down. This gives the final coordinates of (−2, −4).

E. For the x-coordinate, move 3 to the right from the origin. For the y-coordinate, move 2 down. This gives the final coordinate of (3, −2).

Example 3.1.2

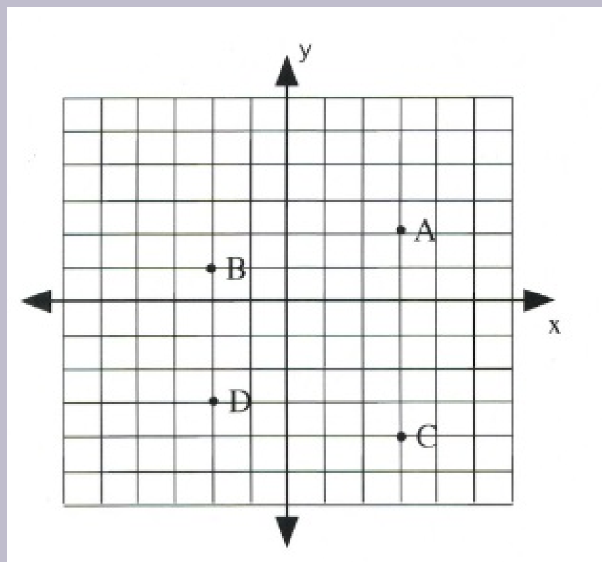
Graph the points A(3, 2), B(−2, 1), C(3, −4), and D(−2, −3).

The first point, A, is at (3, 2). This means $x = 3$ (3 to the right) and $y = 2$ (up 2). Following these instructions, starting from the origin, results in the correct point.

The second point, B(−2, 1), is left 2 for the x-coordinate and up 1 for the y-coordinate.

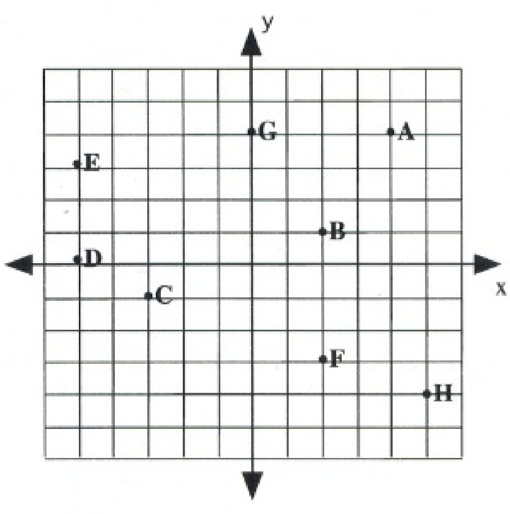
The third point, C(3 ,−4), is right 3, down 4.

The fourth point, D(−2, −3), is left 2, down 3.

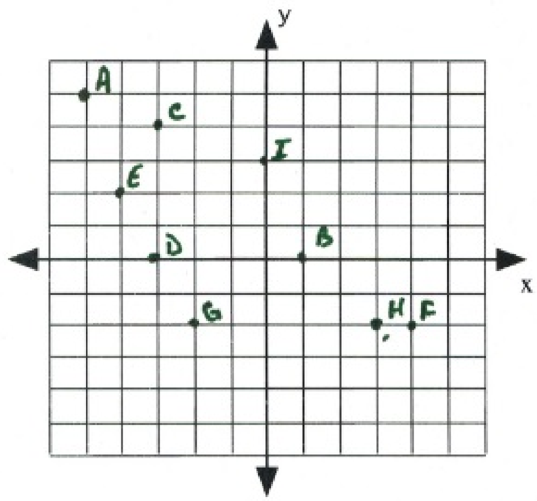


Questions

1. What are the coordinates of each point on the graph below?



2. Plot and label the following points on the graph.



A $(-5,5)$	B $(1,0)$	C $(-3,4)$
D $(-3,0)$	E $(-4,2)$	F $(4,-2)$
G $(-2,-2)$	H $(3,-2)$	I $(0,3)$

Answer Key 3.1

Long Descriptions

Latitude and longitude long description: Two views of the globe that show the landmark points of the latitude and longitude system.

The first globe demonstrates the lines of latitude. The centre line of latitude is called the equator and represents 0° latitude. It wraps around the centre of the Earth from west to east. The globe shows North and South America, and the equator runs through the northern part of South America. The North Pole is at 90° latitude and the South Pole is at −90° latitude. Positive latitude is above the equator, and negative latitude is below it.

The second globe demonstrates the lines of longitude. The centre line of longitude is called the prime meridian and represents 0° longitude. It wraps around the centre of the Earth from north to south. It passes through Greenwich, England, by convention, as well as parts of France, Spain, and western Africa. Positive longitude is east of the prime meridian, and negative longitude is west of it. [Return to Latitude and longitude]

17. 3.2 Midpoint and Distance Between Points

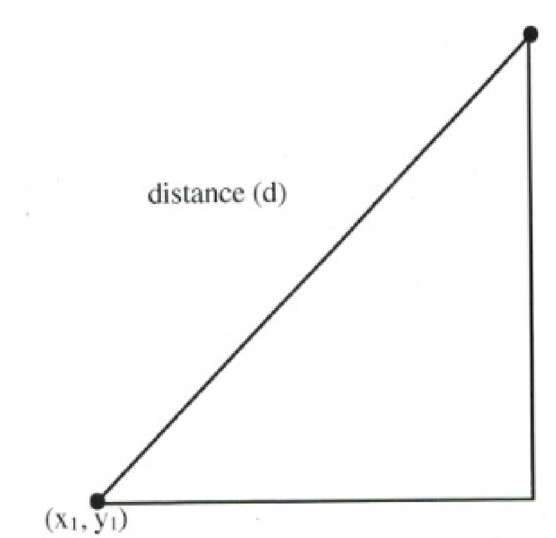
Finding the Distance Between Two Points

The logic used to find the distance between two data points on a graph involves the construction of a right triangle using the two data points and the Pythagorean theorem $(a^2 + b^2 = c^2)$ to find the distance.

To do this for the two data points (x_1, y_1) and (x_2, y_2), the distance between these two points (d) will be found using $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$.

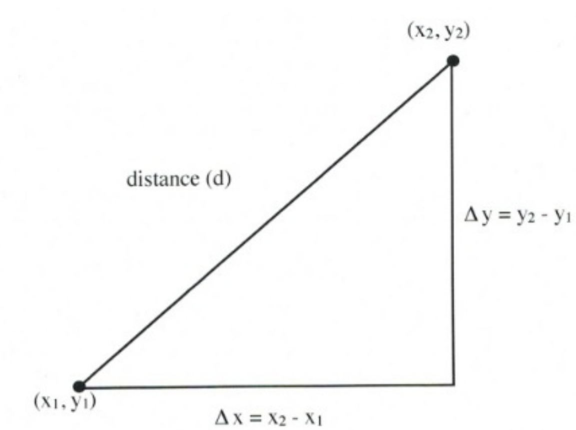
Using the Pythagorean theorem, this will end up looking like:

$$d^2 = \Delta x^2 + \Delta y^2$$

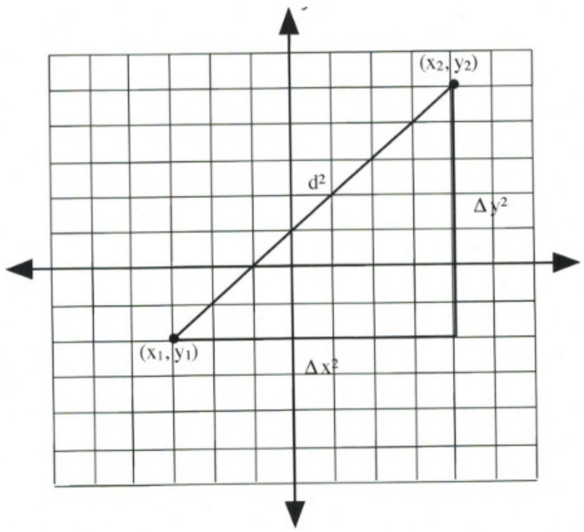


or, in expanded form:

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$



On graph paper, this looks like the following. For this illustration, both Δx and Δy are 7 units long, making the distance $d^2 = 7^2 + 7^2$ or $d^2 = 98$.



The square root of 98 is approximately 9.899 units long.

Example 3.2.1

Find the distance between the points $(-6, -4)$ and $(6, 5)$.

Start by identifying which are the two data points (x_1, y_1) and (x_2, y_2). Let (x_1, y_1) be $(-6, -4)$ and (x_2, y_2) be $(6, 5)$.

Now:

$$\Delta x^2 = (x_2 - x_1)^2 \text{ or } [6 - (-6)]^2 \text{ and } \Delta y^2 = (y_2 - y_1)^2 \text{ or } [5 - (-4)]^2.$$

This means that

$$d^2 = [6 - (-6)]^2 + [5 - (-4)]^2$$

or

$$d^2 = [12]^2 + [9]^2$$

which reduces to

$$d^2 = 144 + 81$$

or

$$d^2 = 225$$

Taking the square root, the result is $d = 15$.

Finding the Midway Between Two Points (Midpoint)

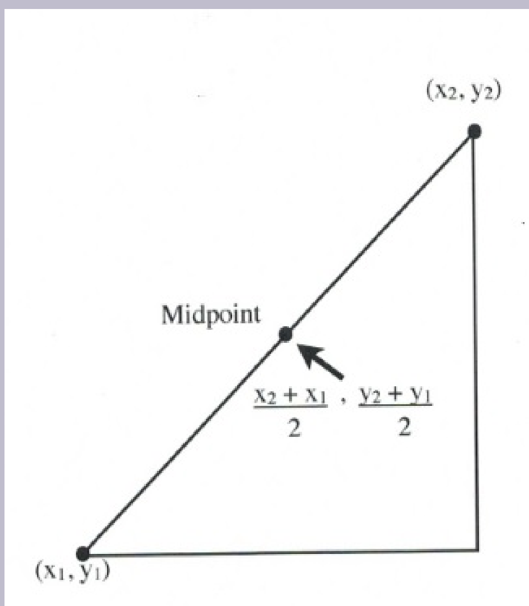
The logic used to find the midpoint between two data points (x_1, y_1) and (x_2, y_2) on a graph involves finding the average values of the x data points (x_1, x_2) and the of the y data points (y_1, y_2). The averages are found by adding both data points together and dividing them by 2.

In an equation, this looks like:

$$x_{\text{mid}} = \frac{x_2 + x_1}{2} \text{ and } y_{\text{mid}} = \frac{y_2 + y_1}{2}$$

Example 3.2.2

Find the midpoint between the points $(-2, 3)$ and $(6, 9)$.



We start by adding the two x data points $(x_1 + x_2)$ and then dividing this result by 2.

$$x_{\text{mid}} = \frac{(-2+6)}{2}$$

or

$$\frac{4}{2} = 2$$

The midpoint's y-coordinate is found by adding the two y data points $(y_1 + y_2)$ and then dividing this result by 2.

$$y_{\text{mid}} = \frac{(9+3)}{2}$$

or

$$\frac{12}{2} = 6$$

The midpoint between the points $(-2, 3)$ and $(6, 9)$ is at the data point $(2, 6)$.

Questions

For questions 1 to 8, find the distance between the points.

1. (−6, −1) and (6, 4)
2. (1, −4) and (5, −1)
3. (−5, −1) and (3, 5)
4. (6, −4) and (12, 4)
5. (−8, −2) and (4, 3)
6. (3, −2) and (7, 1)
7. (−10, −6) and (−2, 0)
8. (8, −2) and (14, 6)

For questions 9 to 16, find the midpoint between the points.

9. (−6, −1) and (6, 5)
10. (1, −4) and (5, −2)
11. (−5, −1) and (3, 5)
12. (6, −4) and (12, 4)
13. (−8, −1) and (6, 7)
14. (1, −6) and (3, −2)
15. (−7, −1) and (3, 9)
16. (2, −2) and (12, 4)

Answer Key 3.2

18. 3.3 Slopes and Their Graphs

Another important property of any line or linear function is slope. Slope is a measure of steepness and indicates in some situations how fast something is changing—specifically, its rate of change. A line with a large slope, such as 10, is very steep. A line with a small slope, such as $\frac{1}{10}$, is very flat or nearly level. Lines that rise from left to right are called positive slopes and lines that sink are called negative slopes. Slope can also be used to describe the direction of a line. A line that goes up as it moves from from left to right is described as having a positive slope whereas a line that goes downward has a negative slope. Slope, therefore, will define a line as rising or falling.

Slopes in real life have significance. For instance, roads with slopes that are potentially dangerous often carry warning signs. For steep slopes that are rising, extra slow moving lanes are generally provided for large trucks. For roads that have steep down slopes, runaway lanes are often provided for vehicles that lose their ability to brake.

When quantifying slope, use the measure of the rise of the line divided by its run. The symbol that represents slope is the letter m, which has unknown origins. Its first recorded usage is in an 1844 text by Matthew O'Brian, "A Treatise on Plane Co-Ordinate Geometry,"[1] which was quickly followed by George Salmon's "A Treatise on Conic Sections" (1848), in which he used m in the equation $y = mx + b$.

$$\text{slope } = \frac{\text{rise of the line}}{\text{run of the line}}$$

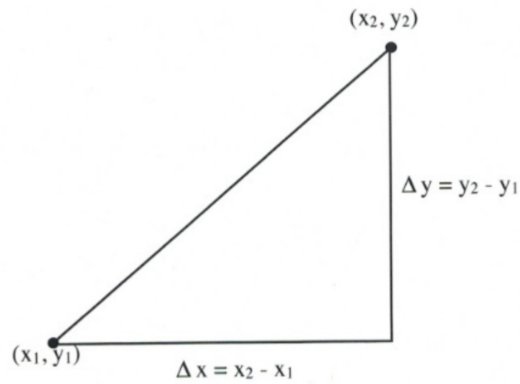
Since the rise of a line is shown by the change in the y-value and the run is shown by the change in the x-value, this equation is shortened to:

$$m = \frac{\Delta y}{\Delta x}, \text{ where } \Delta \text{ is the symbol for change and means final value } - \text{ initial value}$$

This equation is often expanded to:

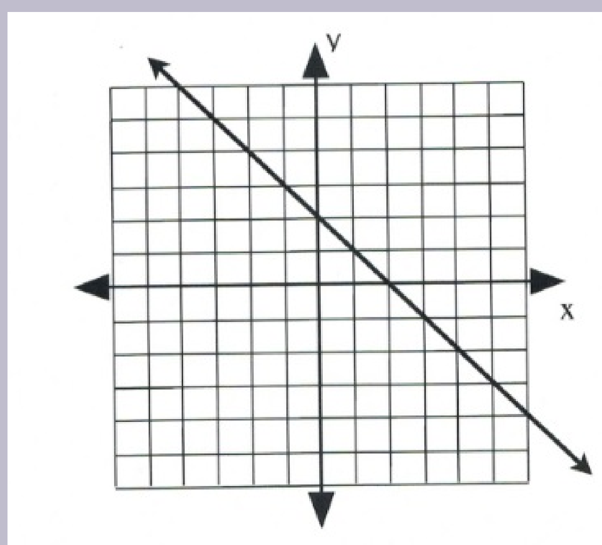
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

1. Derivation of Slope: https://services.math.duke.edu//education/webfeats/Slope/Slopederiv.html



Example 3.3.1

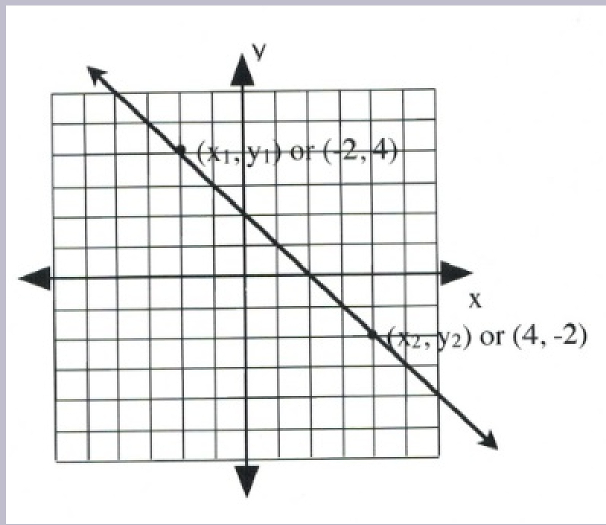
Find the slope of the following line.



First, choose two points on the line on this graph. Any points can be chosen, but they should fall on one of the corner grids. These are labelled (x_1, y_1) and (x_2, y_2).

To find the slope of this line, consider the rise, or vertical change, and the run, or horizontal change. Observe in this example that the Δy-value (the rise) goes from 4 to −2.

Therefore, $\Delta y = y_2 - y_1$, or (4 − −2), which equals (4 + 2), or 6.



The Δx-value (the run) goes from −2 to 4.

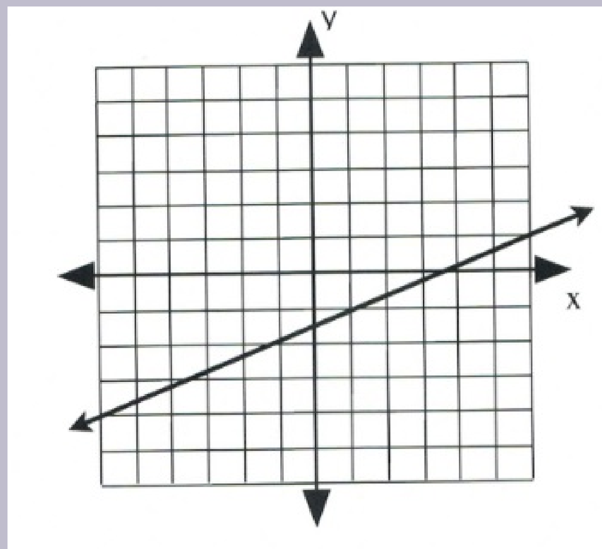
Therefore, $\Delta x = x_2 - x_1$, or (−2 − 4), which equals (−2 + −4), or −6.

This means the slope of this line is $m = \frac{\Delta y}{\Delta x}$, or $\frac{6}{-6}$, or −1.

$m = -1$

Example 3.3.2

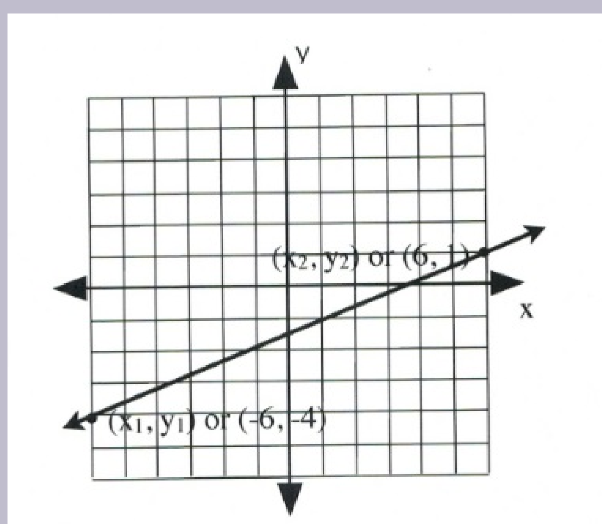
Find the slope of the following line.



First, choose two points on the line on this graph. Any points can be chosen, but to fall on a corner grid, they should be on opposite sides of the graph. These are (x_1, y_1) and (x_2, y_2).

To find the slope of this line, consider the rise, or vertical change, and the run, or horizontal change. Observe in this example that the Δy-value (the rise) goes from −4 to 1.

Therefore, $\Delta y = y_2 - y_1$, or (1 − −4), which equals 5.



The Δx-value (the run) goes from −6 to 6.

Therefore, $\Delta x = x_2 - x_1$ or (6 − −6), which equals 12.

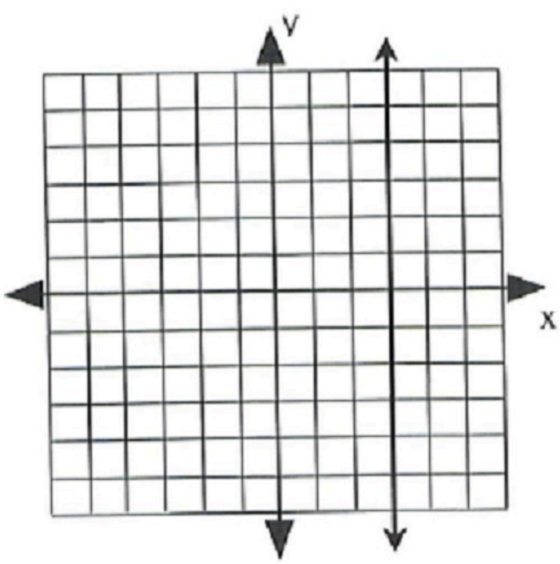
This means the slope of this line is $m = \dfrac{\Delta y}{\Delta x}$, or $\dfrac{5}{12}$, which cannot be further simplified.

$$m = \frac{5}{12}$$

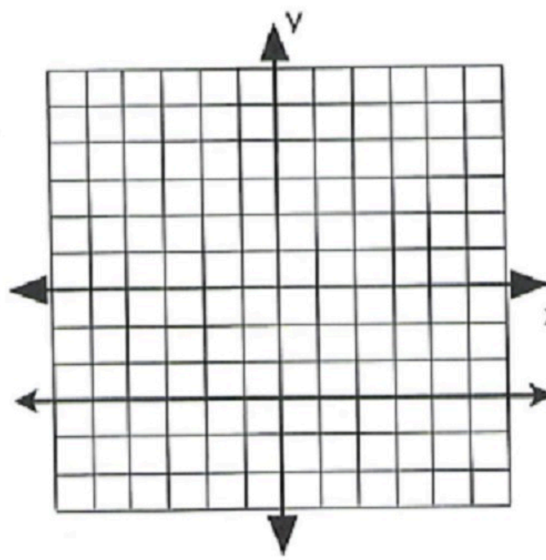
There are two special lines that have unique slopes that must be noted: lines with slopes equal to zero and slopes that are undefined.

Undefined slopes arise when the line on the graph is vertical, going straight up and down. In this case, $\Delta x = 0$, which means that zero is divided by while calculating the slope, which makes it undefined.

Zero slopes are flat, horizontal lines that do not rise or fall; therefore, $\Delta y = 0$. In this case, the slope is simply 0.



Undefined Slope since $\Delta x = 0$



Zero Slope since $\Delta y = 0$

Most often, the slope of the line must be found using data points rather than graphs. In this case, two data points are generally given, and the slope m is found by dividing Δy by Δx. This is usually done using the expanded slope equation of:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 3.3.3

Find the slope of a line that would connect the data points $(-4, 3)$ and $(2, -9)$.

Choose Point 1 to be $(-4, 3)$ and Point 2 to be $(2, -9)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-9 - 3}{2 - -4}$$

$$m = \frac{-12}{6} \text{ or } -2$$

Example 3.3.4

Find the slope of a line that would connect the data points $(-5, 3)$ and $(2, 3)$.

Choose Point 1 to be $(-5, 3)$ and Point 2 to be $(2, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{3 - 3}{2 - -5}$$

$$m = \frac{0}{7} \text{ or } 0$$

This is an example of a flat, horizontal line.

Example 3.3.5

Find the slope of a line that would connect the data points $(4, 3)$ and $(4, -5)$.

Choose Point 1 to be $(4, 3)$ and Point 2 to be $(4, -5)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-5 - 3}{4 - 4}$$

$$m = \frac{-8}{0} \text{ or undefined}$$

This is a vertical line.

Example 3.3.6

Find the slope of a line that would connect the data points $(-4, -3)$ and $(2, 6)$.

Choose Point 1 to be $(-4, -3)$ and Point 2 to be $(2, 6)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

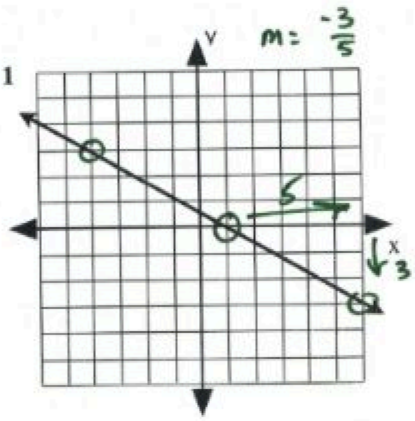
$$m = \frac{6 - -3}{2 - -4}$$

$$m = \frac{9}{6} \text{ or } \frac{3}{2}$$

Questions

For questions 1 to 6, find the slope of each line shown on the graph.

1



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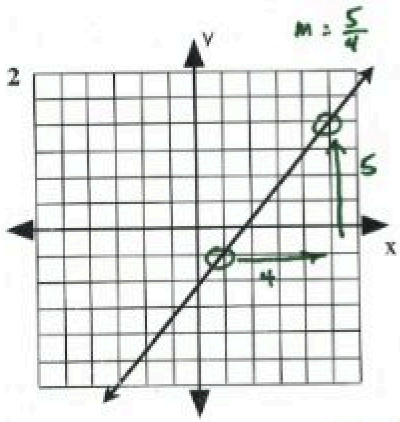

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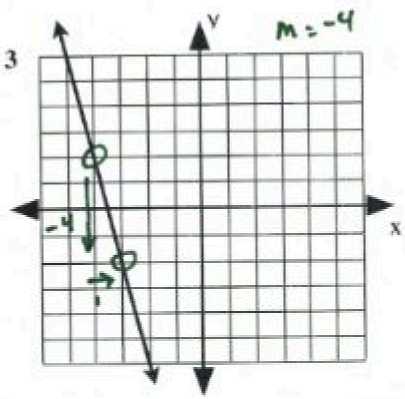

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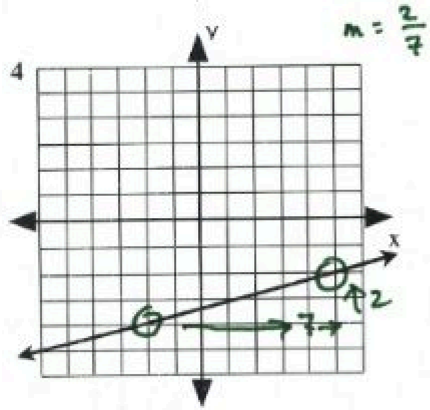

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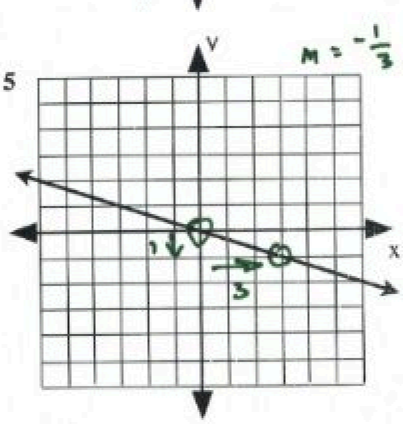

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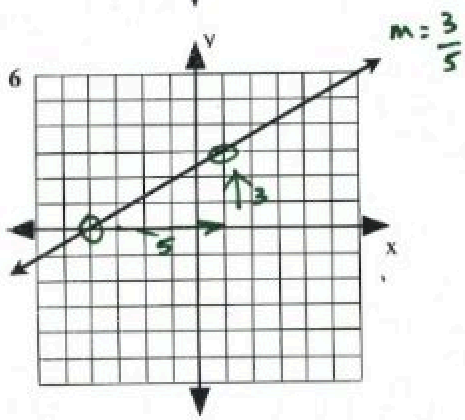


For questions 7 to 26, find the slope of the line that would connect each pair of points.

7. (2, 10), (−2, 15)
8. (1, 2), (−6, −12)
9. (−5, 10), (0, 0)
10. (2, −2), (7, 8)
11. (4, 6), (−8, −10)
12. (−3, 6), (9, −6)
13. (−2 −4), (10, −4)
14. (3, 5), (2, 0)
15. (−4, 4), (−6, 8)
16. (9, −6), (−7, −7)
17. (2, −9), (6, 4)
18. (−6, 2), (5, 0)

19. $(-5, 0), (-5, 0)$
20. $(8, 11), (-3, -13)$
21. $(-7, 9), (1, -7)$
22. $(1, -2), (1, 7)$
23. $(7, -4), (-8, -9)$
24. $(-8, -5), (4, -3)$
25. $(-5, 7), (-8, 4)$
26. $(9, 5), (5, 1)$

Answer Key 3.3

19. 3.4 Graphing Linear Equations

There are two common procedures that are used to draw the line represented by a linear equation. The first one is called the slope-intercept method and involves using the slope and intercept given in the equation.

If the equation is given in the form $y = mx + b$, then m gives the rise over run value and the value b gives the point where the line crosses the y-axis, also known as the y-intercept.

Example 3.4.1

Given the following equations, identify the slope and the y-intercept.

1. $y = 2x - 3 \quad \text{Slope } (m) = 2 \quad y\text{-intercept } (b) = -3$
2. $y = \dfrac{1}{2}x - 1 \quad \text{Slope } (m) = \dfrac{1}{2} \quad y\text{-intercept } (b) = -1$
3. $y = -3x + 4 \quad \text{Slope } (m) = -3 \quad y\text{-intercept } (b) = 4$
4. $y = \dfrac{2}{3}x \quad \text{Slope } (m) = \dfrac{2}{3} \quad y\text{-intercept } (b) = 0$

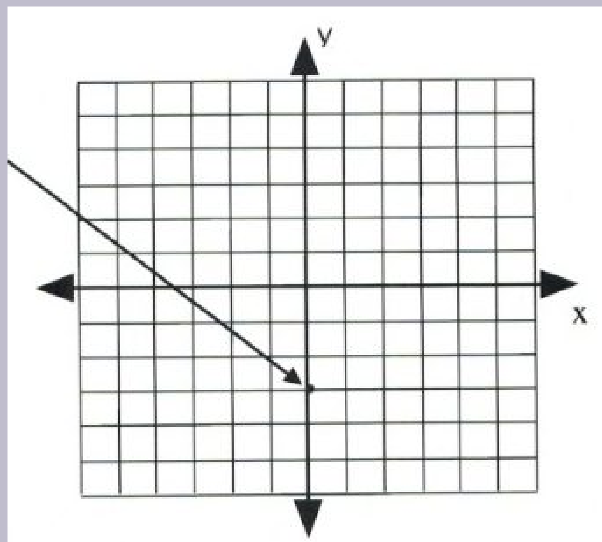
When graphing a linear equation using the slope-intercept method, start by using the value given for the y-intercept. After this point is marked, then identify other points using the slope.

This is shown in the following example.

Example 3.4.2

Graph the equation $y = 2x - 3$.

First, place a dot on the y-intercept, $y = -3$, which is placed on the coordinate $(0, -3)$.

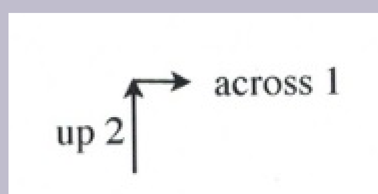


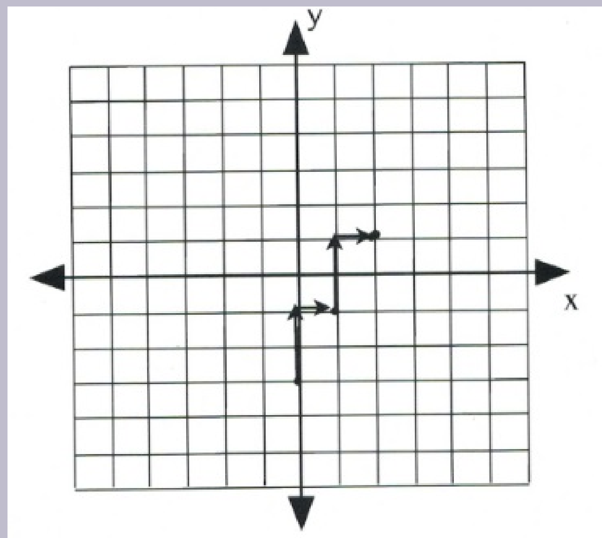
Now, place the next dot using the slope of 2.

A slope of 2 means that the line rises 2 for every 1 across.

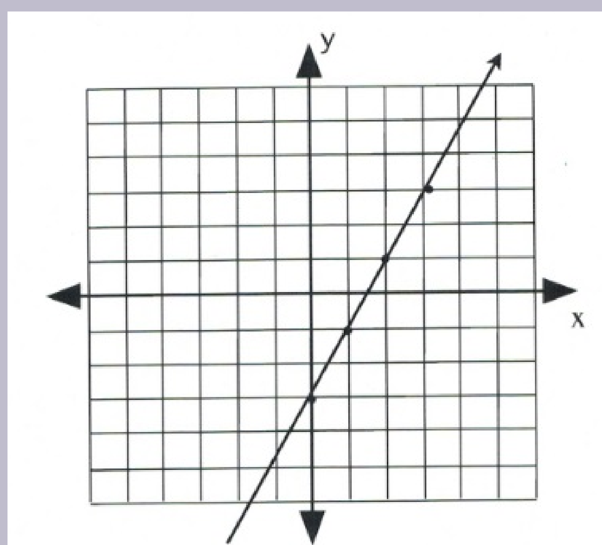
Simply, $m = 2$ is the same as $m = \frac{2}{1}$, where $\Delta y = 2$ and $\Delta x = 1$.

Placing these points on the graph becomes a simple counting exercise, which is done as follows:



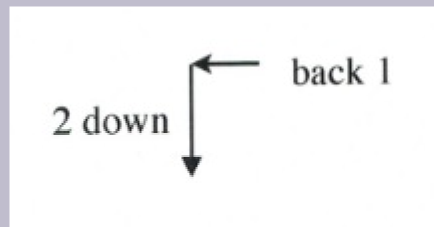


Once several dots have been drawn, draw a line through them, like so:



Note that dots can also be drawn in the reverse of what has been drawn here.

Slope is 2 when rise over run is $\frac{2}{1}$ or $\frac{-2}{-1}$, which would be drawn as follows:

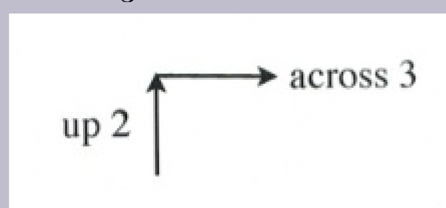


Example 3.4.3

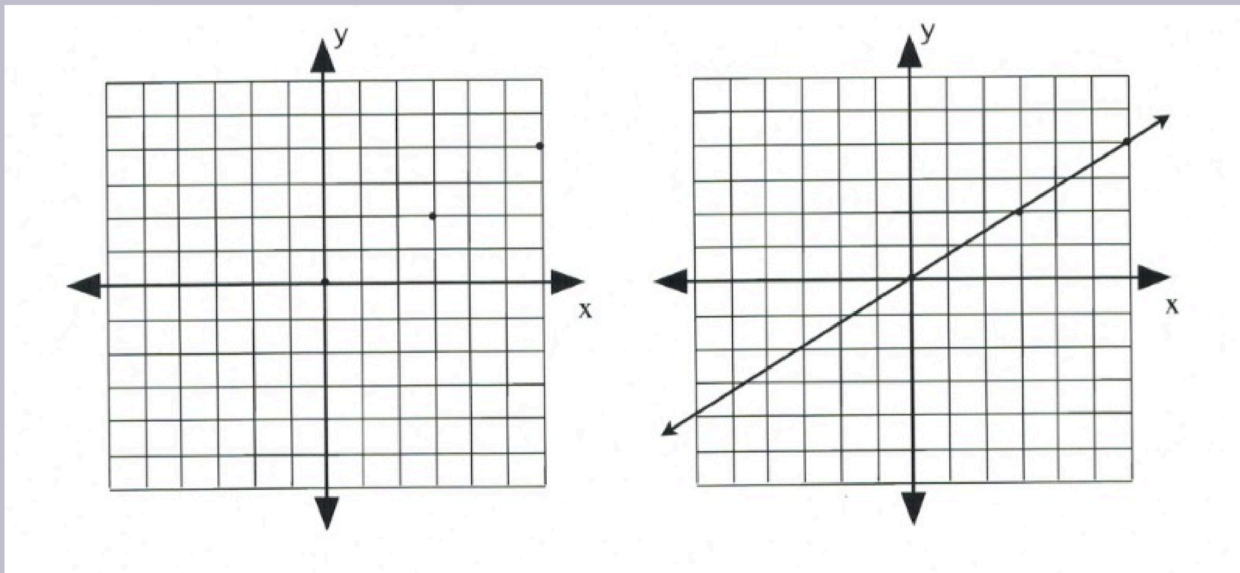
Graph the equation $y = \frac{2}{3}x$.

First, place a dot on the y-intercept, $(0, 0)$.

Now, place the dots according to the slope, $\frac{2}{3}$.



This will generate the following set of dots on the graph. All that remains is to draw a line through the dots.



The second method of drawing lines represented by linear equations and functions is to identify the two intercepts of the linear equation. Specifically, find x when $y = 0$ and find y when $x = 0$.

Example 3.4.4

Graph the equation $2x + y = 6$.

To find the first coordinate, choose $x = 0$.

This yields:

$$\begin{array}{rcrcl} 2(0) & + & y & = & 6 \\ & & y & = & 6 \end{array}$$

Coordinate is $(0, 6)$.

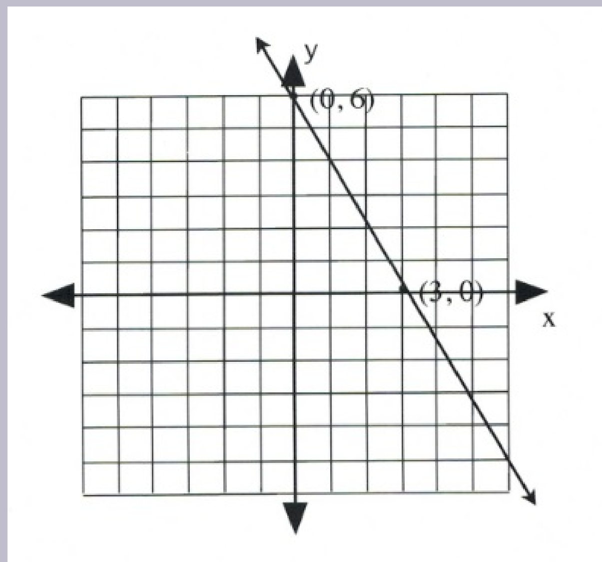
Now choose $y = 0$.

This yields:

$$\begin{array}{rcrcl} 2x & + & 0 & = & 6 \\ & & 2x & = & 6 \\ & & x & = & \frac{6}{2} \text{ or } 3 \end{array}$$

Coordinate is $(3, 0)$.

Draw these coordinates on the graph and draw a line through them.



Example 3.4.5

Graph the equation $x + 2y = 4$.

To find the first coordinate, choose $x = 0$.

This yields:

$$\begin{array}{rcrcl} (0) & + & 2y & = & 4 \\ & & y & = & \frac{4}{2} \text{ or } 2 \end{array}$$

Coordinate is $(0, 2)$.

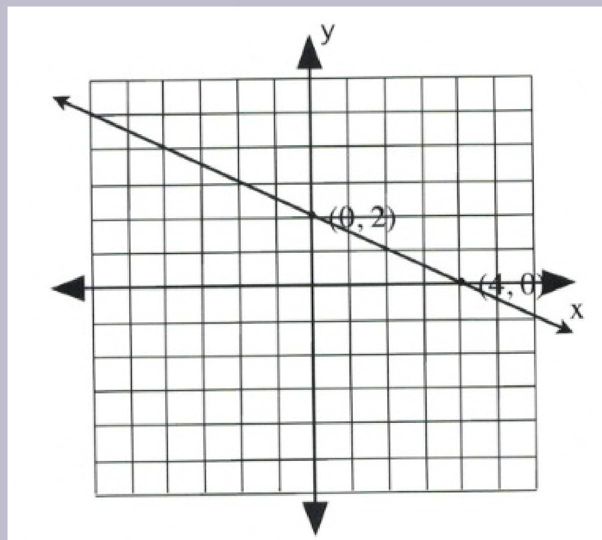
Now choose $y = 0$.

This yields:

$$\begin{array}{rcrcl} x & + & 2(0) & = & 4 \\ & & x & = & 4 \end{array}$$

Coordinate is $(4, 0)$.

Draw these coordinates on the graph and draw a line through them.



Example 3.4.6

Graph the equation $2x + y = 0$.

To find the first coordinate, choose $x = 0$.

This yields:

$$\begin{array}{rcrcr} 2(0) & + & y & = & 0 \\ & & y & = & 0 \end{array}$$

Coordinate is $(0, 0)$.

Since the intercept is $(0, 0)$, finding the other intercept yields the same coordinate. In this case, choose any value of convenience.

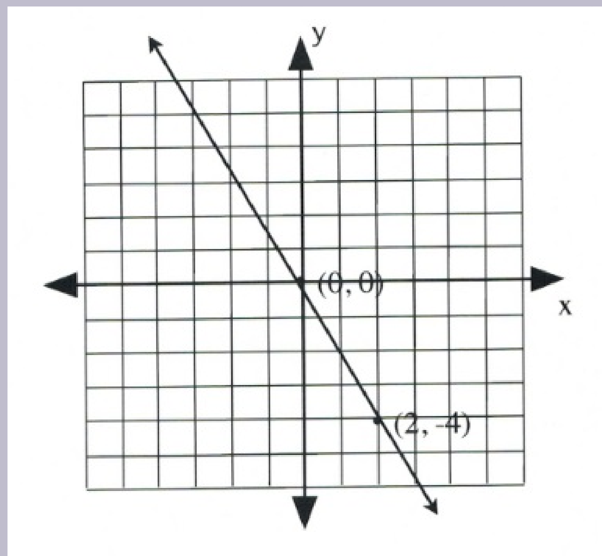
Choose $x = 2$.

This yields:

$$\begin{array}{rcrcr} 2(2) & + & y & = & 0 \\ 4 & + & y & = & 0 \\ -4 & & & & -4 \\ & & y & = & -4 \end{array}$$

Coordinate is $(2, -4)$.

Draw these coordinates on the graph and draw a line through them.



Questions

For questions 1 to 10, sketch each linear equation using the slope-intercept method.

1. $y = -\frac{1}{4}x - 3$
2. $y = \frac{3}{2}x - 1$
3. $y = -\frac{5}{4}x - 4$
4. $y = -\frac{3}{5}x + 1$
5. $y = -\frac{4}{3}x + 2$
6. $y = \frac{5}{3}x + 4$
7. $y = \frac{3}{2}x - 5$
8. $y = -\frac{2}{3}x - 2$
9. $y = -\frac{4}{5}x - 3$

10. $y = \frac{1}{2}x$

For questions 11 to 20, sketch each linear equation using the x- and y-intercepts.

11. $x + 4y = -4$
12. $2x - y = 2$
13. $2x + y = 4$
14. $3x + 4y = 12$
15. $2x - y = 2$
16. $4x + 3y = -12$
17. $x + y = -5$
18. $3x + 2y = 6$
19. $x - y = -2$
20. $4x - y = -4$

For questions 21 to 28, sketch each linear equation using any method.

21. $y = -\frac{1}{2}x + 3$
22. $y = 2x - 1$
23. $y = -\frac{5}{4}x$
24. $y = -3x + 2$
25. $y = -\frac{3}{2}x + 1$
26. $y = \frac{1}{3}x - 3$
27. $y = \frac{3}{2}x + 2$
28. $y = 2x - 2$

For questions 29 to 40, reduce and sketch each linear equation using any method.

29. $y + 3 = -\frac{4}{5}x + 3$
30. $y - 4 = \frac{1}{2}x$
31. $x + 5y = -3 + 2y$
32. $3x - y = 4 + x - 2y$
33. $4x + 3y = 5(x + y)$
34. $3x + 4y = 12 - 2y$
35. $2x - y = 2 - y \text{ (tricky)}$
36. $7x + 3y = 2(2x + 2y) + 6$
37. $x + y = -2x + 3$
38. $3x + 4y = 3y + 6$
39. $2(x + y) = -3(x + y) + 5$
40. $9x - y = 4x + 5$

Answer Key 3.4

20. 3.5 Constructing Linear Equations

Quite often, students are required to find the equation of a line given only a data point and a slope or two data points. The simpler of these problems is to find the equation when given a slope and a data point. To do this, use the equation that defines the slope of a line:

$$m=\frac{y_2-y_1}{x_2-x_1},\text{ where }y_2\text{ is replaced by }y\text{ and }x_2\text{ is replaced by }x$$

This becomes:

$$m=\frac{y-y_1}{x-x_1},\text{ where }x_1\text{ and }y_1\text{ are replaced by the coordinates and }m\text{ by the given slope}$$

To illustrate this method, consider the following example.

Example 3.5.1

Find the equation that has slope $m=2$ and passes through the point $(2,5)$.

First, replace m with 2 and x_1,y_1 with $(2,5)$.

$$m=\frac{y_2-y_1}{x_2-x_1}\text{ becomes }2=\frac{y-5}{x-2}$$

Next, reduce the resulting equation.

First, multiply both sides by the denominator to eliminate the fraction:

$$(x-2)2=\frac{y-5}{x-2}(x-2)$$

This leaves:

$$(x-2)2=y-5$$

Which simplifies to:

$$2x-4=y-5$$

This can be written in the y-intercept form by isolating the variable y:

$$\begin{array}{ccccccc}2x&-&4&=&y&-&5\\&+&5&&&+&5\\2x&+&1&=&y&&\end{array}$$

$$\text{or}$$

$$\begin{array}{ccccc}y&=&2x&+&1\end{array}$$

It is also useful to write the equation in the general form of $Ax+By+C=0$, where A, B, and C are integers and A is positive.

In general form, $2x-4=y-5$ becomes:

$$\begin{array}{rrrrrrrrr} 2x & - & 4 & & = & y & - & 5 \\ & - & y & + & 5 & -y & + & 5 \\ 2x & - & y & + & 1 & = & 0 \end{array}$$

The standard form of a linear equation is written as $Ax + By = C$.

In standard form, $2x - 4 = y - 5$ becomes:

$$\begin{array}{rrrrrrr} 2x & - & 4 & = & y & - & 5 \\ -y & + & 4 & & -y & + & 4 \\ 2x & - & y & = & -1 \end{array}$$

The three common forms that a linear equation can be written in are:

$$\begin{aligned} \text{Slope-intercept form:} \quad & y = mx + b \\ \text{General form:} \quad & Ax + By + C = 0 \\ \text{Standard form:} \quad & Ax + By = C \end{aligned}$$

Example 3.5.2

Find the equation having slope $m = -\dfrac{1}{2}$ that passes though the point $(1, -4)$.

Write the solutions in slope-intercept form and in both general and standard forms.

First, replace m with $-\dfrac{1}{2}$ and x_1, y_1 with $(1, -4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ becomes } -\frac{1}{2} = \frac{y - -4}{x - 1}$$

Multiplying both sides by $2(x - 1)$ to eliminate the denominators yields:

$$-1(x - 1) = 2(y + 4)$$

Which simplifies to:

$$-x + 1 = 2y + 8$$

Writing this solution in all three forms looks like:

$$\begin{aligned}\text{Slope-intercept form:}\quad & y = -\frac{1}{2}x - 7\\ \text{General form:}\quad & x + 2y + 7 = 0\\ \text{Standard form:}\quad & x + 2y = -7\end{aligned}$$

The more difficult variant of this type of problem is that in which the equation of a line that connects two data points must be found. However, this is simpler than it may seem.

The first step is to find the slope of the line that would connect those two points. Use the slope equation, as has been done previously in this textbook. After this is done, use this slope and one of the two data points given at the beginning of the problem. The following example illustrates this.

Example 3.5.3

Find the equation of the line that runs through $(-1, -2)$ and $(3, 8)$.

First, find the slope:

$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1}\\ m &= \frac{8 - -2}{3 - -1} \text{ or } \frac{8+2}{3+1}\\ m &= \frac{10}{4} \text{ or } \frac{5}{2}\end{aligned}$$

Now, treat this as a problem of finding a line with a given slope m running through a point (x, y).

The slope is $\frac{5}{2}$, and there are two points to choose from: $(-1, -2)$ and $(3, 8)$. Choose the simplest point to work with. For this problem, either point works. For this example, choose $(3, 8)$.

$$\begin{aligned} m &= \frac{y-y_1}{x-x_1} \\ \frac{5}{2} &= \frac{y-8}{x-3} \end{aligned}$$

Eliminate the fraction by multiplying both sides by $2(x-3)$.

This leaves $5(x-3)=2(y-8)$, which must be simplified:

$$\begin{array}{rcrcrcr} 5(x & - & 3) & = & 2(y & - & 8) \\ 5x & - & 15 & = & 2y & - & 16 \\ -2y & + & 15 & & -2y & + & 15 \\ 5x & - & 2y & = & -1 & & \end{array}$$

This answer is in standard form, but it can easily be converted to the y-intercept form or general form if desired.

Questions

For questions 1 to 12, write the slope-intercept form of each linear equation using the given point and slope.

1. $(2,3)$ and $m=\frac{2}{3}$
2. $(1,2)$ and $m=4$
3. $(2,2)$ and $m=\frac{1}{2}$
4. $(2,1)$ and $m=-\frac{1}{2}$
5. $(-1,-5)$ and $m=9$
6. $(2,-2)$ and $m=-2$
7. $(-4,1)$ and $m=\frac{3}{4}$
8. $(4,-3)$ and $m=-2$
9. $(0,-2)$ and $m=-3$
10. $(-1,1)$ and $m=4$
11. $(0,-5)$ and $m=-\frac{1}{4}$
12. $(0,2)$ and $m=-\frac{5}{4}$

For questions 13 to 24, write the general form of each linear equation using the given point and slope.

13. $(-1,-5)$ and $m=2$
14. $(2,-2)$ and $m=-2$
15. $(5,-1)$ and $m=-\frac{3}{5}$
16. $(-2,-2)$ and $m=-\frac{2}{3}$
17. $(-4,1)$ and $m=\frac{1}{2}$
18. $(4,-3)$ and $m=-\frac{7}{4}$
19. $(4,-2)$ and $m=-\frac{3}{2}$
20. $(-2,0)$ and $m=-\frac{5}{2}$
21. $(-5,-3)$ and $m=-\frac{2}{5}$
22. $(3,3)$ and $m=\frac{7}{3}$
23. $(2,-2)$ and $m=1$
24. $(-3,4)$ and $m=-\frac{1}{3}$

For questions 25 and 32, write the slope-intercept form of each linear equation using the given points.

25. $(-4,3)$ and $(-3,1)$
26. $(1,3)$ and $(-3,-3)$
27. $(5,1)$ and $(-3,0)$
28. $(-4,5)$ and $(4,4)$
29. $(-4,-2)$ and $(0,4)$
30. $(-4,1)$ and $(4,4)$
31. $(3,5)$ and $(-5,3)$
32. $(-1,-4)$ and $(-5,0)$

For questions 33 to 40, write the general form of each linear equation using the given points.

33. $(3,-3)$ and $(-4,5)$
34. $(-1,-5)$ and $(-5,-4)$
35. $(3,-3)$ and $(-2,4)$
36. $(-6,-7)$ and $(-3,-4)$
37. $(-5,1)$ and $(-1,-2)$
38. $(-5,-1)$ and $(5,-2)$
39. $(-5,5)$ and $(2,-3)$
40. $(1,-1)$ and $(-5,-4)$

Answer Key 3.5

21. 3.6 Perpendicular and Parallel Lines

Perpendicular, parallel, horizontal, and vertical lines are special lines that have properties unique to each type. Parallel lines, for instance, have the same slope, whereas perpendicular lines are the opposite and have negative reciprocal slopes. Vertical lines have a constant x-value, and horizontal lines have a constant y-value.

Two equations govern perpendicular and parallel lines:

For parallel lines, the slope of the first line is the same as the slope for the second line. If the slopes of these two lines are called m_1 and m_2, then $m_1 = m_2$.

$$\text{The rule for parallel lines is } m_1 = m_2$$

Perpendicular lines are slightly more difficult to understand. If one line is rising, then the other must be falling, so both lines have slopes going in opposite directions. Thus, the slopes will always be negative to one another. The other feature is that the slope at which one is rising or falling will be exactly flipped for the other one. This means that the slopes will always be negative reciprocals to each other. If the slopes of these two lines are called m_1 and m_2, then $m_1 = \dfrac{-1}{m_2}$.

$$\text{The rule for perpendicular lines is } m_1 = \frac{-1}{m_2}$$

Example 3.6.1

Find the slopes of the lines that are parallel and perpendicular to $y = 3x + 5$.

The parallel line has the identical slope, so its slope is also 3.

The perpendicular line has the negative reciprocal to the other slope, so it is $-\dfrac{1}{3}$.

Example 3.6.2

Find the slopes of the lines that are parallel and perpendicular to $y = -\dfrac{2}{3}x - 4$.

The parallel line has the identical slope, so its slope is also $-\dfrac{2}{3}$.

The perpendicular line has the negative reciprocal to the other slope, so it is $\dfrac{3}{2}$.

Typically, questions that are asked of students in this topic are written in the form of “Find the equation of a line passing through point (x, y) that is perpendicular/parallel to $y = mx + b$.” The first step is to identify the slope that is to be used to solve this equation, and the second is to use the described methods to arrive at the solution like previously done. For instance:

Example 3.6.3

Find the equation of the line passing through the point $(2, 4)$ that is parallel to the line $y = 2x - 3.$

The first step is to identify the slope, which here is the same as in the given equation, $m = 2.$

Now, simply use the methods from before:

$$\begin{array}{rcl} m & = & \dfrac{y - y_1}{x - x_1} \\ 2 & = & \dfrac{y - 4}{x - 2} \end{array}$$

Clearing the fraction by multiplying both sides by $(x - 2)$ leaves:

$$2(x - 2) = y - 4 \text{ or } 2x - 4 = y - 4$$

Now put this equation in one of the three forms. For this example, use the standard form:

$$\begin{array}{rrrrrrr} 2x & - & 4 & = & y & - & 4 \\ -y & + & 4 & & -y & + & 4 \\ 2x & - & y & = & 0 & & \end{array}$$

Example 3.6.4

Find the equation of the line passing through the point $(1, 3)$ that is perpendicular to the line $y = \dfrac{3}{2}x + 4.$

The first step is to identify the slope, which here is the negative reciprocal to the one in the given equation, so $m = -\dfrac{2}{3}.$

Now, simply use the methods from before:

$$\begin{aligned} m &= \frac{y - y_1}{x - x_1} \\ -\frac{2}{3} &= \frac{y-3}{x-1} \end{aligned}$$

First, clear the fraction by multiplying both sides by $3(x-1)$. This leaves:

$$-2(x-1) = 3(y-3)$$

which reduces to:

$$-2x + 2 = 3y - 9$$

Now put this equation in one of the three forms. For this example, choose the general form:

$$\begin{array}{rrrrrrrrr} -2x & & & + & 2 & = & 3y & - & 9 \\ & & -3y & + & 9 & & -3y & + & 9 \\ -2x & - & 3y & + & 11 & = & 0 & & \end{array}$$

For the general form, the coefficient in front of the x must be positive. So for this equation, multiply the entire equation by −1 to make $-2x$ positive.

$$\begin{gathered} (-2x - 3y + 11 = 0)(-1) \\ 2x + 3y - 11 = 0 \end{gathered}$$

Questions that are looking for the vertical or horizontal line through a given point are the easiest to do and the most commonly confused.

Vertical lines always have a single x-value, yielding an equation like $x = \text{constant}$.

Horizontal lines always have a single y-value, yielding an equation like $y = \text{constant}$.

Example 3.6.5

Find the equation of the vertical and horizontal lines through the point $(-2, 4)$.

The vertical line has the same x-value, so the equation is $x = -2$.

The horizontal line has the same y-value, so the equation is $y = 4$.

Questions

For questions 1 to 6, find the slope of any line that would be parallel to each given line.

1. $y = 2x + 4$
2. $y = -\frac{2}{3}x + 5$
3. $y = 4x - 5$
4. $y = -10x - 5$
5. $x - y = 4$
6. $6x - 5y = 20$

For questions 7 to 12, find the slope of any line that would be perpendicular to each given line.

7. $y = \frac{1}{3}x$
8. $y = -\frac{1}{2}x - 1$
9. $y = -\frac{1}{3}x$
10. $y = \frac{4}{5}x$
11. $x - 3y = -6$
12. $3x - y = -3$

For questions 13 to 18, write the slope-intercept form of the equation of each line using the given point and line.

13. (1, 4) and parallel to $y = \frac{2}{5}x + 2$
14. (5, 2) and perpendicular to $y = \frac{1}{3}x + 4$
15. (3, 4) and parallel to $y = \frac{1}{2}x - 5$
16. (1, −1) and perpendicular to $y = -\frac{3}{4}x + 3$
17. (2, 3) and parallel to $y = -\frac{3}{5}x + 4$
18. (−1, 3) and perpendicular to $y = -3x - 1$

For questions 19 to 24, write the general form of the equation of each line using the given point and line.

19. (1, −5) and parallel to $-x + y = 1$
20. (1, −2) and perpendicular to $-x + 2y = 2$
21. (5, 2) and parallel to $5x + y = -3$
22. (1, 3) and perpendicular to $-x + y = 1$
23. (4, 2) and parallel to $-4x + y = 0$
24. (3, −5) and perpendicular to $3x + 7y = 0$

For questions 25 to 36, write the equation of either the horizontal or the vertical line that runs through each point.

25. Horizontal line through $(4, -3)$
26. Vertical line through $(-5, 2)$
27. Vertical line through $(-3,1)$
28. Horizontal line through $(-4, 0)$
29. Horizontal line through $(-4, -1)$
30. Vertical line through $(2, 3)$
31. Vertical line through $(-2, -1)$
32. Horizontal line through $(-5, -4)$
33. Horizontal line through $(4, 3)$
34. Vertical line through $(-3, -5)$
35. Vertical line through $(5, 2)$
36. Horizontal line through $(5, -1)$

Answer Key 3.6

22. 3.7 Numeric Word Problems

Number-based word problems can be very confusing, and it takes practice to convert a word-based sentence into a mathematical equation. The best strategy to solve these problems is to identify keywords that can be pulled out of a sentence and use them to set up an algebraic equation.

Variables that are to be solved for are often written as "a number," "an unknown," or "a value."

"Equal" is generally represented by the words "is," "was," "will be," or "are."

Addition is often stated as "more than," "the sum of," "added to," "increased by," "plus," "all," or "total." Addition statements are quite often written backwards. An example of this is "three more than an unknown number," which is written as $x+3$.

Subtraction is often written as "less than," "minus," "decreased by," "reduced by," "subtracted from," or "the difference of." Subtraction statements are quite often written backwards. An example of this is "three less than an unknown number," which is written as $x-3$.

Multiplication can be seen in written problems with the words "times," "the product of," or "multiplied by."

Division is generally found by a statement such as "divided by," "the quotient of," or "per."

Example 3.7.1

28 less than five times a certain number is 232. What is the number?

- **28 less** means that it is subtracted from the unknown number (write this as −28)
- **five times an unknown number** is written as $5x$
- **is 232** means it equals 232 (write this as = 232)

Putting these pieces together and solving gives:

$$\begin{array}{rrrcr} 5x & - & 28 & = & 232 \\ & + & 28 & & +28 \\ & & 5x & = & 260 \\ \\ & & x & = & \dfrac{260}{5} \quad \text{or } 52 \end{array}$$

Example 3.7.2

Fifteen more than three times a number is the same as nine less than six times the number. What is the number?

- **Fifteen more than three times a number** is $3x + 15$ or $15 + 3x$
- **is** means =
- **nine less than six times the number** is $6x - 9$

Putting these parts together gives:

$$\begin{array}{rrrrrrr} 3x & + & 15 & = & 6x & - & 9 \\ -6x & - & 15 & = & -6x & - & 15 \\ & & -3x & = & -24 & & \\ & & & & & & \\ & & x & = & \dfrac{-24}{-3} & \text{or } 8 & \end{array}$$

Another type of number problem involves consecutive integers, consecutive odd integers, or consecutive even integers. Consecutive integers are numbers that come one after the other, such as 3, 4, 5, 6, 7. The equation that relates consecutive integers is:

$$x, x+1, x+2, x+3, x+4$$

Consecutive odd integers and consecutive even integers both share the same equation, since every second number must be skipped to remain either odd (such as 3, 5, 7, 9) or even (2, 4, 6, 8). The equation that is used to represent consecutive odd or even integers is:

$$x, x+2, x+4, x+6, x+8$$

Example 3.7.3

The sum of three consecutive integers is 93. What are the integers?

The relationships described in equation form are as follows:

$$x + x + 1 + x + 2 = 93$$

Which reduces to:

$$\begin{array}{rrrrr} 3x & + & 3 & = & 93 \\ & - & 3 & & -3 \\ & & 3x & = & 90 \\ \\ & & x & = & \dfrac{90}{3} \quad \text{or } 30 \end{array}$$

This means that the three consecutive integers are 30, 31, and 32.

Example 3.7.4

The sum of three consecutive even integers is 246. What are the integers?

The relationships described in equation form are as follows:

$$x + x + 2 + x + 4 = 246$$

Which reduces to:

$$\begin{array}{rrrrr} 3x & + & 6 & = & 246 \\ & - & 6 & & -6 \\ & & 3x & = & 240 \\ \\ & & x & = & \dfrac{240}{3} \quad \text{or } 80 \end{array}$$

This means that the three consecutive even integers are 80, 82, and 84.

Questions

For questions 1 to 8, write the formula defining each relationship. **Do not solve.**

1. Five more than twice an unknown number is 25.
2. Twelve more than 4 times an unknown number is 36.
3. Three times an unknown number decreased by 8 is 22.

4. Six times an unknown number less 8 is 22.
5. When an unknown number is decreased by 8, the difference is half the unknown number.
6. When an unknown number is decreased by 4, the difference is half the unknown number.
7. The sum of three consecutive integers is 21.
8. The sum of the first two of three odd consecutive integers, less the third, is 5.

For questions 9 to 16, write and solve the equation describing each relationship.

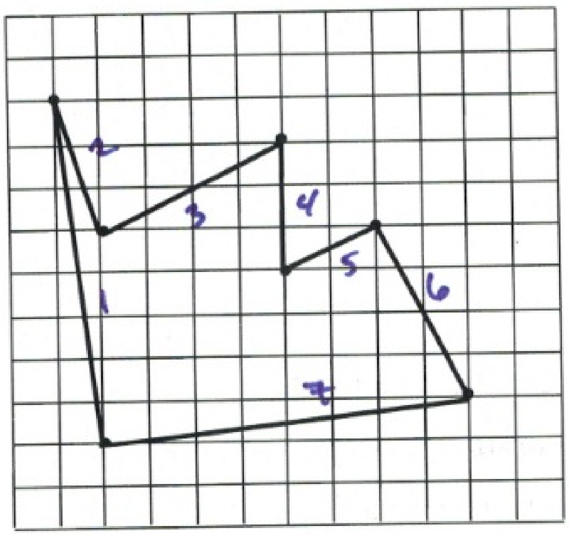
9. When five is added to three times a certain number, the result is 17. What is the number?
10. If five is subtracted from three times a certain number, the result is 10. What is the number?
11. Sixty more than nine times a number is the same as two less than ten times the number. What is the number?
12. Eleven less than seven times a number is five more than six times the number. Find the number.
13. The sum of three consecutive integers is 108. What are the integers?
14. The sum of three consecutive integers is −126. What are the integers?
15. Find three consecutive integers such that the sum of the first, twice the second, and three times the third is −76.
16. Find three consecutive odd integers such that the sum of the first, two times the second, and three times the third is 70.

Answer Key 3.7

23. 3.8 The Newspaper Delivery Puzzle

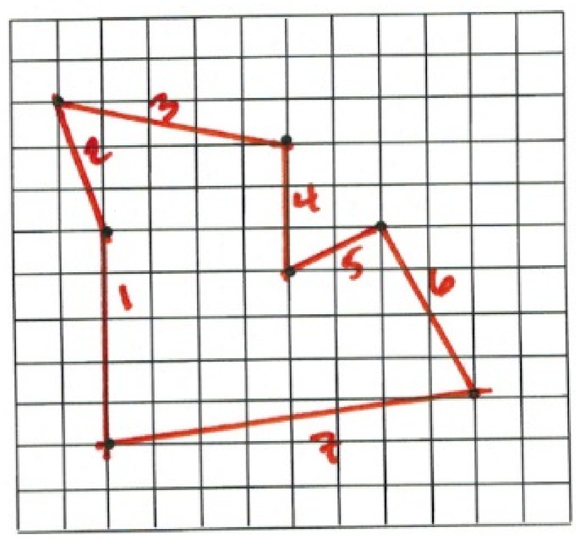
A newspaper delivery driver wants to visit of number of specific stores in a city to drop off bundles of newspapers and return to the starting position by taking the shortest distance. Since there are a number of stores and choices of what path is best to take, what is the shortest distance you can find?

One possible path is shown below. Can you find the distance covered in this path if each square represents 1 km?



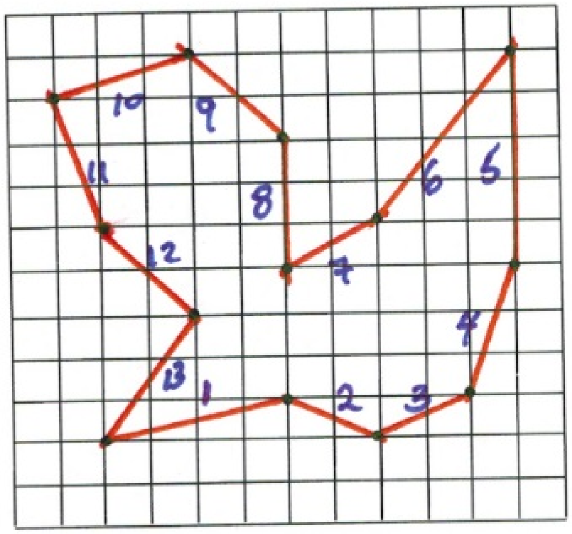
$$
\begin{aligned}
d_1 &= \sqrt{8^2+1^2} &&= 8.062\\
d_2 &= \sqrt{3^2+1^2} &&= 3.612\\
d_3 &= \sqrt{4^2+2^2} &&= 4.472\\
d_4 &= 3 &&= 3\\
d_5 &= \sqrt{1^2+2^2} &&= 2.236\\
d_6 &= \sqrt{4^2+2^2} &&= 4.472\\
d_7 &= \sqrt{8^2+1^2} &&= 8.062\\
& && \quad 33.916 \text{ km}
\end{aligned}
$$

Can you find a shorter path? How many km of driving would you save?



$$
\begin{aligned}
d_1 &= 5 &&= 5\\
d_2 &= \sqrt{3^2+1^2} &&= 3.612\\
d_3 &= \sqrt{5^2+1^2} &&= 5.099\\
d_4 &= 3 &&= 3\\
d_5 &= \sqrt{1^2+2^2} &&= 2.236\\
d_6 &= \sqrt{4^2+2^2} &&= 4.472\\
d_7 &= \sqrt{8^2+1^2} &&= 8.062\\
& && \quad 31.481 \text{ km}
\end{aligned}
$$

These problems can rapidly become more difficult as the number of stops are increased. An example of this is as follows. What is the shortest distance that you can find for this puzzle?



$$
\begin{array}{lclcl}
d_1 & = & \sqrt{4^2+1^2} & = & 4.123 \\
d_2 & = & \sqrt{1^2+2^2} & = & 2.236 \\
d_3 & = & \sqrt{1^2+2^2} & = & 2.236 \\
d_4 & = & \sqrt{1^2+3^2} & = & 3.162 \\
d_5 & = & 5 & = & 5 \\
d_6 & = & \sqrt{3^2+4^2} & = & 5 \\
d_7 & = & \sqrt{2^2+1^2} & = & 2.236 \\
d_8 & = & 3 & = & 3 \\
d_9 & = & \sqrt{2^2+2^2} & = & 2.828 \\
d_{10} & = & \sqrt{1^2+3^2} & = & 3.612 \\
d_{11} & = & \sqrt{1^2+3^2} & = & 3.612 \\
d_{12} & = & \sqrt{2^2+2^2} & = & 2.828 \\
d_{13} & = & \sqrt{2^2+3^2} & = & 3.606 \\
 & & & & 43.929 \text{ km}
\end{array}
$$

Why UPS Drivers Don’t Turn Left and You Probably Shouldn’t Either

The company UPS has analyzed delivery routs thoroughly. Read this article by Graham Kendall[1] to find out how this affects the routes their drivers take.

Why UPS drivers don’t turn left and you probably shouldn’t either

1. Kendall, Graham. Why UPS Drivers Don't Turn Right and Why You Probably Shouldn't Either. https://theconversation.com/why-ups-drivers-dont-turn-left-and-you-probably-shouldnt-either-71432

PART IV

CHAPTER 4: INEQUALITIES

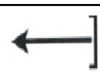
Learning Objectives

This chapter covers:

- Solving and Graphing Linear Equations
- Compound Inequalities
- Linear Absolute Value Inequalities
- 2D Inequality & Absolute Value Graphs
- Geometric Word Problems

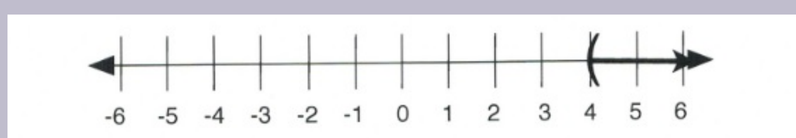
24. 4.1 Solve and Graph Linear Inequalities

When given an equation, such as $x = 4$ or $x = -5$, there are specific values for the variable. However, with inequalities, there is a range of values for the variable rather than a defined value. To write the inequality, use the following notation and symbols:

Symbol	**Meaning**
	> Greater than
	≤ Greater than or equal to
	< Less than
	≥ Less than or equal to

Example 4.1.1

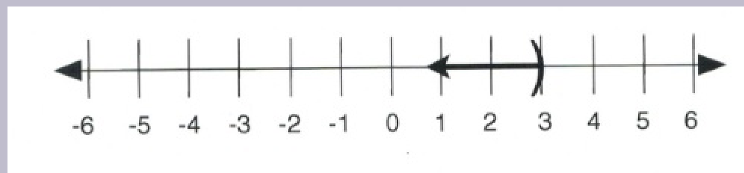
Given a variable x such that $x > 4$, this means that x can be as close to 4 as possible but always larger. For $x > 4$, x can equal 5, 6, 7, 199. Even $x =$ 4.000000000000001 is true, since x is larger than 4, so all of these are solutions to the inequality. The line graph of this inequality is shown below:



Written in interval notation, $x > 4$ is shown as $(4, \infty)$.

Example 4.1.2

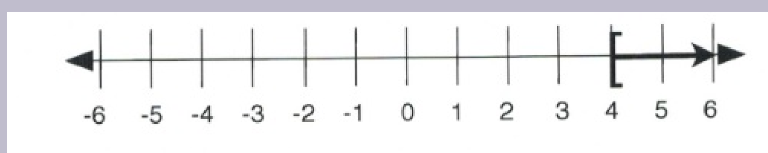
Likewise, if $x < 3$, then x can be any value less than 3, such as 2, 1, −102, even 2.99999999999. The line graph of this inequality is shown below:



Written in interval notation, $x < 3$ is shown as $(-\infty, 3)$.

Example 4.1.3

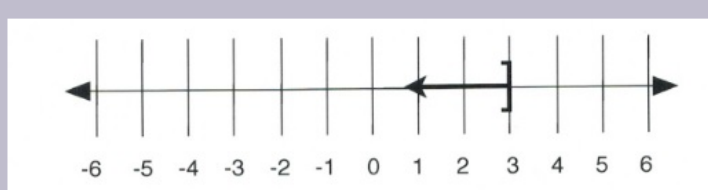
For greater than or equal (≥) and less than or equal (≤), the inequality starts at a defined number and then grows larger or smaller. For $x \geq 4$, x can equal 5, 6, 7, 199, or 4. The line graph of this inequality is shown below:



Written in interval notation, $x \geq 4$ is shown as $[4, \infty)$.

Example 4.1.4

If $x \leq 3$, then x can be any value less than or equal to 3, such as 2, 1, −102, or 3. The line graph of this inequality is shown below:



Written in interval notation, $x \leq 3$ is shown as $(-\infty, 3]$.

When solving inequalities, the direction of the inequality sign (called the sense) can flip over. The sense will flip under two conditions:

First, the sense flips when the inequality is divided or multiplied by a negative. For instance, in reducing $-3x < 12$, it is necessary to divide both sides by −3. This leaves $x > -4.$

Second, the sense will flip over if the entire equation is flipped over. For instance, $x > 2$, when flipped over, would look like $2 < x.$ In both cases, the 2 must be shown to be smaller than the x, or the x is always greater than 2, no matter which side each term is on.

Example 4.1.5

Solve the inequality $5 - 2x > 11$ and show the solution on both a number line and in interval notation.

First, subtract 5 from both sides:

$$\begin{array}{rrrrr} 5 & - & 2x & \geq & 11 \\ -5 & & & & -5 \\ & & -2x & \geq & 6 \end{array}$$

Divide both sides by −2:

$$\dfrac{-2x}{-2} \geq \dfrac{6}{-2}$$

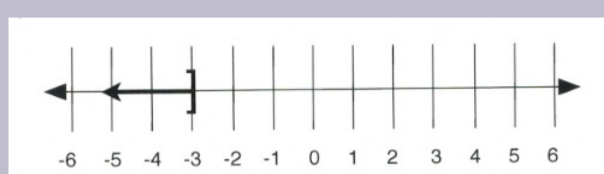
Since the inequality is divided by a negative, it is necessary to flip the direction of the sense.

This leaves:

$$x \leq -3$$

In interval notation, the solution is written as $(-\infty, -3]$.

On a number line, the solution looks like:



Inequalities can get as complex as the linear equations previously solved in this textbook. All the same patterns for solving inequalities are used for solving linear equations.

Example 4.1.6

Solve and give interval notation of $3(2x-4)+4x<4(3x-7)+8$.

Multiply out the parentheses:

$$6x-12+4x<12x-28+8$$

Simplify both sides:

$$10x-12<12x-20$$

Combine like terms:

$$\begin{array}{rrrrrr} 10x & - & 12 & < & 12x & - & 20 \\ -12x & + & 12 & & -12x & + & 12 \\ & & -2x & < & -8 & & \end{array}$$

The last thing to do is to isolate x from the −2. This is done by dividing both sides by −2. Because both sides are divided by a negative, the direction of the sense must be flipped.

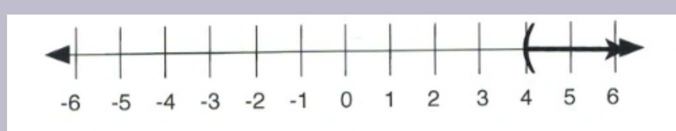
This means:

$$\frac{-2x}{-2}<\frac{-8}{-2}$$

Will end up looking like:

$$x>4$$

The solution written on a number line is:



Written in interval notation, $x>4$ is shown as $(4,\infty)$.

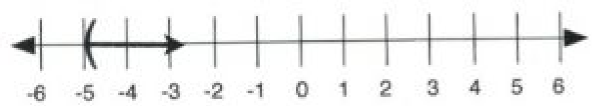
Questions

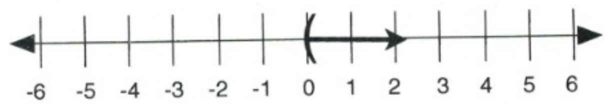
For questions 1 to 6, draw a graph for each inequality and give its interval notation.

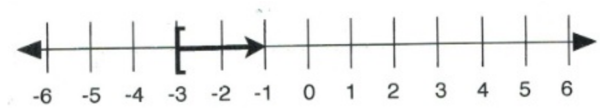
1. $n>-5$
2. $n>4$
3. $-2\leq k$

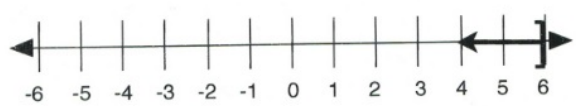
4. $1 \geq k$
5. $5 \geq x$
6. $-5 < x$

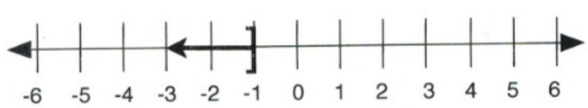
For questions 7 to 12, write the inequality represented on each number line and give its interval notation.

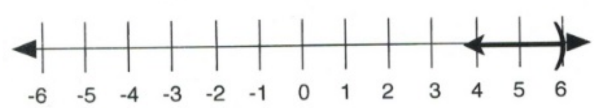
7. 

8. 

9. 

10. 

11. 

12. 

For questions 13 to 38, draw a graph for each inequality and give its interval notation.

13. $\dfrac{x}{11} \geq 10$
14. $-2 \leq \dfrac{n}{13}$
15. $2 + r < 3$
16. $\dfrac{m}{5} \leq -\dfrac{6}{5}$
17. $8 + \dfrac{n}{3} \geq 6$
18. $11 > 8 + \dfrac{x}{2}$
19. $2 > \dfrac{(a-2)}{5}$
20. $\dfrac{(v-9)}{-4} \leq 2$
21. $-47 \geq 8 - 5x$

22. $\dfrac{(6+x)}{12} \leq -1$
23. $-2(3+k) < -44$
24. $-7n - 10 \geq 60$
25. $18 < -2(-8+p)$
26. $5 \geq \dfrac{x}{5} + 1$
27. $24 \geq -6(m-6)$
28. $-8(n-5) \geq 0$
29. $-r - 5(r-6) < -18$
30. $-60 \geq -4(-6x-3)$
31. $24 + 4b < 4(1+6b)$
32. $-8(2-2n) \geq -16 + n$
33. $-5v - 5 < -5(4v+1)$
34. $-36 + 6x > -8(x+2) + 4x$
35. $4 + 2(a+5) < -2(-a-4)$
36. $3(n+3) + 7(8-8n) < 5n + 5 + 2$
37. $-(k-2) > -k - 20$
38. $-(4-5p) + 3 \geq -2(8-5p)$

Answer Key 4.1

25. 4.2 Compound Inequalities

Several inequalities can be combined together to form what are called compound inequalities.

The first type of compound inequality is the "or" inequality, which is true when either inequality results in a true statement. When graphing this type of inequality, one useful trick is to graph each individual inequality above the number line before moving them both down together onto the actual number line.

When giving interval notation for a solution, if there are two different parts to the graph, put a $\cup$ (union) symbol between two sets of interval notation, one for each part.

Example 4.2.1

Solve the inequality $2x - 5 > 3 \text{ or } 4 - x > 6$. Graph the solution and write it in interval notation.

Isolate the variables from the numbers:

$$\begin{array}{rrrrrrrrrr} 2x & - & 5 & > & 3 & \text{or} & 4 & - & x & > & 6 \\ & + & 5 & & +5 & & -4 & & & & -4 \\ & & 2x & > & 8 & \text{or} & & & -x & > & 2 \end{array}$$

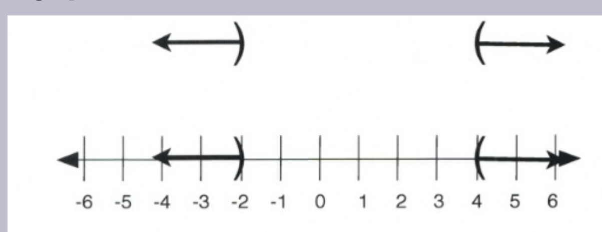
Isolate the variable x (remember to flip the sense where necessary):

$$\frac{2x}{2} > \frac{8}{2} \quad \text{or} \quad \frac{-x}{-1} > \frac{2}{-1}$$

Solution:

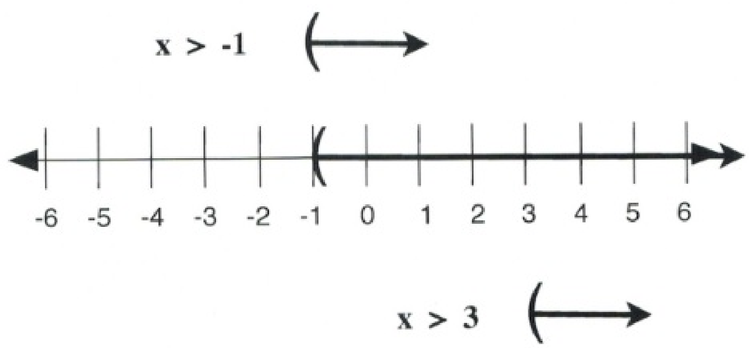
$$x > 4 \text{ or } x < -2$$

Position the inequalities and graph:



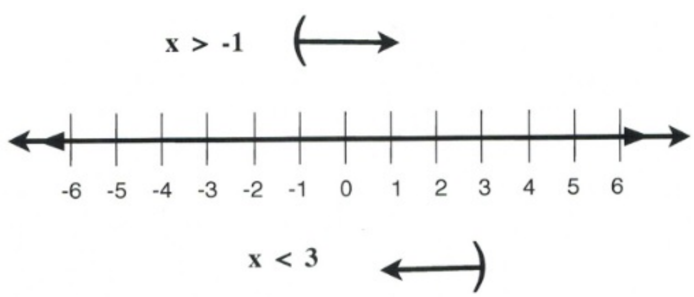
In interval notation, the solution is written as $(-\infty, -2) \cup (4, \infty)$.

Note: there are several possible results that result from an "or" statement. The graphs could be pointing different directions, as in the graph above, or pointing in the same direction, as in the graph representing $x > -1 \text{ or } x > 3$ that is shown below.



In interval notation, this solution is written as $(-1, \infty)$.

It is also possible to have solutions that point in opposite directions but are overlapping, as shown by the solutions and graph below.



In interval notation, this solution is written as $(-\infty, \infty)$, or simply $x \in \mathbb{R}$, since the graph is all possible numbers.

The second type of compound inequality is the "and" inequality. "And" inequalities require both inequality statements to be true. If one part is false, the whole inequality is false. When graphing these inequalities, follow a similar process as before, sketching both solutions for both inequalities above the number line. However, this time, it is only the overlapping portion that is drawn onto the number line. When a solution for an "and" compound inequality is given in interval notation, it will be expressed in a manner very similar to single inequalities. The symbol that can be used for "and" is the intersection symbol, ∩.

Example 4.2.2

Solve the compound inequality $2x + 8 \geq 5x - 7 \text{ and } 5x - 3 > 3x + 1$. Graph the solution and express it in interval notation.

Move all variables to the right side and all numbers to the left:

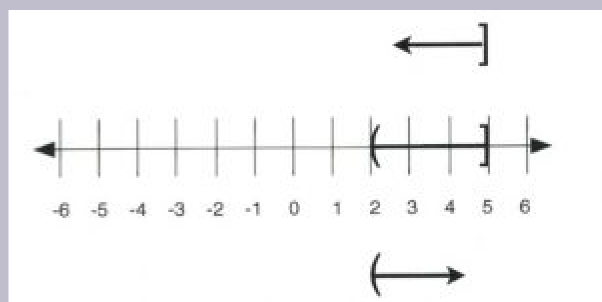
$$\begin{array}{rrrcrrrcrrrcrrr}
2x & + & 8 & \geq & 5x & - & 7 & \text{and} & 5x & - & 3 & > & 3x & + & 1 \\
-5x & - & 8 & & -5x & - & 8 & & -3x & + & 3 & & -3x & + & 3 \\
& & -3x & \geq & -15 & & & \text{and} & & & 2x & > & 4 & &
\end{array}$$

Isolate the variable x for both (flip the sense for the negative):

$$\frac{-3x}{-3} \geq \frac{-15}{-3} \quad \text{and} \quad \frac{2x}{2} > \frac{4}{2}$$

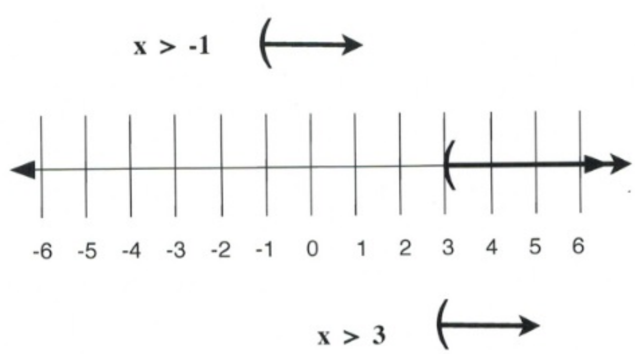
Solution:

$$x \leq 5 \text{ and } x > 2$$



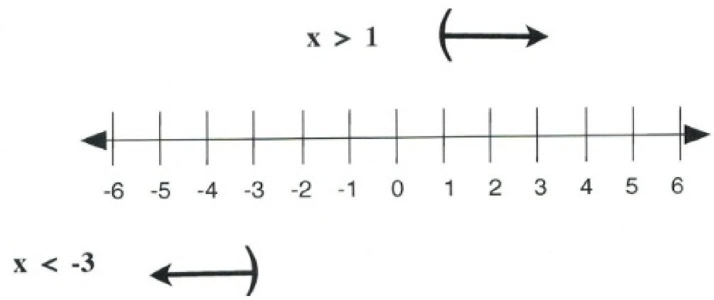
In interval notation, this solution is written as $(2, 5]$.

Note: there are several different results that could result from an "and" statement. The graphs could be pointing towards each other as in the graph above, or pointing in the same direction, as in the graph representing $x > -1$ and $x > 3$ (shown below). In this case, the solution must be true for both inequalities, which make a combined graph of:



In interval notation, this solution is written as $(3, \infty)$.

It is also possible to have solutions that point in opposite directions but do not overlap, as shown by the solutions and graph below. Since there is no overlap, there is no real solution.



In interval notation, this solution is written as no solution, $x = \{\}$ or $x = \emptyset$.

The third type of compound inequality is a special type of "and" inequality. When the variable (or expression containing the variable) is between two numbers, write it as a single math sentence with three parts, such as $5 < x \leq 8,$ to show x is greater than 5 and less than or equal to 8. To stay balanced when solving this type of inequality, because there are three parts to work with, it is necessary to perform the same operation on all three parts. The graph, then, is of the values between the benchmark numbers with appropriate brackets on the ends.

Example 4.2.3

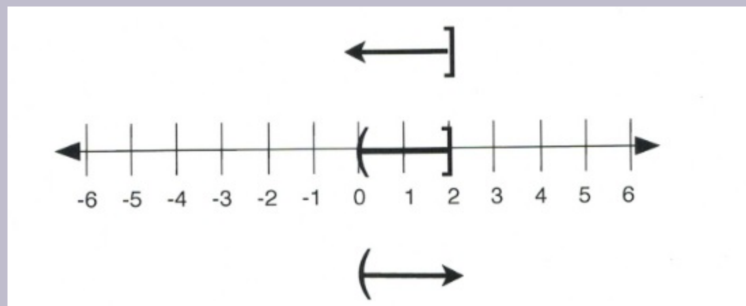
Solve the inequality $-6 \leq -4x + 2 < 2.$ Graph the solution and write it in interval notation.

Isolate the variable $-4x$ by subtracting 2 from all three parts:

$$\begin{array}{rrrrrrr} -6 & \leq & -4x & + & 2 & < & 2 \\ -2 & & & - & 2 & & -2 \\ -8 & \leq & & -4x & & < & 0 \end{array}$$

Isolate the variable x by dividing all three parts by −4 (remember to flip the sense):

$$\begin{array}{ccccc} \dfrac{-8}{-4} & \leq & \dfrac{-4x}{-4} & < & \dfrac{0}{-4} \\ \\ 2 & \geq & x & > & 0 \end{array}$$



In interval notation, this is written as $(0, 2]$.

Questions

For questions 1 to 32, solve each compound inequality, graph its solution, and write it in interval notation.

1. $\dfrac{n}{3} < 3 \text{ or } -5n < -10$
2. $6m \geq -24 \text{ or } m - 7 < -12$
3. $x + 7 \geq 12 \text{ or } 9x < -45$
4. $10r > 0 \text{ or } r - 5 < -12$
5. $x - 6 < -13 \text{ or } 6x < -60$
6. $9 + n < 2 \text{ or } 5n > 40$
7. $\dfrac{v}{8} > -1 \text{ and } v - 2 < 1$
8. $-9x < 63 \text{ and } \dfrac{x}{4} < 1$
9. $-8 + b < -3 \text{ and } 4b < 20$
10. $-6n < 12 \text{ and } \dfrac{n}{3} < 2$
11. $a + 10 \geq 3 \text{ and } 8a < 48$
12. $-6 + v \geq 0 \text{ and } 2v > 4$
13. $3 < 9 + x < 7$
14. $0 \geq \dfrac{x}{9} \geq -1$
15. $11 < 8 + k < 12$
16. $-11 < n - 9 < -5$
17. $-3 < x - 1 < 1$
18. $-1 < \dfrac{p}{8} < 0$
19. $-4 < 8 - 3m < 11$
20. $3 + 7r > 59 \text{ or } -6r - 3 > 33$
21. $-16 < 2n - 10 < -2$
22. $-6 - 8x \geq -6 \text{ or } 2 + 10x > 82$

23. $-5b+10<30$ and $7b+2<-40$
24. $n+10\geq 15$ or $4n-5<-1$
25. $3x-9<2x+10$ and $5+7x<10x-10$
26. $4n+8<3n-6$ or $10n-8\geq 9+9n$
27. $-8-6v<8-8v$ and $7v+9<6+10v$
28. $5-2a\geq 2a+1$ or $10a-10\geq 9a+9$
29. $1+5k\geq 7k-3$ or $k-10>2k+10$
30. $8-10r<8+4r$ or $-6+8r<2+8r$
31. $2x+9\geq 10x+1$ and $3x-2<7x+2$
32. $-9m+2<-10-6m$ or $-m+5\geq 10+4m$

Answer Key 4.2

26. 4.3 Linear Absolute Value Inequalities

Absolute values are positive magnitudes, which means that they represent the positive value of any number.

For instance, | −5 | and | +5 | are the same, with both having the same value of 5, and | −99 | and | +99 | both share the same value of 99.

When used in inequalities, absolute values become a boundary limit to a number.

Example 4.3.1

Consider $|x| < 4.$

This means that the unknown x value is less than 4, so $|x| < 4$ becomes $x < 4.$ However, there is more to this with regards to negative values for $x.$

| −1 | is a value that is a solution, since 1 < 4.

However, | −5 | < 4 is not a solution, since 5 > 4.

The boundary of $|x| < 4$ works out to be between −4 and +4.

This means that $|x| < 4$ ends up being bounded as $-4 < x < 4.$

If the inequality is written as $|x| \leq 4,$ then little changes, except that x can then equal −4 and +4, rather than having to be larger or smaller.

This means that $|x| \leq 4$ ends up being bounded as $-4 \leq x \leq 4.$

Example 4.3.2

Consider $|x| > 4.$

This means that the unknown x value is greater than 4, so $|x| > 4$ becomes $x > 4.$ However, the negative values for x must still be considered.

The boundary of $|x| > 4$ works out to be smaller than −4 and larger than +4.

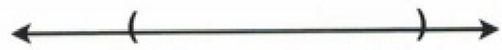
This means that $|x| > 4$ ends up being bounded as $x < -4 \text{ or } 4 < x.$

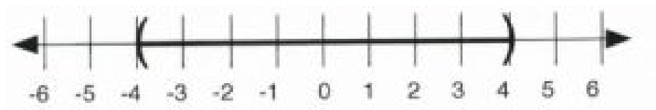
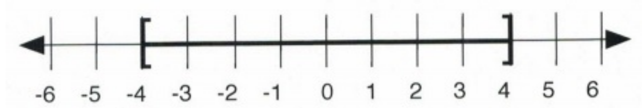
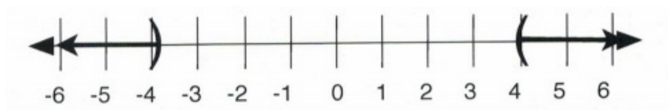
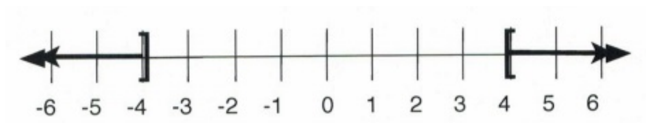
If the inequality is written as $|x| \geq 4,$ then little changes, except that x can then equal −4 and +4, rather than having to be larger or smaller.

This means that $|x| \geq 4$ ends up being bounded as $x \leq -4 \text{ or } 4 \leq x.$

When drawing the boundaries for inequalities on a number line graph, use the following conventions:

For ≤ or ≥, use [brackets] as boundary limits. 

For < or >, use (parentheses) as boundary limits. 

Equation	**Number Line**
$\lvert x\rvert < 4$	
$\lvert x\rvert \le 4$	
$\lvert x\rvert > 4$	
$\lvert x\rvert \ge 4$	

When an inequality has an absolute value, isolate the absolute value first in order to graph a solution and/or write it in interval notation. The following examples will illustrate isolating and solving an inequality with an absolute value.

Example 4.3.3

Solve, graph, and give interval notation for the inequality $-4 - 3|x| \geq -16.$

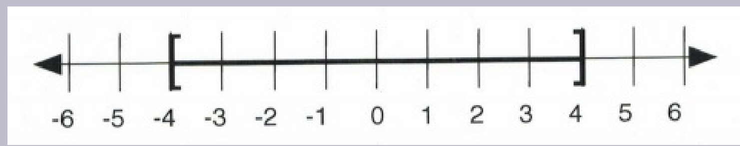
First, isolate the inequality:

$$\begin{array}{rrrcrl} -4 & - & 3|x| & \geq & -16 & \\ +4 & & & & +4 & \text{add 4 to both sides} \\ & & \dfrac{-3|x|}{-3} & \geq & \dfrac{-12}{-3} & \text{divide by } -3 \text{ and flip the sense} \\ & & & & & \\ & & |x| & \leq & 4 & \end{array}$$

At this point, it is known that the inequality is bounded by 4. Specifically, it is between −4 and 4.

This means that $-4 \leq |x| \leq 4.$

This solution on a number line looks like:



To write the solution in interval notation, use the symbols and numbers on the number line: $[-4, 4]$.

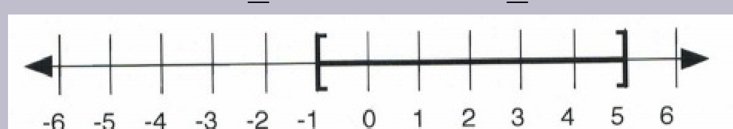
Other examples of absolute value inequalities result in an algebraic expression that is bounded by an inequality.

Example 4.3.4

Solve, graph, and give interval notation for the inequality $|2x - 4| \leq 6$.

This means that the inequality to solve is $-6 \leq 2x - 4 \leq 6$:

$$\begin{array}{rcrcrcr} -6 & \leq & 2x & - & 4 & \leq & 6 \\ +4 & & & + & 4 & & +4 \\ \dfrac{-2}{2} & \leq & & \dfrac{2x}{2} & & \leq & \dfrac{10}{2} \\ \\ -1 & \leq & & x & & \leq & 5 \end{array}$$



To write the solution in interval notation, use the symbols and numbers on the number line: $[-1, 5]$.

Example 4.3.5

Solve, graph, and give interval notation for the inequality $9 - 2|4x + 1| > 3$.

First, isolate the inequality by subtracting 9 from both sides:

$$\begin{array}{rrrrrrr} 9 & - & 2|4x & + & 1| & > & 3 \\ -9 & & & & & & -9 \\ & & -2|4x & + & 1| & > & -6 \end{array}$$

Divide both sides by −2 and flip the sense:

$$\frac{-2|4x+1|}{-2} > \frac{-6}{-2}$$

$$|4x+1| < 3$$

At this point, it is known that the inequality expression is between −3 and 3, so $-3 < 4x + 1 < 3.$

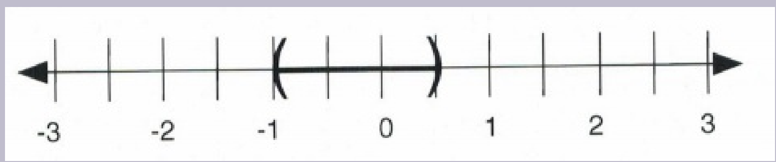
All that is left is to isolate x. First, subtract 1 from all three parts:

$$\begin{array}{rrrrrrr} -3 & < & 4x & + & 1 & < & 3 \\ -1 & & & - & 1 & & -1 \\ -4 & < & & 4x & & < & 2 \end{array}$$

Then, divide all three parts by 4:

$$\frac{-4}{4} < \frac{4x}{4} < \frac{2}{4}$$

$$-1 < x < \frac{1}{2}$$



In interval notation, this is written as $\left(-1, \dfrac{1}{2}\right)$.

It is important to remember when solving these equations that the absolute value is always positive. If given an absolute value that is less than a negative number, there will be no solution because absolute value will always be positive, i.e., greater than a negative. Similarly, if absolute value is greater than a negative, the answer will be all real numbers.

This means that:

$$|2x-4| < -6 \text{ has no possible solution } (x \neq \mathbb{R})$$

and

$$|2x-4| > -6 \text{ has every number as a solution and is written as } (-\infty, \infty)$$

Note: since infinity can never be reached, use parentheses instead of brackets when writing infinity (positive or negative) in interval notation.

Questions

For questions 1 to 33, solve each inequality, graph its solution, and give interval notation.

1. $|x| < 3$
2. $|x| \leq 8$
3. $|2x| < 6$
4. $|x+3| < 4$
5. $|x-2| < 6$
6. $|x-8| < 12$
7. $|x-7| < 3$
8. $|x+3| \leq 4$
9. $|3x-2| < 9$
10. $|2x+5| < 9$
11. $1+2|x-1| \leq 9$
12. $10-3|x-2| \geq 4$
13. $6-|2x-5| > 3$
14. $|x| > 5$
15. $|3x| > 5$
16. $|x-4| > 5$
17. $|x+3| > 3$
18. $|2x-4| > 6$
19. $|x-5| > 3$
20. $3-|2-x| < 1$
21. $4+3|x-1| < 10$
22. $3-2|3x-1| \geq -7$
23. $3-2|x-5| \leq -15$
24. $4-6|-6-3x| \leq -5$
25. $-2-3|4-2x| \geq -8$
26. $-3-2|4x-5| \geq 1$
27. $4-5|-2x-7| < -1$
28. $-2+3|5-x| \leq 4$
29. $3-2|4x-5| \geq 1$
30. $-2-3|-3x-5| \geq -5$
31. $-5-2|3x-6| < -8$
32. $6-3|1-4x| < -3$
33. $4-4|-2x+6| > -4$

Answer Key 4.3

27. 4.4 2D Inequality and Absolute Value Graphs

Graphing a 2D Inequality

To graph an inequality, borrow the strategy used to draw a line graph in 2D. To do this, replace the inequality with an equal sign.

Example 4.4.1

Consider the following inequalities:

$$\begin{array}{rcrcr} 3x & + & 2y & < & 12 \\ 3x & + & 2y & \leq & 12 \\ 3x & + & 2y & > & 12 \\ 3x & + & 2y & \geq & 12 \end{array}$$

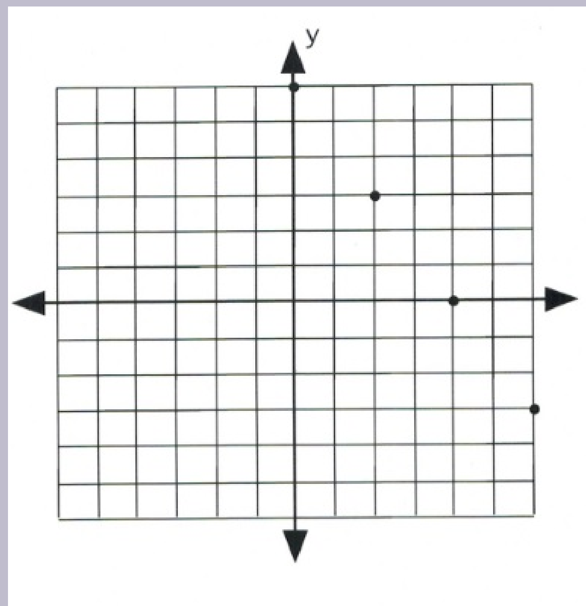
All can be changed to $3x + 2y = 12$ by replacing the inequality sign with =.

It is then possible to create a data table using the new equation.

Create a data table of values for the equation $3x + 2y = 12$.

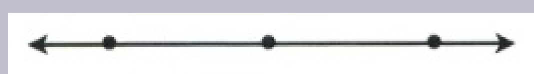
x	y
0	6
2	3
4	0
6	−3

Using these values, plot the data points on a graph.



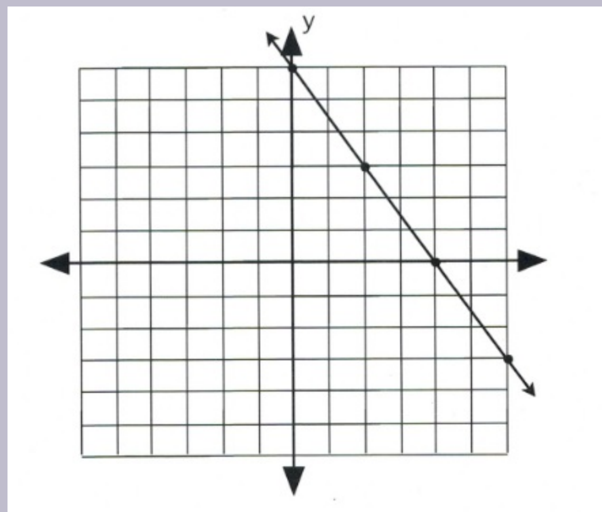
Once the data points are plotted, draw a line that connects them all. The type of line drawn depends on the original inequality that was replaced.

If the inequality had ≤ or ≥, then draw a solid line to represent data points that are on the line.

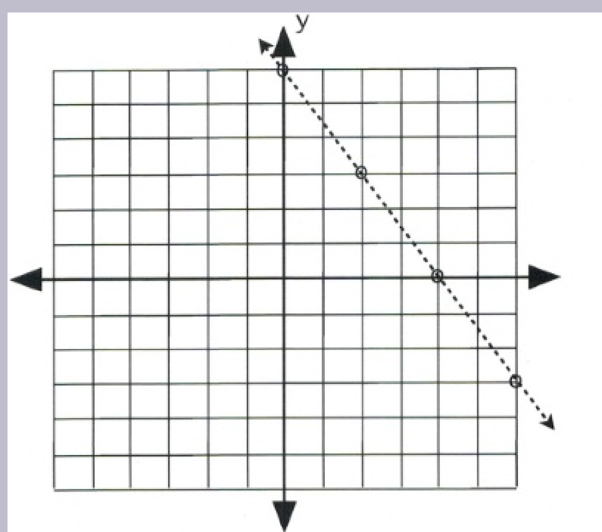


If the inequality had < or >, then draw a dashed line instead to indicate that some data points are excluded.

If the inequality is either $3x + 2y \leq 12$ or $3x + 2y \geq 12$, then draw its graph using a solid line and solid dots.

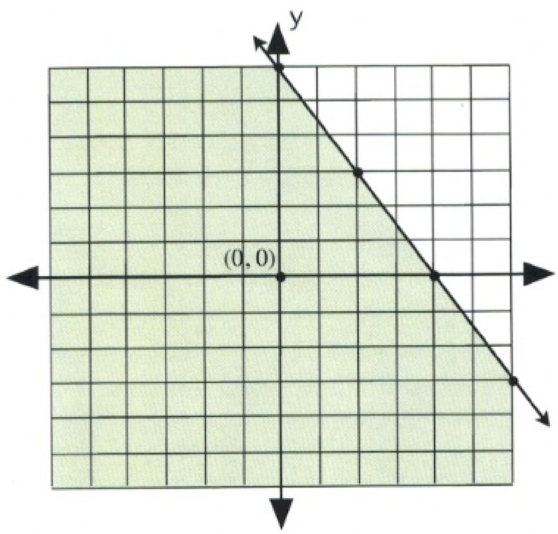


If the inequality is either $3x + 2y < 12$ or $3x + 2y > 12$, then draw its graph using a dashed line and hollow dots.



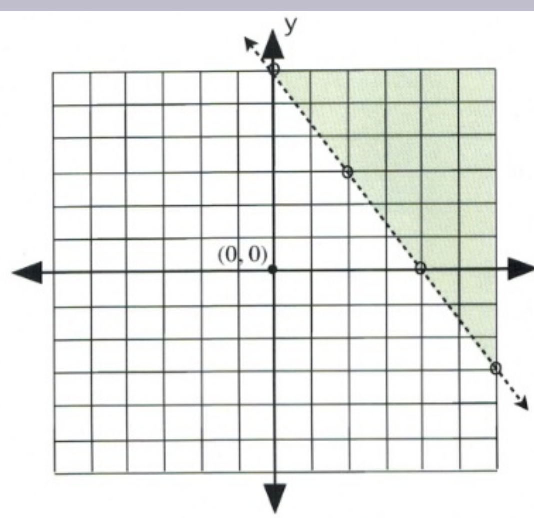
There remains only one step to complete this graph: finding which side of the line makes the inequality true and shading it. The easiest way to do this is to choose the data point $(0, 0)$.

It is evident that, for $3(0) + 2(0) \leq 12$ and $3(0) + 2(0) < 12$, the data point $(0, 0)$ is true for the inequality. In this case, shade the side of the line that contains the data point $(0, 0)$.



Completed Graph of 3x + 2y ≤ 12

It is also clear that, for $3(0) + 2(0) \geq 12$ and $3(0) + 2(0) > 12$, the data point $(0, 0)$ is false for the inequality. In this case, shade the side of the line that does not contain the data point $(0, 0)$.



Completed Graph of 3x + 2y > 12

Graphing an Absolute Value Function

To graph an absolute value function, first create a data table using the absolute value part of the equation.

The data point that is started with is the one that makes the absolute value equal to 0 (this is the x-value of the vertex). Calculating the value of this point is quite simple.

For example, for $|x-3|$, the value $x=3$ makes the absolute value equal to 0.

Examples of others are:

$$\begin{array}{rrrrrrrr} |x & + & 2| & = & 0 & \text{when} & x & = & -2 \\ |x & - & 11| & = & 0 & \text{when} & x & = & 11 \\ |x & + & 9| & = & 0 & \text{when} & x & = & -9 \end{array}$$

The graph of an absolute value equation will be a V-shape that opens upward for any positive absolute function and opens downward for any negative absolute value function.

Example 4.4.2

Plot the graph of $y=|x+2|-3$.

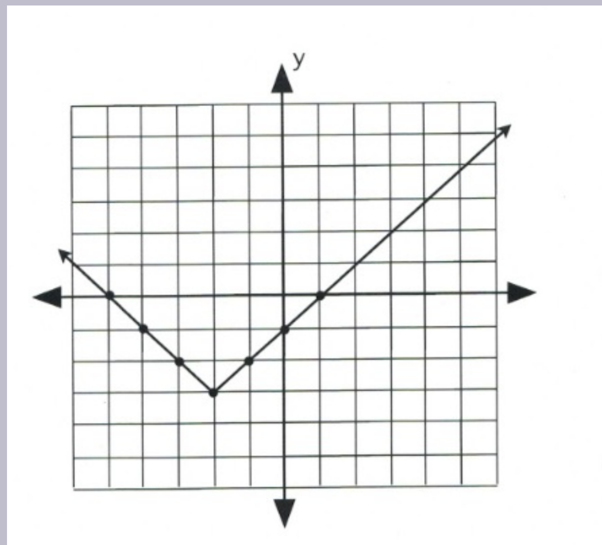
The data point that gives the x-value of the vertex is $|x+2|=0$, in which $x=-2$. This is the first value.

For $x=-2, y=|-2+2|-3$, which yields $y=-3$.

Now pick x-values that are larger and less than −2 to get three data points on both sides of the vertex, $(-2,-3)$.

x	y
1	0
0	−1
−1	−2
−2	−3
−3	−2
−4	−1
−5	0

Once there are three data points on either side of the vertex, plot and connect them in a line. The graph is complete.



Example 4.4.3

Plot the graph of $y = -|x-2|+1.$

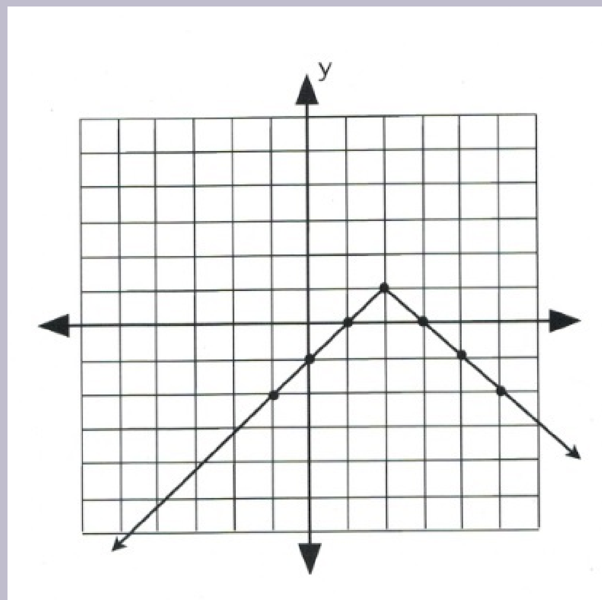
The data point that gives the x-value of the vertex is $|x-2|=0,$ in which $x=2.$ This is the first value.

For $x=2, y=-|2-2|+1,$ which yields $y=1.$

Now pick x-values that are larger and less than 2 to get three data points on both sides of the vertex, $(2,1).$

x	y
5	−2
4	−1
3	0
2	1
1	0
0	−1
−1	−2

Once there are three data points on either side of the vertex, plot and connect them in a line. The graph is complete.



Questions

For questions 1 to 8, graph each linear inequality.

1. $y > 3x + 2$
2. $3x - 4y > 12$
3. $2y \geq 3x + 6$
4. $3x - 2y \geq 6$
5. $2y > 5x + 10$
6. $5x + 4y > -20$
7. $4y \geq 5x + 20$
8. $5x + 2y \geq -10$

For questions 9 to 16, graph each absolute value equation.

9. $y = |x - 4|$
10. $y = |x - 3| + 3$
11. $y = |x - 2|$
12. $y = |x - 2| + 2$
13. $y = -|x - 2|$
14. $y = -|x - 2| + 2$
15. $y = -|x + 2|$
16. $y = -|x + 2| + 2$

Answer Key 4.4

28. 4.5 Geometric Word Problems

It is common to run into geometry-based word problems that look at either the interior angles, perimeter, or area of shapes. When looking at interior angles, the sum of the angles of any polygon can be found by taking the number of sides, subtracting 2, and then multiplying the result by 180°. In other words:

$$\text{sum of interior angles} = 180^\circ \times (\text{number of sides} - 2)$$

This means the interior angles of a triangle add up to 180° × (3 − 2), or 180°. Any four-sided polygon will have interior angles adding to 180° × (4 − 2), or 360°. A chart can be made of these:

$$\begin{array}{rrcrcr}
3\text{ sides:} & 180^\circ & \times & (3-2) & = & 180^\circ \\
4\text{ sides:} & 180^\circ & \times & (4-2) & = & 360^\circ \\
5\text{ sides:} & 180^\circ & \times & (5-2) & = & 540^\circ \\
6\text{ sides:} & 180^\circ & \times & (6-2) & = & 720^\circ \\
7\text{ sides:} & 180^\circ & \times & (7-2) & = & 900^\circ \\
8\text{ sides:} & 180^\circ & \times & (8-2) & = & 1080^\circ
\end{array}$$

Example 4.5.1

The second angle (A_2) of a triangle is double the first (A_1). The third angle (A_3) is 40° less than the first (A_1). Find the three angles.

The relationships described in equation form are as follows:

$$A_2 = 2A_1 \text{ and } A_3 = A_1 - 40^\circ$$

Because the shape in question is a triangle, the interior angles add up to 180°. Therefore:

$$A_1 + A_2 + A_3 = 180^\circ$$

Which can be simplified using substitutions:

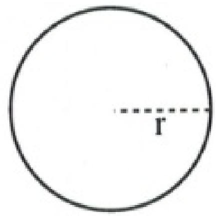
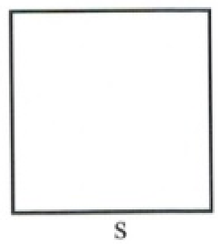
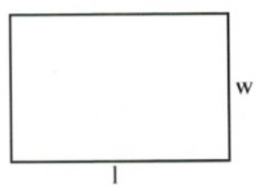
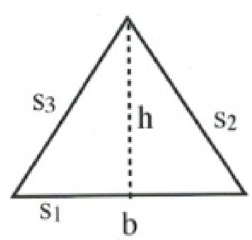
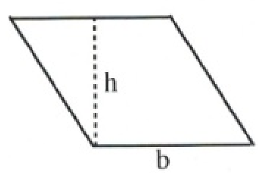
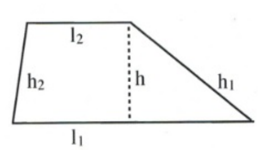
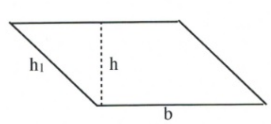
$$A_1 + (2A_1) + (A_1 - 40^\circ) = 180^\circ$$

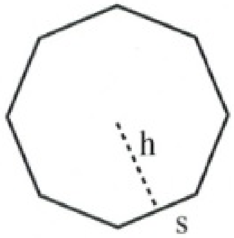
Which leaves:

$$
\begin{aligned}
2A_1 + A_1 + A_1 - 40^\circ &= 180^\circ \\
4A_1 - 40^\circ &= 180^\circ \\
4A_1 &= 180^\circ + 40^\circ \\
A_1 &= \frac{220^\circ}{4} \quad \text{or} \quad 55^\circ
\end{aligned}
$$

This means $A_2 = 2(55^\circ)$ or 110° and $A_3 = 55^\circ - 40^\circ$ or 15°.

Common Geometric Shapes with Related Area and Perimeter Equations

Shape	Picture	Area	Perimeter
Circle		πr^2	$2\pi r$
Square		s^2	$4s$
Rectangle		lw	$2l + 2w$
Triangle		$\frac{1}{2}bh$	$s_1 + s_2 + s_3$
Rhombus		bh	$4b$
Trapezoid		$\frac{1}{2}(l_1 + l_2)\,h$	$l_1 + l_2 + h_1 + h_2$
Parallelogram		bh	$2h_1 + 2b$

Regular polygon (n-gon)		$\left(\frac{1}{2}sh\right)$ (number of sides)	s(number of sides)

Another common geometry word problem involves perimeter, or the distance around an object. For example, consider a rectangle, for which $\text{perimeter} = 2l + 2w$.

Example 4.5.2

If the length of a rectangle is 5 m less than twice the width, and the perimeter is 44 m long, find its length and width.

The relationships described in equation form are as follows:

$$L = 2W - 5 \text{ and } P = 44$$

For a rectangle, the perimeter is defined by:

$$P = 2W + 2L$$

Substituting for L and the value for the perimeter yields:

$$44 = 2W + 2(2W - 5)$$

Which simplifies to:

$$44 = 2W + 4W - 10$$

Further simplify to find the length and width:

$$\begin{array}{rrrrl} 44 & + & 10 & = & 6W \\ & & 54 & = & 6W \\ & & W & = & \frac{54}{6} \quad \text{or} \quad 9 \\ \text{So} & & L & = & 2(9) \quad - \quad 5 \quad \text{or} \quad 13 \end{array}$$

The width is 9 m and the length is 13 m.

Other common geometric problems are:

Example 4.5.3

A 15 m cable is cut into two pieces such that the first piece is four times larger than the second. Find the length of each piece.

The relationships described in equation form are as follows:

$$P_1 + P_2 = 15 \text{ and } P_1 = 4P_2$$

Combining these yields:

$$\begin{array}{rrrl} 4P_2 & + & P_2 & = & 15 \\ & & 5P_2 & = & 15 \\ & & P_2 & = & \dfrac{15}{5} \quad \text{or} \quad 3 \end{array}$$

This means that $P_2 = 3$ m and $P_1 = 4(3)$, or 12 m.

Questions

For questions 1 to 8, write the formula defining each relation. **Do not solve.**

1. The length of a rectangle is 3 cm less than double the width, and the perimeter is 54 cm.
2. The length of a rectangle is 8 cm less than double its width, and the perimeter is 64 cm.
3. The length of a rectangle is 4 cm more than double its width, and the perimeter is 32 cm.
4. The first angle of a triangle is twice as large as the second and 10° larger than the third.
5. The first angle of a triangle is half as large as the second and 20° larger than the third.
6. The sum of the first and second angles of a triangle is half the amount of the third angle.
7. A 140 cm cable is cut into two pieces. The first piece is five times as long as the second.
8. A 48 m piece of hose is to be cut into two pieces such that the second piece is 5 m longer than the first.

For questions 9 to 18, write and solve the equation describing each relationship.

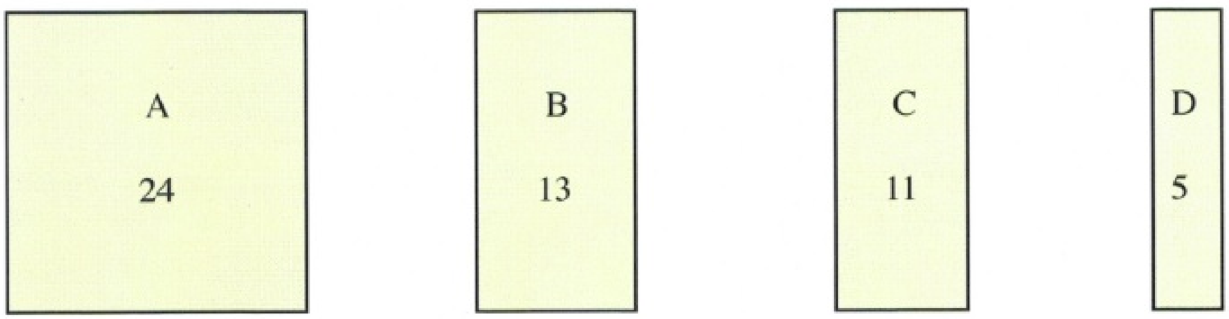
9. The second angle of a triangle is the same size as the first angle. The third angle is 12° larger than the first angle. How large are the angles?

10. Two angles of a triangle are the same size. The third angle is 12° smaller than the first angle. Find the measure of the angles.
11. Two angles of a triangle are the same size. The third angle is three times as large as the first. How large are the angles?
12. The second angle of a triangle is twice as large as the first. The measure of the third angle is 20° greater than the first. How large are the angles?
13. Find the dimensions of a rectangle if the perimeter is 150 cm and the length is 15 cm greater than the width.
14. If the perimeter of a rectangle is 304 cm and the length is 40 cm longer than the width, find the length and width.
15. Find the length and width of a rectangular garden if the perimeter is 152 m and the width is 22 m less than the length.
16. If the perimeter of a rectangle is 280 m and the width is 26 m less than the length, find its length and width.
17. A lab technician cuts a 12 cm piece of tubing into two pieces such that one piece is two times longer than the other. How long are the pieces?
18. An electrician cuts a 30 m piece of cable into two pieces. One piece is 2 m longer than the other. How long are the pieces?

Answer Key 4.5

29. 4.6 The Cook Oil Sharing Puzzle

Cally, Katy, and Jasnah buy a bulk container of 24 litres of cooking oil. The problem is that they intend to divide it evenly into 8-litre amounts in three different containers. The other two containers are 13 litres and 11 litres. They have a small 5 litre container that can be used. Katy comes up with a solution, and they all agree that dividing the oil will require seven careful steps. Can you find Katy's solution or find one that takes fewer steps?



	Action taken:	Each container holds:			
Step 1.	________	$A =$ __	$B =$ __	$C =$ __	$D =$ __
Step 2.	________	$A =$ __	$B =$ __	$C =$ __	$D =$ __
Step 3.	________	$A =$ __	$B =$ __	$C =$ __	$D =$ __
Step 4.	________	$A =$ __	$B =$ __	$C =$ __	$D =$ __
Step 5.	________	$A =$ __	$B =$ __	$C =$ __	$D =$ __
Step 6.	________	$A =$ __	$B =$ __	$C =$ __	$D =$ __
Step 7.	________	$A =$ __	$B =$ __	$C =$ __	$D =$ __

Answer Key 4.6

30. 4.7 Mathematics in Life: The Eiffel Tower

The Eiffel Tower was completed in 1889 by Gustave Eiffel as the entrance for the world's fair held in Paris the same year. It has since been visited by an estimated quarter billion people and remains the tallest structure in Paris with a height of 324 m.

Some of the particulars to this construction are that it is comprised of 18,038 pieces of metal held together by some 2.5 million rivets. There are 1,710 steps to the top of the tower, and the iron from which it was fashioned has a mass of 7300 tonnes and requires painting (60 tonnes) every seven years to prevent rusting. The top of the tower (the sun-facing side) can shift up to 18 cm due to thermal expansion. The density of the wrought iron used to create this tower is 7.70 g/cm^3 and was allegedly sourced from Reșița in Romania.

Question: If one were to dismantle the Eiffel Tower and melt it to create an iron sphere, what would be its radius?

First, the volume:

$$V = 7,300,000 \text{ kg} \times \frac{1 \text{ cm}^3}{7.70 \times 10^{-3} \text{ kg}}$$

$$V = 9.48 \times 10^8 \text{ cm}^3$$

Next, the radius:

$$V = \frac{4}{3}\pi r^3 \quad \text{or} \quad r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$= \sqrt[3]{\frac{3(9.48 \times 10^8 \text{ cm}^3)}{4\pi}}$$

$$= 609 \text{ cm}$$

$$= \sim 6.1 \text{ m}$$

PART V

MIDTERM 1 PREPARATION

This chapter contains several sections that will help you prepare for the midterm exam:

Midterm 1 Composition
Midterm 1 Review
Midterm Sample A
Midterm Sample B
Midterm Sample C
Midterm Sample D
Midterm Sample E

Midterm One: Composition

The first midterm will be composed of fifteen questions covering chapters 1 to 4.

Twelve questions will be algebra questions, and three questions will be word problems.

- **Questions 1–4 will be drawn from Chapter 1: Algebra Review and Chapter 2: Linear Equations**
    - 1.1 Integers
    - 1.2 Fractions
    - 1.3 Order of Operations
    - 1.4 Properties of Algebra
    - 1.5 Terms and Definitions
    - 2.1 Elementary Linear Equations
    - 2.2 Solving Linear Equations
    - 2.3 Intermediate Linear Equations
    - 2.4 Fractional Linear Equations
    - 2.5 Absolute Value Equations
    - 2.6 Working with Formulas
- **Questions 5–8 will be drawn from Chapter 3: Graphing**
    - 3.1 Points and Coordinates
    - 3.2 Midpoint and Distance Between Points
    - 3.3 Slopes and Their Graphs
    - 3.4 Graphing Linear Equations
    - 3.5 Constructing Linear Equations
    - 3.6 Perpendicular and Parallel Lines
- **Questions 9–12 will be drawn from Chapter 4: Inequalities**
    - 4.1 Solve and Graph Linear Inequalities
    - 4.2 Compound Inequalities
    - 4.3 Linear Absolute Value Inequalities
    - 4.4 2D Inequality and Absolute Value Graphs

- **Questions 13–15 will be drawn from:**
  - 1.6 Unit Conversion Word Problems
  - 2.7 Variation Word Problems
  - 3.7 Numeric Word Problems
  - 4.5 Geometric Word Problems

Students will be allowed to use MATQ 1099 Data Booklets for both Midterms and Final Exam.

Midterm One Review Questions

Questions 1–12 are for review only and will not show up on exams.

1. $\frac{0}{5} = 0$ (true or false)
2. $(-5+4) \div 0$
3. $-3(-5)$
4. $12 \div 3 \cdot 4$
5. $24 \div (6 \div 3)$
6. $5(2+3) - (2 \cdot 3)$
7. $3(3+4) = 3 \cdot 3 + 3 \cdot 4$ (true or false)
8. $2 \cdot \sqrt{9} \cdot (-3)$
9. $|21 + -3|$
10. $-(4)^2$
11. $(-4)^2$
12. -4^2

Questions similar to 13–20 will show up on exams.

13. Evaluate: $-b\sqrt{b^2 - 4ac}$ if $a = 8, b = -6$, and $c = -2$.

Chapter 2: Linear Equations (Exam Type Questions)

14. Solve for x in the equation $3(x-4) - 27 = 7 - 5(x+6)$.
15. Isolate the variable R in the following equation: $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$
16. Solve for x in the equation $\frac{x+3}{8} - \frac{3}{4} = \frac{x+6}{10}$

Chapter 3: Graphing (Exam Type Questions)

17. Find the equation of the horizontal line that passes through the point $(-2, 5)$.

18. Find the equation that has a slope of $\frac{2}{3}$ and passes through the point $(-1, 2)$.
19. Find the equation of the line passing through the points $(-2, -1)$ and $(2, 11)$.
20. Graph the relation $y = \frac{2}{3}x - 1$.
21. Find the distance between the data points $(7, -3)$ and $(15, 3)$.
22. Find the midpoint between the data points $(7, -3)$ and $(15, 3)$.

Chapter 4: Inequalities (Exam Type Questions)

Questions 23–26 are for review only and will not show up on exams.

23. 0 is a whole number (true or false)
24. The elements of the set of integers are elements of the set of rational numbers (true or false)
25. If $A = \{r, s\}, B = \{s, t\}$ and $C = \{r, t, w\}$, then $A \cup (B \cap C)$ equals what?
26. $-8 < 2 < 3$ (true or false)

Questions similar to 27–37 will show up on exams.

For questions 27 to 30, find each solution set and graph it.

27. $3x - 6(1 + 6x) > 60$
28. $-18 \leq 4x - 6 \leq 2$
29. $|2x - 1| > 7$
30. $\left|\frac{3x - 4}{5}\right| < 1$
31. $3x - 2y < 10$
32. Graph $y = |x - 1| - 2$.

Chapter 1–3 Word Problems (Exam Type Questions)

33. Find two numbers such that 6 times the larger number plus 2 times the smaller is 38, and 4 times the larger minus twice the smaller is 12.
34. Karl is going to cut a 36 cm cable into 2 pieces. If the first piece is to be 5 times as long as the second piece, find the length of each piece.
35. Kyra gave her brother Mark a logic question to solve: if she has 14 coins in her pocket worth $\$2.60$, and if the coins are only dimes and quarters, how many of each kind of coin does she have?
36. Find two consecutive even integers such that their sum is 10 less than the first integer.
37. y varies jointly with m and the square of n and inversely with d. If $y = 12$ when $m = 3, n = 4$, and $d = 8$, find the constant k, then use k to find y when $m = -3, n = 3$, and $d = 6$.

Midterm 1: Practice Questions Answer Key

31. Midterm 1: Version A

1. Evaluate: $-b-\sqrt{b^2-4ac}$ if $a=4, b=-3$, and $c=-1$.
2. Solve for x in the equation $2(x-5)-85=3-9(x+6)$.
3. Isolate the variable b in the equation $A=\dfrac{h}{B-b}$.
4. Solve for x in the equation $\dfrac{x+1}{4}-\dfrac{5}{8}=\dfrac{x-1}{8}$.
5. Write an equation of the vertical line that passes through the point $(-2,5)$.
6. Find the equation that has a slope of $\dfrac{2}{5}$ and passes through the point $(-1,-2)$.
7. Find the equation of the line passing through the points $(-2,0)$ and $(6,4)$.
8. Graph the relation $y=\dfrac{2}{3}x-1$.

For questions 9 to 11, find each solution set and graph it.

9. $6x-5(1+6x)>67$
10. $-10\leq 4x-2\leq 14$
11. $\left|\dfrac{3x+2}{5}\right|=2$
12. Graph the relation $5x+2y<15$.
13. Find two numbers such that 5 times the larger number plus 3 times the smaller is 47, and 4 times the larger minus twice the smaller is 20.
14. Karl is going to cut a 36 cm cable into 2 pieces. If the first piece is to be 5 times as long as the second piece, find the length of each piece.
15. y varies jointly with m and n and inversely with the square of d. If $y=3$ when $m=2, n=8$, and $d=4$, find the constant k, then use k to find y when $m=15, n=10$, and $d=5$.

Midterm 1: Version A Answer Key

32. Midterm 1: Version B

1. Evaluate: $-b-\sqrt{b^2-4ac}$ if $a=5, b=6,$ and $c=1.$
2. Solve for x in the equation $3(5x-6)=4\left[-3(2-x)\right].$
3. Isolate the variable b in the equation $A=\dfrac{h}{B\cdot b}.$
4. Solve for x in the equation $\dfrac{x+3}{5}-\dfrac{x}{2}=\dfrac{5-3x}{10}.$
5. Find the equation of the horizontal line that passes through the point $(-3,4).$
6. Find the equation that has a slope of $\dfrac{1}{3}$ and passes through the point $(-1,4).$
7. Find the equation of the line passing through the points $(0,4)$ and $(-3,5).$
8. Graph the relation $y=\dfrac{1}{3}x-2.$

For questions 9 to 11, find each solution set and graph it.

9. $6x-4(3-2x)>5(3-4x)+7$
10. $-3\leq 2x+3<9$
11. $\left|\dfrac{3x+2}{5}\right|<2$
12. Graph the relation $5x+2y<10.$
13. Find two consecutive even integers such that their sum is 16 less than five times the first integer.
14. Karl is going to cut a 40 cm cable into 2 pieces. If the first piece is to be 4 times as long as the second piece, find the length of each piece.
15. P varies directly as T and inversely as V. If $P=100$ when $T=200$ and $V=500,$ find the constant $k,$ then use this to find P when $T=100$ and $V=500.$

Midterm 1: Version B Answer Key

33. Midterm 1: Version C

1. Evaluate $-b - \sqrt{b^2 - 4ac}$ if $a = 4, b = 4,$ and $c = 1.$
2. Solve for x in the equation $2(x - 4) + 8 = 3 - 7(x + 3).$
3. Isolate the variable B in the equation $A = \dfrac{h}{B + b}.$
4. Solve for x in the equation $\dfrac{x}{15} - \dfrac{x - 3}{3} = \dfrac{1}{5}.$
5. Write the equation of the vertical line that passes through the point $(-2, -2).$
6. Find the equation that has a slope of $\dfrac{2}{3}$ and passes through the point $(0, -3).$
7. Find the equation of the line passing through the points $(14, -6)$ and $(-1, 4).$
8. Graph the relation $2x - y = -2.$

For questions 9 to 11, find each solution set and graph it.

9. $0 \leq 2x + 4 < 8$
10. $y - 1 > 3 \text{ or } y - 1 < -3$
11. $|2x - 3| > 5$
12. Graph the relation $y = |x| - 3.$
13. Find two numbers such that 5 times the larger number plus 3 times the smaller is 49, and 4 times the larger minus twice the smaller is 26.
14. Karl is going to cut a 42 cm cable into 2 pieces. If the first piece is to be 5 times as long as the second piece, find the length of each piece.
15. y varies jointly with m and inversely with the square of d. If $y = 3$ when $m = 2$ and $d = 4,$ find the constant k, then use k to find y when $m = 25$ and $d = 5.$

Midterm 1: Version C Answer Key

34. Midterm 1: Version D

1. Evaluate: $3b - \sqrt{b^2 - 4ac}$ if $a = 4, b = 4,$ and $c = 1.$
2. Solve for x in the equation $2(x - 4) + 8 = -6 + 3(x + 3).$
3. Isolate the variable r_2 in the equation $\dfrac{1}{R} = \dfrac{1}{r_1} + \dfrac{1}{r_2}.$
4. Solve for x in the equation $\dfrac{x}{15} - \dfrac{x-3}{3} = \dfrac{1}{3}.$
5. Write the equation of the horizontal line that passes through the point $(-2, 5).$
6. Find the equation that has a slope of $\dfrac{2}{3}$ and passes through the point $(-2, 4).$
7. Find the equation of the line passing through the points $(12, -7)$ and $(8, -9).$
8. Graph the relation $y = \dfrac{2}{3}x - 2.$

For questions 9 to 11, find each solution set and graph it.

9. $-27 \leq 6x - 9 \leq 3$
10. $\left|\dfrac{2x+2}{6}\right| = 2$
11. $|2x - 1| > 6$
12. Graph the relation $y = |2x| - 1.$
13. For a given triangle, the first and second angles are equal, but the third angle is 10° less than twice the first angle. What are the measures of the three angles?
14. Find two consecutive even integers such that their sum is 20 less than the first integer.
15. y varies jointly with m and the square of n and inversely with d. If $y = 16$ when $m = 3, n = 4,$ and $d = 6,$ find the constant k, then use k to find y when $m = -2, n = 4,$ and $d = 8.$

Midterm 1: Version D Answer Key

35. Midterm 1: Version E

1. Simplify the following:
   a. $-(3)^2$
   b. $(-3)^2$
   c. -3^2
   d. $3(2+4)-(2\cdot 4)$
   e. $-|-5+8|$
2. Solve for x in the equation $2(x-4)+18=-12+4(x+3)$.
3. Isolate the variable r_1 in the equation $\dfrac{1}{R}-\dfrac{1}{r_1}=\dfrac{1}{r_2}$.
4. Solve for x in the equation $\dfrac{x}{12}-\dfrac{x-4}{3}=\dfrac{2}{3}$.
5. Find the equation of the horizontal line that passes through the point $(-4,-6)$.
6. Find the equation that has a slope of $\dfrac{2}{5}$ and passes through the point $(-1,1)$.
7. Find the equation of the line passing through the points $(0,-1)$ and $(2,5)$.
8. Graph the relation $y=\dfrac{2}{3}x+1$.

For questions 9 to 11, find each solution set and graph it.

9. $-20\le 8x-4\le 28$
10. $\left|\dfrac{2x+2}{6}\right|\le 2$
11. $\left|\dfrac{3x-4}{5}\right|>1$
12. Graph $3x-2y<12$.
13. Find three consecutive odd integers such that the sum of the first integer, two times the second integer, and three times the third integer is 94.
14. Karl is going to cut a 800 cm cable into 2 pieces. If the first piece is to be 3 times as long as the second piece, find the length of each piece.
15. y varies jointly with m and inversely with the square of n. If $y=12$ when $m=3$ and $n=4$, find the constant k, then use k to find y when $m=3$ and $n=-3$.

Midterm 1: Version E Answer Key

PART VI

CHAPTER 5: SYSTEMS OF EQUATIONS

Learning Objectives

This chapter covers:

- Graphed Solutions
- Substitution Solutions
- Adding & Subtraction Solutions
- Solving for Three Variables
- Monetary Word Problems
- Solving for Four Variables
- Solving Internet Game Puzzles

36. 5.1 Graphed Solutions

Often, it is necessary to find the coordinates that are shared by two or more equations. There are multiple methods to find these shared values. While not giving values that are precise, graphing these equations make it possible to see the approximate solution and what type of solution it is.

Problems like $3x - 4 = 11$ have been solved in this textbook by adding 4 to both sides and then dividing by 3 (solution is $x = 5$). There are also methods to solve equations with more than one variable in them. It turns out that, to solve for more than one variable, it is necessary to have the same number of equations as variables. For example, to solve for two variables such as x and y, two equations are required. When there are several equations that must be solved, that is called a system of equations. When solving a system of equations, the solution must work in both equations. This solution is usually given as an ordered pair, (x, y). The following example illustrates a solution working in both equations.

Example 5.1.1

Show $(2, 1)$ is a solution to the equations $3x - y = 5$ and $x + y = 3$.

Using the coordinates $(2, 1)$, check to see if $x = 2, y = 1$ is a solution for both equations.

$$\begin{array}{rrrr} \text{For } 3x - y = 5 & 3(2) - (1) = 5 & \text{For } x + y = 3 & (2) + (1) = 3 \\ & 6 - 1 = 5 & & 3 = 3 \\ & 5 = 5 & & \end{array}$$

The coordinate $(2, 1)$ is a solution for both equations, which means that the lines that they represent will intersect at $(2, 1)$ when they are drawn on a graph.

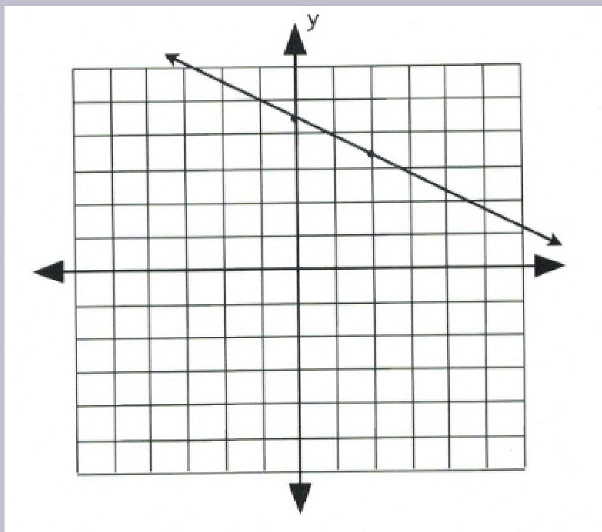
Note that the graph of an equation yields a picture of all its solutions. When two equations are graphed on the same coordinate plane, this displays not only the solutions of both equations, but where these solutions intersect. For instance, these solutions will show if the linear equations intersect at a point, over a line, or not at all.

In the following examples, each of the three possible types of solutions will be explored.

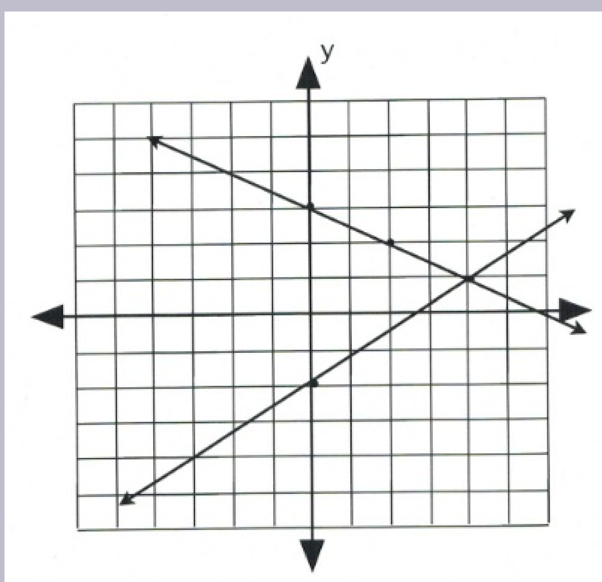
Example 5.1.2

Identify if any common intersection exists between the following linear equations: $y = -\dfrac{1}{2}x + 3$ and $y = \dfrac{3}{4}x - 2.$

First, graph $y = -\dfrac{1}{2}x + 3.$ The slope is $-\dfrac{1}{2}$ and the y-intercept is 3. For the first point, choose the intercept, and choose the second one using the slope. Draw a line through these two points to generate the first line.



Now, graph the second equation, $y = \frac{3}{4}x - 2$, using its slope of $\frac{3}{4}$ and intercept of −2.



The lines cross at the coordinate $(4, 1)$.

To check to see if this intersection is correct, substitute $x = 4$ and $y = 1$ into the two original equations.

$$
\begin{array}{lrcllrcl}
\text{Check:} & y & = & -\dfrac{1}{2}x+3 & & y & = & \dfrac{3}{4}x-2 \\
& (1) & = & -\dfrac{1}{2}(4)+3 & & (1) & = & \dfrac{3}{4}(4)-2 \\
& 1 & = & -2+3 & & 1 & = & 3-2 \\
& 1 & = & 1 & & 1 & = & 1
\end{array}
$$

The coordinate $(4, 1)$ is the shared point between both linear equations. This type of intersection is a unique solution that is called consistent and independent.

It is also possible to have a situation in which the same linear equation is graphed twice. Such an equation is easy to create.

Take the equation $y = 2x + 1$:

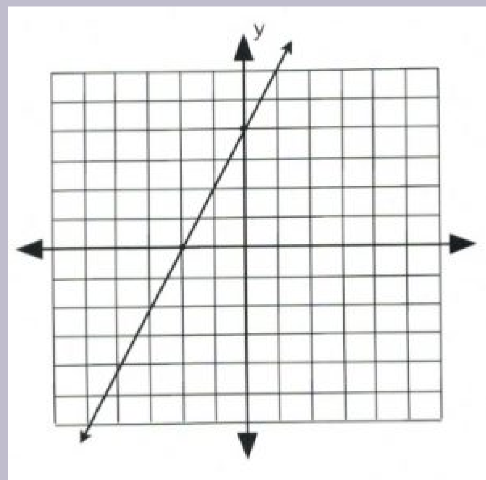
$$
\begin{array}{l}
\text{Multiplying by } 2 \text{ results in } 2y = 4x + 2 \\
\text{Multiplying by } 3 \text{ results in } 3y = 6x + 3 \\
\text{Multiplying by } \dfrac{1}{2} \text{ results in } \dfrac{1}{2}y = x + \dfrac{1}{2} \\
\text{Multiplying by } -2 \text{ results in } -2y = -4x - 2
\end{array}
$$

These are all the same equation, and if any two of them were graphed, the result would be the exact same line. This type of intersection has many solutions and is called consistent and dependent.

Example 5.1.3

Find the intersection of the linear equations $2x - y = -4$ and $4x - 2y = -8$.

Plot these equations using their intercepts:



For $2x - y = -4$:

when $x = 0, y = 4 \qquad (0, 4)$
when $y = 0, x = -2 \qquad (-2, 0)$

For $4x - 2y = -8$:

when $x = 0, y = 4 \qquad (0, 4)$
when $y = 0, x = -2 \qquad (-2, 0)$

The two lines from the previous example have the exact same intercepts and, when graphed, draw the exact same line. Since the two graphs have solutions, it is defined as being consistent. However, the many solutions means they are dependent.

Lines that are parallel will never intersect and, as a result, will never have a solution or a shared intersection. Any system of equations having no solution is defined as being inconsistent. Parallel equations are identical except for having different intercepts. This means that the equation $2x + 3y = 5$ is parallel to:

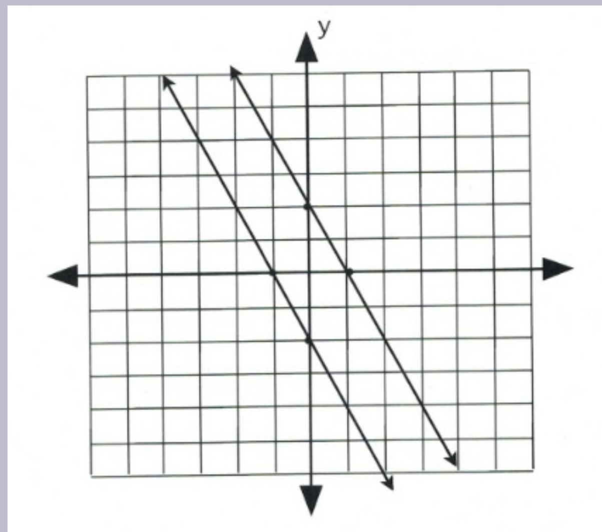
$$2x+3y = 6 \qquad 2x+3y = 7 \qquad 2x+3y = 8 \qquad 2x+3y = 0 \qquad 2x+3y = -5$$

Each of the above equations is parallel and will never intersect with each other.

Example 5.1.4

Find the intersection of the linear equations $2x + y = -2$ and $2x + y = 2$.

Plot these equations using their intercepts:



For $2x + y = -2$:
when $x = 0, y = -2 \qquad (0, -2)$
when $y = 0, x = -1 \qquad (-1, 0)$

For $2x + y = 2$:
when $x = 0, y = 2 \qquad (0, 2)$
when $y = 0, x = 1 \qquad (1, 0)$

Questions

For questions 1 to 12, find the intersection point of each system of equations.

1. $\begin{cases} y = -x + 1 \\ y = -5x - 3 \end{cases}$
2. $\begin{cases} y = -\frac{5}{4}x - 2 \\ y = -\frac{1}{4}x + 2 \end{cases}$
3. $\begin{cases} y = -3 \\ y = -x - 4 \end{cases}$
4. $\begin{cases} y = -x - 2 \\ y = \frac{2}{3}x + 3 \end{cases}$

5. $$\begin{cases} y = -\frac{3}{4}x + 1 \\ y = -\frac{3}{4}x + 2 \end{cases}$$

6. $$\begin{cases} y = 2x + 2 \\ y = -x - 4 \end{cases}$$

7. $$\begin{cases} y = \frac{1}{3}x + 2 \\ y = -\frac{5}{3}x - 4 \end{cases}$$

8. $$\begin{cases} y = 2x - 4 \\ y = -4x + 2 \end{cases}$$

9. $$\begin{cases} y = \frac{5}{3}x + 4 \\ y = -\frac{2}{3}x - 3 \end{cases}$$

10. $$\begin{cases} y = -\frac{1}{2}x + 4 \\ y = -\frac{1}{2}x + 1 \end{cases}$$

11. $$\begin{cases} x + 3y = -9 \\ 5x + 3y = 3 \end{cases}$$

12. $$\begin{cases} x + 4y = -12 \\ 2x + y = 4 \end{cases}$$

Chapter 5.1 Answer Key

37. 5.2 Substitution Solutions

While solving a system by graphing has advantages, it also has several limitations. First, it requires the graph to be perfectly drawn: if the lines are not straight, it may result in the wrong answer. Second, graphing is challenging if the values are really large—over 100, for example—or if the answer is a decimal that the graph will not be able to depict accurately, like 3.2134. For these reasons, graphing is rarely used to solve systems of equations. Commonly, algebraic approaches such as substitution are used instead.

Example 5.2.1

Find the intersection of the equations $2x-3y=7$ and $y=3x-7.$

Since $y=3x-7$, substitute $3x-7$ for the y in $2x-3y=7.$

The result of this looks like:

$$2x-3(3x-7)=7$$

Now solve for the variable x:

$$\begin{array}{rrrrrrr} 2x & - & 9x & + & 21 & = & 7 \\ & & & - & 21 & & -21 \\ & & & & \dfrac{-7x}{-7} & = & \dfrac{-14}{-7} \\ \\ & & & & x & = & 2 \end{array}$$

Once the x-coordinate is known, the y-coordinate is easily found.

To find y, use the equations $y=3x-7$ and $x=2$:

$$\begin{aligned} y &= 3(2)-7 \\ &= 6-7 \\ &= -1 \end{aligned}$$

These lines intersect at $x=2$ and $y=-1$, or at the coordinate $(2,-1)$.

This means the intersection is both consistent and independent.

Example 5.2.2

Find the intersection of the equations $y+4=3x$ and $2y-6x=-8.$

To solve this using substitution, y or x must be isolated. The first equation is the easiest in which to isolate a variable:

$$\begin{array}{rrrrrrr} y & + & 4 & = & 3x & & \\ & - & 4 & & -4 & & \\ & & y & = & 3x & - & 4 \end{array}$$

Substituting this value for y into the second equation yields:

$$\begin{array}{rrrrrrr} 2(3x & - & 4) & - & 6x & = & -8 \\ 6x & - & 8 & - & 6x & = & -8 \\ & + & 8 & & & & +8 \\ & & & & 0 & = & 0 \end{array}$$

The equations are identical, and when they are combined, they completely cancel out. This is an example of a consistent and dependent set of equations that has many solutions.

Example 5.2.3

Find the intersection of the equations $6x - 3y = -9$ and $-2x + y = 5$.

The second equation looks to be the easiest in which to isolate a variable, so:

$$\begin{array}{rrrrrrr} -2x & + & y & = & 5 & & \\ +2x & & & & +2x & & \\ & & y & = & 2x & + & 5 \end{array}$$

Substituting this into the first equation yields:

$$\begin{array}{rrrrrrr} 6x & - & 3(2x & + & 5) & = & -9 \\ 6x & - & 6x & - & 15 & = & -9 \\ & & & & -15 & = & -9 \end{array}$$

The variables cancel out, resulting in an untrue statement. These are parallel lines that have identical variables but different intercepts. There is no solution, and these are inconsistent equations.

Questions

For questions 1 to 20, solve each system of equations by substitution.

1. $\begin{cases} y = -3x \\ y = 6x - 9 \end{cases}$

2. $\begin{cases} y = x + 5 \\ y = -2x - 4 \end{cases}$

3. $\begin{cases} y = -2x - 9 \\ y = 2x - 1 \end{cases}$

4. $\begin{cases} y = -6x + 3 \\ y = 6x + 3 \end{cases}$

5. $\begin{cases} y = 6x + 4 \\ y = -3x - 5 \end{cases}$

6. $\begin{cases} y = 3x + 13 \\ y = -2x - 22 \end{cases}$

7. $\begin{cases} y = 3x + 2 \\ y = -3x + 8 \end{cases}$

8. $\begin{cases} y = -2x - 9 \\ y = -5x - 21 \end{cases}$

9. $\begin{cases} y = 2x - 3 \\ y = -2x + 9 \end{cases}$

10. $\begin{cases} y = 7x - 24 \\ y = -3x + 16 \end{cases}$

11. $\begin{cases} y = 3x - 4 \\ 3x - 3y = -6 \end{cases}$

12. $\begin{cases} -x + 3y = 12 \\ y = 6x + 21 \end{cases}$

13. $\begin{cases} y = -6 \\ 3x - 6y = 30 \end{cases}$

14. $\begin{cases} 6x - 4y = -8 \\ y = -6x + 2 \end{cases}$

15. $\begin{cases} y = -5 \\ 3x + 4y = -17 \end{cases}$

16. $\begin{cases} 7x + 2y = -7 \\ y = 5x + 5 \end{cases}$

17. $\begin{cases} -6x + 6y = -12 \\ 8x - 3y = 16 \end{cases}$

18. $\begin{cases} -8x + 2y = -6 \\ -2x + 3y = 11 \end{cases}$

19. $\begin{cases} 2x + 3y = 16 \\ -7x - y = 20 \end{cases}$

20. $\begin{cases} -x - 4y = -14 \\ -6x + 8y = 12 \end{cases}$

Answer Key 5.2

38. 5.3 Addition and Subtraction Solutions

One of the most powerful methods for solving systems of equations (finding their intersection points) is in adding and subtracting equations. In later math courses, this process is the foundation of matrix algebra, but for now, consider only equations.

The objective in finding the solutions to the these systems of equations is to isolate variables and find what they are equal to. Adding and subtracting equations can make this process quite fast and easy.

Example 5.3.1

Find the solution to the following system of equations: $3x - 4y = 8$ and $5x + 4y = -24.$

First, line them up over top of each other, since they will be added or subtracted. Notice that, when added, the $-4y$ and $+4y$ cancel each other out:

$$\begin{array}{rrrrrr} & 3x & - & 4y & = & 8 \\ + & 5x & + & 4y & = & -24 \\ & & & \dfrac{8x}{8} & = & \dfrac{-16}{8} \\ \\ & & & x & = & -2 \end{array}$$

It is now known that these equations intersect at the value where $x = -2.$ Now choose one of the two original equations (generally, the simplest to work with) and substitute $x = -2$ to find y:

$$\begin{array}{rrrrr} 3(-2) & - & 4y & = & 8 \\ -6 & - & 4y & = & 8 \\ +6 & & & & +6 \\ & & \dfrac{-4y}{-4} & = & \dfrac{14}{-4} \\ \\ & & y & = & \dfrac{14}{-4} \\ \\ & & y & = & -\dfrac{7}{2} \end{array}$$

The intersection point of these two linear equations is $x = -2$ and $y = -\dfrac{1}{2}$, or at the coordinate $\left(2, -\dfrac{1}{2}\right)$.

Generally, it takes a little more work than just placing the equations on top of each other and having a variable cancel out. In most cases, there is typically one variable that, once multiplied, is cancelled out when the equations are added to each other. For instance:

Example 5.3.2

Find the solution to the following system of equations: $-6x+5y=22$ and $2x+3y=2.$

First, line up the equations and choose the variable that shall be eliminated:

$$\left\{\begin{array}{rcrcr} -6x & + & 5y & = & 22 \\ 2x & + & 3y & = & 2 \end{array}\right.$$

The x variable could be eliminated if the bottom $2x$ were $6x.$ For this to happen, the entire bottom equation would have to be multiplied by 3:

$$\begin{array}{rrcrcrr} & (2x & + & 3y & = & 2) & (3) \\ \\ & -6x & + & 5y & = & 22 & \\ + & 6x & + & 9y & = & 6 & \\ & & & \dfrac{14y}{14} & = & \dfrac{28}{14} & \\ \\ & & & y & = & 2 & \end{array}$$

It is now known that these equations intersect at the value where $y=2.$ Now choose one of the two original equations (the simplest looks to be $2x+3y=2$) and substitute $y=2$ to find x:

$$\begin{array}{rcrcr} 2x & + & 3(2) & = & 2 \\ 2x & + & 6 & = & 2 \\ & - & 6 & & -6 \\ & & \dfrac{2x}{2} & = & \dfrac{-4}{2} \\ \\ & & x & = & -2 \end{array}$$

The intersection point of these two linear equations is $x=-2$ and $y=2$, or at the coordinate $(-2,2)$.

The more difficult of systems of two linear equations generally require the manipulation of both equations to eliminate one of the variables. For example, consider the following pair of linear equations:

Example 5.3.3

Find the solution to the following system of equations: $2x + 3y = -4$ and $3x - 4y = 11.$

First, line up the equations and choose the variable that shall be eliminated:

$$\begin{cases} 2x & + & 3y & = & -4 \\ 3x & - & 4y & = & 11 \end{cases}$$

It looks the simplest to eliminate the x variable. This means the top equation needs to be multiplied by 3 and the bottom equation multiplied by −2. Then, the two equations are added together, and each side is divided by 17:

$$\begin{array}{rrcrcrr} & (2x & + & 3y & = & -4) & (3) \\ & (3x & - & 4y & = & 11) & (-2) \\ \\ & 6x & + & 9y & = & -12 & \\ + & -6x & + & 8y & = & -22 & \\ & & & \dfrac{17y}{17} & = & \dfrac{-34}{17} & \\ \\ & & & y & = & -2 & \end{array}$$

It is now known that these equations intersect at the value where $y = -2.$ Now choose one of the two original equations (choose $2x + 3y = -4$) and substitute $y = -2$ to find x:

$$\begin{array}{rcccr} 2x & + & 3(-2) & = & -4 \\ 2x & - & 6 & = & -4 \\ & + & 6 & & +6 \\ & & \dfrac{2x}{2} & = & \dfrac{2}{2} \\ \\ & & x & = & 1 \end{array}$$

The intersection point of these two linear equations is $x = 1$ and $y = -2$, or at the coordinate $(1, -2)$.

The last examples that will be done for this topic are equations having no solution or infinite solutions.

Example 5.3.4

Find the solution to the following system of equations: $2x - 5y = 3$ and $-6x + 15y = -9.$

First, line up the equations to choose the variable that shall be eliminated:

$$\begin{cases} 2x & - & 5y & = & 3 \\ -6x & + & 15y & = & -9 \end{cases}$$

To eliminate the x variable, the top equation needs to be multiplied by 3:

$$\begin{array}{rrrrrl} & (2x & - & 5y & = & 3) \quad (3) \\ \\ & 6x & - & 15y & = & 9 \\ + & -6x & + & 15y & = & -9 \\ & & & 0 & = & 0 \end{array}$$

Everything cancels out because the two equations are identical. Therefore, there are infinite solutions.

Example 5.3.5

Find the solution to the following system of equations: $4x - 6y = 8$ and $4x - 6y = -4$.

Once these two equations are aligned, it is easy to see they are identical except for their intercepts. They are parallel lines. To cancel the variables out, one of the two equations must be multiplied by −1:

$$\begin{array}{rrrrrl} & 4x & - & 6y & = & 8 \\ & 4x & - & 6y & = & -4 \quad (-1) \\ \\ & 4x & - & 6y & = & 8 \\ + & -4x & + & 6y & = & 4 \\ & & & 0 & = & 12 \end{array}$$

The result is all the variables cancelling out to 0 and falsely equalling some number. There is no solution, since these equations will never intercept each other.

Questions

For questions 1 to 24, solve each system of equations by elimination.

1. $\left\{\begin{array}{rrrrr} 4x & + & 2y & = & 0 \\ -4x & - & 9y & = & -28 \end{array}\right.$
2. $\left\{\begin{array}{rrrrr} -7x & + & y & = & -10 \\ -9x & - & y & = & -22 \end{array}\right.$
3. $\left\{\begin{array}{rrrrr} -9x & + & 5y & = & -22 \\ 9x & - & 5y & = & 13 \end{array}\right.$
4. $\left\{\begin{array}{rrrrr} -x & - & 2y & = & -7 \\ x & + & 2y & = & 7 \end{array}\right.$

5. $\begin{cases} -6x + 9y = 3 \\ 6x - 9y = -9 \end{cases}$

6. $\begin{cases} 5x - 5y = -15 \\ x - y = -3 \end{cases}$

7. $\begin{cases} 4x - 6y = -10 \\ 4x + 6y = -14 \end{cases}$

8. $\begin{cases} -3x + 3y = -12 \\ -3x + 9y = -24 \end{cases}$

9. $\begin{cases} -x - 5y = 28 \\ -x + 4y = -17 \end{cases}$

10. $\begin{cases} -10x - 5y = 0 \\ -10x - 10y = -30 \end{cases}$

11. $\begin{cases} 2x - y = 5 \\ 5x + 2y = -28 \end{cases}$

12. $\begin{cases} -5x + 6y = -17 \\ x - 2y = 5 \end{cases}$

13. $\begin{cases} 10x + 6y = 24 \\ -6x + y = 4 \end{cases}$

14. $\begin{cases} x + 3y = -1 \\ 10x + 6y = -10 \end{cases}$

15. $\begin{cases} 2x + 4y = 24 \\ 4x - 12y = 8 \end{cases}$

16. $\begin{cases} -6x + 4y = 12 \\ 12x + 6y = 18 \end{cases}$

17. $\begin{cases} -7x + 4y = -4 \\ 10x - 8y = -8 \end{cases}$

18. $\begin{cases} -6x + 4y = 4 \\ 3x - y = 26 \end{cases}$

19. $\begin{cases} 5x + 10y = 20 \\ -6x - 5y = -3 \end{cases}$

20. $\begin{cases} -9x - 5y = -19 \\ 3x - 7y = -11 \end{cases}$

21. $\begin{cases} -7x + 5y = -8 \\ -3x - 3y = 12 \end{cases}$

22. $\begin{cases} 8x + 7y = -24 \\ 6x + 3y = -18 \end{cases}$

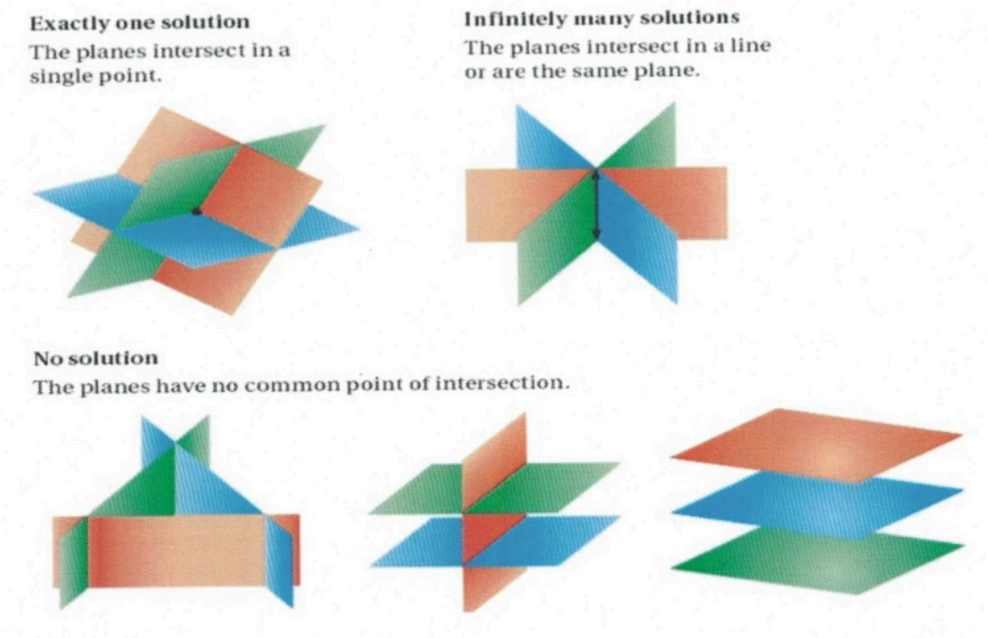
23. $\begin{cases} -8x - 8y = -8 \\ 10x + 9y = 1 \end{cases}$

24. $\begin{cases} -7x + 10y = 13 \\ 4x + 9y = 22 \end{cases}$

Answer Key 5.3

39. 5.4 Solving for Three Variables

When given three variables, you are given the equation for a plane or a flat surface similar to a sheet of paper. Some of the possible solutions to the intersections of these equations can be visualized below.



In solving systems of equations with three variables, use the strategies that are used to solve systems of two equations. One recommended method is to eliminate one variable at the onset, thus turning the set of three equations with three unknowns into two equations with two unknowns. The standard method to work with three equations or more is to use subtraction and/or addition.

Example 5.4.1

Find the intersection or the solution to the following system of equations: $3x + 2y - z = -1, -2x - 2y + 3z = 5$, and $5x + 2y - z = 3$.

As we did with a set of two equations, first line up the equations to choose the variable that we wish to eliminate:

$$\left\{\begin{array}{rcrcrcr} 3x & + & 2y & - & z & = & -1 \\ -2x & - & 2y & + & 3z & = & 5 \\ 5x & + & 2y & - & z & = & 3 \end{array}\right.$$

For these equations, it looks easiest to eliminate the y-variable. To do this, add the first and second equations together and then add the second and third equations together:

$$\begin{array}{rrrrrrrr} & 3x & + & 2y & - & z & = & -1 \\ + & -2x & - & 2y & + & 3z & = & 5 \\ & & & x & + & 2z & = & 4 \end{array} \qquad \begin{array}{rrrrrrrr} & -2x & - & 2y & + & 3z & = & 5 \\ + & 5x & + & 2y & - & z & = & 3 \\ & & & 3x & + & 2z & = & 8 \end{array}$$

Now, you are left with $x + 2z = 4$ and $3x + 2z = 8$. We now solve these as done previously with a set of two equations:

$$\begin{cases} x + 2z = 4 \\ 3x + 2z = 8 \end{cases}$$

Multiply either the top or the bottom equation by −1 to eliminate the z-variable.

$$\begin{array}{rrrrr} (x & + & 2z & = & 4) \quad (-1) \\ 3x & + & 2z & = & 8 \end{array}$$

$$\begin{array}{rrrrrr} & -x & - & 2z & = & -4 \\ + & 3x & + & 2z & = & 8 \\ & & & \dfrac{2x}{2} & = & \dfrac{4}{2} \\ \\ & & & x & = & 2 \end{array}$$

Next, find z using one of $x + 2z = 4$ or $3x + 2z = 8$ and the solution $x = 2$. $x + 2z = 4$ looks to be the easiest to work with.

$$\begin{array}{rrrrr} x & + & 2z & = & 4 \\ 2 & + & 2z & = & 4 \\ -2 & & & & -2 \\ & & \dfrac{2z}{2} & = & \dfrac{2}{2} \\ \\ & & z & = & 1 \end{array}$$

Finally, find y using one of the original three equations:

$$\begin{array}{rrrrrrr} 3x & + & 2y & - & z & = & -1 \\ 3(2) & + & 2y & - & (1) & = & -1 \\ 6 & + & 2y & - & 1 & = & -1 \\ & & 5 & + & 2y & = & -1 \\ & & -5 & & & = & -5 \\ & & & & \dfrac{2y}{2} & = & \dfrac{-6}{2} \\ \\ & & & & y & = & -3 \end{array}$$

These planes intersect at the point $x = 2$, $y = -3$, and $z = 1$, or the coordinate $(2, -3, 1)$.

Sometimes, you are given a set of three equations with missing variables. These systems of equations require slightly more thought to solve than the previous problems.

Example 5.4.2

Find the intersection or the solution to the following system of equations: $x + 2y - z = 0, 3x - 2y = -2,$ and $y + z = 3.$

First, line up the equations to choose the variable that we wish to eliminate:

$$\begin{cases} \begin{array}{rcrcrcr} x & + & 2y & - & z & = & 0 \\ 3x & - & 2y & & & = & -2 \\ & & y & + & z & = & 3 \end{array} \end{cases}$$

In this example, adding the first and last equations eliminates the variable z, without having to modify any of the equations:

$$\begin{array}{rrcrcrcr} & x & + & 2y & - & z & = & 0 \\ + & & & y & + & z & = & 3 \\ & & & x & + & 3y & = & 3 \end{array}$$

Now, there are two equations left:

$$\begin{cases} \begin{array}{rcrcr} 3x & - & 2y & = & -2 \\ x & + & 3y & = & 3 \end{array} \end{cases}$$

First multiply the bottom equation by −3, then add it to the top equation, to eliminate the variable x:

$$\begin{array}{rrcrcrl} & (x & + & 3y & = & 3) & (-3) \\ \\ & 3x & - & 2y & = & -2 & \\ + & -3x & - & 9y & = & -9 & \\ & & & -11y & = & -11 & \\ & & & y & = & 1 & \end{array}$$

Now choose one of the two remaining equations, $3x - 2y = -2$ or $x + 3y = 3,$ to find the variable x. Choosing $x + 3y = 3,$ leaves:

$$\begin{array}{rcrcr} x & + & 3(1) & = & 3 \\ x & + & 3 & = & 3 \\ & - & 3 & = & -3 \\ & & x & = & 0 \end{array}$$

Finally, to find the third variable, use one of the original three equations: $x + 2y - z = 0, 3x - 2y = -2,$ or $y + z = 3.$ Choosing $y + z = 3,$ gives:

$$\begin{array}{rcrcr} (1) & + & z & = & 3 \\ & & z & = & 2 \end{array}$$

These planes intersect at the point $x = 0, y = 1,$ and $z = 2,$ or the coordinate $(0, 1, 2).$

Questions

Solve each of the following systems of equations.

1. $\begin{cases} a - b + 2c = 2 \\ 2a + b - c = 2 \\ a + b + c = 3 \end{cases}$

2. $\begin{cases} 2a + 3b - c = 12 \\ 3a + 4b + c = 19 \\ a - 2b + c = -3 \end{cases}$

3. $\begin{cases} 3x + y - z = 7 \\ x + 3y - z = 5 \\ x + y + 2z = 3 \end{cases}$

4. $\begin{cases} x + y + z = 4 \\ x + 2y + 3z = 10 \\ x - y + 4z = 20 \end{cases}$

5. $\begin{cases} x + 2y - z = 0 \\ 2x - y + z = 15 \\ 3x - 2y - 4z = -5 \end{cases}$

6. $\begin{cases} x - y + 2z = -3 \\ x + 2y + 3z = 4 \\ 2x + y + z = -3 \end{cases}$

7. $\begin{cases} x + y + z = 6 \\ 2x - y - z = -3 \\ x - 2y + 3z = 6 \end{cases}$

8. tricky: $\begin{cases} x + y - z = 0 \\ x + 2y - 4z = 0 \\ 2x + y + z = 0 \end{cases}$

9. $\begin{cases} x + y + z = 2 \\ 2x - y + 3z = 9 \\ y - z = -3 \end{cases}$

10. $\begin{cases} 6x - y - 2z = -1 \\ 4x + z = 3 \\ -2x + 3y = 5 \end{cases}$

11. $\begin{cases} y + z = 5 \\ 2x - 3y + z = -1 \\ x - z = -2 \end{cases}$

12. $\begin{cases} 3x + 4y - z = 11 \\ y + 2z = -4 \\ -2x + y = -6 \end{cases}$

13. $$\begin{cases} x + 6y + 3z = 30 \\ 2x + 2z = 4 \\ -2y + z = -6 \end{cases}$$

14. $$\begin{cases} x - y + 2z = 0 \\ x + 2y = 1 \\ 2x + z = 4 \end{cases}$$

15. $$\begin{cases} x + y + z = 4 \\ -y - z = -4 \\ x - 2y = 0 \end{cases}$$

16. $$\begin{cases} x + y - z = 2 \\ 2y - 4z = -4 \\ 2x + z = 6 \end{cases}$$

Answer Key 5.4

40. 5.5 Monetary Word Problems

Solving value problems generally involves the solution of systems of equations. Value problems are ones in which each variable has a value attached to it, such as a nickel being worth 5¢, a dollar worth $\$1.00$, and a stamp worth 85¢. Using a table will help to set up and solve these problems. The basic structure of this table is shown below:

Example Table for Solving Value Problems

Name	Amount	Value	Equation

The first column in the table (Name) is used to identify the objects in the problem. The second column (Amount) identifies the amounts of each of the objects. The third column (Value) is used for the value of each object. The last column (Equation) is the product of the Amount times the Value.

Example 5.5.1

In Cally's piggy bank, there are 11 coins having a value of $\$1.85$. Each coin is either a quarter or a dime. Use this data to fill in the chart and find the equation to be solved.

- The objects' names are quarters (Q) and dimes (D)
- $Q+D=11$, which means $Q=11-D$ or $D=11-Q$
- Quarters have a value of $\$0.25$ and dimes have a value of $\$0.10$

Name	Amount	Value	Equation
Quarters (Q)	Q	$\$0.25$	$\$0.25Q$
Dimes (D)	$11-Q$	$\$0.10$	$\$0.10(11-Q)$
Mixture	11	N/A	$\$1.85$

The first two equations generally combine to equal the third equation.

The equation derived from this data is: $\$0.25Q+\$0.10(11-Q)=\$1.85$

Example 5.5.2

Doug and Becky sold 41 tickets for an event. Tickets for children cost $\$1.50$ and tickets for adults cost $\$2.00$. Total receipts for the event were $\$73.50$. How many of each type of ticket was sold?

Name	Amount	Value	Equation
Children (C)	C	$\$1.50$	$\$1.50C$
Adult (A)	$41-C$	$\$2.00$	$\$2.00(41-C)$
Mixture	41	N/A	$\$73.50$

The equation to be solved is:

$$\begin{array}{rrrrrrr} 1.50C & + & 2.00(41 & - & C) & = & 73.50 \\ 1.50C & + & 82.00 & - & 2.00C & = & 73.50 \\ & - & 82.00 & & & & -82.00 \\ & & & & \dfrac{0.50C}{0.50} & = & \dfrac{8.50}{0.50} \\ & & & & C & = & 17 \end{array}$$

This means that the number of adult tickets sold was $A = 41 - 17 = 24.$

Example 5.5.3

Nick has a collection of 5-cent stamps and 8-cent stamps. There are three times as many 8-cent stamps as 5-cent stamps. The total value of all the stamps is $\$3.48.$ How many of each stamp does Nick have?

Name	Amount	Value	Equation
Five-centers (F)	F	$\$0.05$	$\$0.05(F)$
Eight-centers (E)	$E=3F$	$\$0.08$	$\$0.08(3F)$
Mixture	$4F$	N/A	$\$3.48$

The equation to be solved is:

$$\begin{array}{rrrrr} 0.05(F) & + & 0.08(3F) & = & 3.48 \\ 0.05F & + & 0.24F & = & 3.48 \\ & & \dfrac{0.29F}{0.29} & = & \dfrac{3.48}{0.29} \\ & & F & = & 12 \end{array}$$

This means that the number of eight-cent stamps is $E = 3(12) = 36.$

Example 5.5.4

Angela invests $\$4000$ in two accounts, one at 6% interest, the other at 9% interest for one year. At the end of the year, she had earned $\$270$ in interest. How much did she have invested in each account?

Name	**Amount**	**Value**	**Equation**
Account (S)	S	0.06	$(0.06)(S)$
Account (N)	$\$4000 - S$	0.09	$(0.09)(\$4000 - S)$
Total interest	$\$4000$	N/A	$\$270$

The equation to be solved is:

$$\begin{array}{rcrcrcr} (0.06)(S) & + & (0.09)(4000 & - & S) & = & 270 \\ 0.06S & + & 360 & - & 0.09S & = & 270 \\ & - & 360 & & & & -360 \\ & & & & \dfrac{-0.03S}{-0.03} & = & \dfrac{-90}{-0.03} \\ & & & & S & = & \dfrac{-90}{-0.03} = \$3000 \end{array}$$

This means that the amount in the nine-percent account is $\$4000 - \$3000 = \$1000$.

Example 5.5.5

Clark and Kyra invest $\$5000$ in one account and $\$8000$ in an account paying 4% more in interest. They earned $\$1230$ in interest after one year. At what rates did they invest?

Name	**Amount**	**Value**	**Equation**
Account (F)	$\$5000$	X	$X(\$5000)$
Account (E)	$\$8000$	$X + 0.04$	$(X + 0.04)(\$8000)$
Total interest	N/A	N/A	$\$1230$

The equation to be solved is:

$$\begin{array}{rcrcrcl} X(5000) & + & (X & + & 0.04)(8000) & = & 1230 \\ 5000X & + & 8000X & + & 0.04(8000) & = & 1230 \\ 5000X & + & 8000X & + & 320 & = & 1230 \\ & & & - & 320 & & -320 \\ & & & & \dfrac{13000X}{13000} & = & \dfrac{910}{13000} \\ & & & & X & = & \dfrac{910}{13000} = 0.07 = 7\% \end{array}$$

The other interest rate is $0.07 + 0.04 = 0.11 = 11\%.$

This means that $\$5000$ was invested at 7% and $\$8000$ was invested at 11%.

Questions

For questions 1 to 10, find the equations that describe each problem. **Do not solve.**

1. A collection of dimes and quarters is worth $\$15.25.$ There are 103 coins in all. How many of each kind of coin is there?
2. A collection of fifty-cent pieces and nickels is worth $\$13.40.$ There are 34 coins in all. How many of each kind of coin is there?
3. The attendance at a school concert was 578. Admission was $\$2.00$ for adults and $\$1.50$ for children. The total of the receipts was $\$985.00.$ How many adults and how many children attended?
4. Natasha's purse contains $\$3.90$ made up of dimes and quarters. If there are 21 coins in all, how many dimes and how many quarters are there?
5. A boy has $\$2.25$ in nickels and dimes. If there are twice as many dimes as nickels, how many of each kind of coin does he have?
6. $\$3.75$ is made up of quarters and fifty-cent pieces. If the number of quarters exceeds the number of fifty-cent pieces by three, how many coins of each denomination are there?
7. An inheritance of $\$10000$ is invested in two ways, part at 9.5% and the remainder at 11%. The combined annual interest was $\$1038.50.$ How much was invested at each rate?
8. Kerry earned a total of $\$900$ last year on his investments. If $\$7000$ was invested at a certain rate of return and $\$9000$ was invested in a fund with a rate that was 2% higher, find the two rates of interest.
9. Jason earned $\$256$ in interest last year on his investments. If $\$1600$ was invested at a certain rate of return and $\$2400$ was invested in a fund with a rate that was double the rate of the first fund, find the two rates of interest.
10. Millicent earned $\$435$ last year in interest. If $\$3000$ was invested at a certain rate of return and $\$4500$ was invested in a fund with a rate that was 2% lower, find the two rates of interest.

For questions 11 to 25, find and solve the equations that describe each problem.

11. There were 203 tickets sold for a volleyball game. For activity-card holders, the price was $\$1.25$ each, and for non-card holders, the price was $\$2$ each. The total amount of money collected was $\$310.$ How many of each type of ticket was sold?

12. At a local ball game, the hot dogs sold for $\$2.50$ each and the hamburgers sold for $\$2.75$ each. There were 131 total food items sold for a total value of $\$342.$ How many of each item was sold?
13. A piggy bank contains 27 total dimes and quarters. The coins have a total value of $\$4.95.$ Find the number of dimes and quarters in the piggy bank.
14. A coin purse contains 18 total nickels and dimes. The coins have a total value of $\$1.15.$ Find the number of nickels and dimes in the coin purse.
15. Sally bought 40 stamps for $\$9.60.$ The purchase included 25¢ stamps and 20¢ stamps. How many of each type of stamp was bought?
16. A postal clerk sold some 15¢ stamps and some 25¢ stamps. Altogether, 15 stamps were sold for a total cost of $\$3.15.$ How many of each type of stamp was sold?
17. A total of $\$27000$ is invested, part of it at 12% and the rest at 13%. The total interest after one year is $\$3385.$ How much was invested at each rate?
18. A total of $\$50000$ is invested, part of it at 5% and the rest at 7.5%. The total interest after one year is $\$3250.$ How much was invested at each rate?
19. The total value of dimes and quarters in a piggy bank is $\$6.05.$ There are six more quarters than dimes. Find the number of each type of coin in the piggy bank.
20. A piggy bank contains nickels and dimes. The number of dimes is ten less than twice the number of nickels. The total value of all the coins is $\$2.75.$ Find the number of each type of coin in the piggy bank.
21. An investment portfolio earned $\$2010$ in interest last year. If $\$3000$ was invested at a certain rate of return and $\$24000$ was invested in a fund with a rate that was 4% lower, find the two rates of interest.
22. Samantha earned $\$1480$ in interest last year on their investments. If $\$5000$ was invested at a certain rate of return and $\$11000$ was invested in a fund with a rate that was two-thirds the rate of the first fund, find the two rates of interest.
23. A man has $\$5.10$ in nickels, dimes, and quarters. There are twice as many nickels as dimes and three more dimes than quarters. How many coins of each kind are there?
24. A bag containing nickels, dimes, and quarters has a value of $\$3.75.$ If there are 40 coins in all and three times as many dimes as quarters, how many coins of each kind are there?
25. A collection of stamps consists of 22¢ stamps and 40¢ stamps. The number of 22¢ stamps is three more than four times the number of 40¢ stamps. The total value of the stamps is $\$8.34.$ Find the number of 22¢ stamps in the collection.

Answer Key 5.5

41. 5.6 Solving for Four Variables

Three-dimensional space and time problems. Have fun solving these! 🙂

Questions

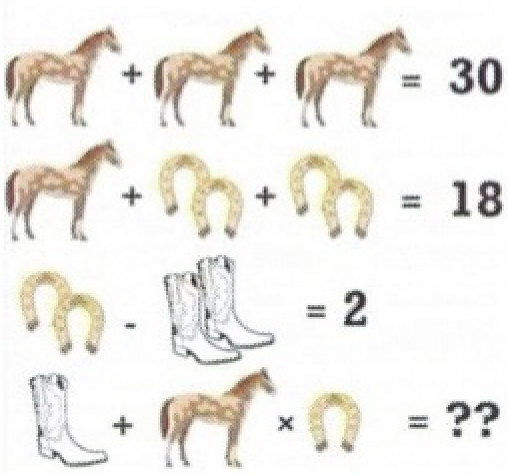
1. $$\begin{cases} t + x + y + z = 6 \\ t + 2x + 2y + 4z = 17 \\ -t + x - y - z = -2 \\ -t + 3x + y - z = 2 \end{cases}$$

2. $$\begin{cases} t + x - y + z = -1 \\ -t + 2x + 2y + z = 3 \\ -t + 3x + y - z = 1 \\ -2t + x + y - 3z = 0 \end{cases}$$

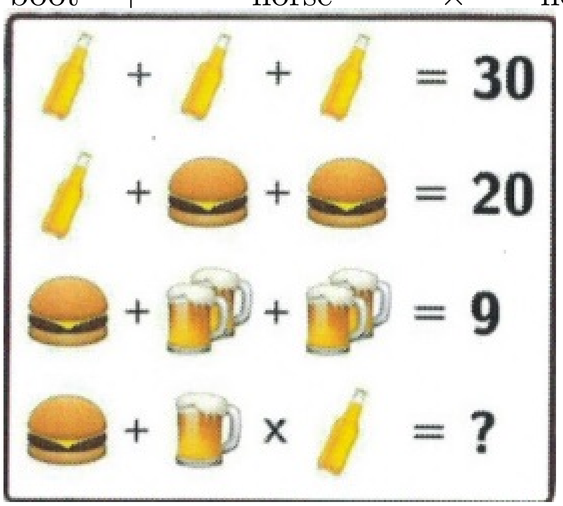
Answer Key 5.6

42. 5.7 Solving Internet Game Puzzles

The internet has many of the following picture number puzzles. All are quickly and easily solved with the strategies used for solving systems of equations. What are the solutions for the following picture puzzles?



1. $$\begin{array}{ccccccc} \text{horse} & + & \text{horse} & + & \text{horse} & = & 30 \\ \text{horse} & + & \text{two horseshoes} & + & \text{two horseshoes} & = & 18 \\ & & \text{two horseshoes} & - & \text{two boots} & = & 2 \\ \text{boot} & + & \text{horse} & \times & \text{horseshoe} & = & ? \end{array}$$



2. $$\begin{array}{ccccccc} \text{bottle} & + & \text{bottle} & + & \text{bottle} & = & 30 \\ \text{bottle} & + & \text{hamburger} & + & \text{hamburger} & = & 20 \\ \text{hamburger} & + & \text{two mugs} & + & \text{two mugs} & = & 9 \\ \text{hamburger} & + & \text{mug} & \times & \text{bottle} & = & ? \end{array}$$
3. A more challenging system of equations is the four-by-four puzzle shown below. Each individual row and column are summed to the number in either row or column 5. Find the five missing numbers and place them in their correct positions.

Rows	Column 1	Column 2	Column 3	Column 4	Column 5
Row 1	a	$2b$	$-c$	d	?
Row 2	$2d$	$-a$	$-2b$	$-2c$	-8
Row 3	c	$-d$	a	b	?
Row 4	$-b$	$-c$	$-d$	$2a$	5
Row 5	-1	?	3	?	?

Answer key 5.7

PART VII

CHAPTER 6: POLYNOMIALS

Learning Objectives

This chapter covers:

- Working with Exponents
- Negative Exponents
- Scientific Notation
- Basic operations Using Polynomials
- Multiplication of Polynomials
- Special Products
- Dividing Polynomials
- Mixture & Solution Word Problems
- Pascal's Triangle & Binomial Expansion

43. 6.1 Working with Exponents

Exponents often can be simplified using a few basic properties, since exponents represent repeated multiplication. The basic structure of writing an exponent looks like x^y, where x is defined as the base and y is termed its exponent. For this instance, y represents the number of times that the variable x is multiplied by itself

When looking at numbers to various powers, the following table gives the numeric value of several numbers to various powers.

Squares	Cubes	4^{th} Power	5^{th} Power	6^{th} Power	7^{th} Power
$2^2 = 4$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$	$2^6 = 64$	$2^7 = 128$
$3^2 = 9$	$3^3 = 27$	$3^4 = 81$	$3^5 = 243$	$3^6 = 729$	$3^7 = 2,187$
$4^2 = 16$	$4^3 = 64$	$4^4 = 256$	$4^5 = 1,024$	$4^6 = 4,096$	$4^7 = 16,384$
$5^2 = 25$	$5^3 = 125$	$5^4 = 625$	$5^5 = 3,125$	$5^6 = 15,625$	$5^7 = 78,125$
$6^2 = 36$	$6^3 = 216$	$6^4 = 1,296$	$6^5 = 7,776$	$6^6 = 46,656$	$6^7 = 279,936$
$7^2 = 49$	$7^3 = 343$	$7^4 = 2,401$	$7^5 = 16,807$	$7^6 = 117,649$	$7^7 = 823,543$
$8^2 = 64$	$8^3 = 512$	$8^4 = 4,096$	$8^5 = 32,768$	$8^6 = 262,144$	$8^7 = 2,097,152$
$9^2 = 81$	$9^3 = 729$	$9^4 = 6,561$	$9^5 = 59,049$	$9^6 = 531,441$	$9^7 = 4,782,969$
$10^2 = 100$	$10^3 = 1,000$	$10^4 = 10,000$	$10^5 = 100,000$	$10^6 = 1,000,000$	$10^7 = 10,000,000$
$11^2 = 121$	$12^2 = 144$	$13^2 = 169$	$14^2 = 196$	$15^2 = 225$	$20^2 = 400$

For this chart, the expanded forms of the base 2 for multiple exponents is shown:

$$\begin{aligned}
2^2 &= 2 \times 2 = 4, \\
2^3 &= 2 \times 2 \times 2 = 8, \\
2^4 &= 2 \times 2 \times 2 \times 2 = 16, \\
2^5 &= 2 \times 2 \times 2 \times 2 \times 2 = 32 \\
2^6 &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64 \quad \text{and so on}
\end{aligned}$$

Once there is an exponent as a base that is multiplied or divided by itself to the number represented by the exponent, it becomes straightforward to identify a number of rules and properties that can be defined.

The following examples outline a number of these rules.

Example 6.1.1

What is the value of $a^2 \times a^3$?

$a^2 \times a^3$ means that you have $(a \times a)(a \times a \times a)$,

which is the same as $(a \times a \times a \times a \times a)$

or a^5

This means that, when there is the same base and exponent that is multiplied by the same base with a different exponent, the total exponent value can be found by adding up the exponents.

$$\text{Product Rule of Exponents: } x^m \times x^n = x^{m+n}$$

Example 6.1.2

What is the value of $(a^2)^3$?

$(a^2)^3$ means that you have $(a^2) \times (a^2) \times (a^2)$,

which is the same as $(a \times a)(a \times a)(a \times a)$

or $(a \times a \times a \times a \times a \times a)$,

which equals a^6

When you have some base and exponent where both are multiplied by another exponent, the total exponent value can be found by multiplying the two different exponents together.

$$\text{Power of a Power Rule of Exponents: } (x^m)^n = x^{mn}$$

Example 6.1.3

What is the value of $(ab)^2$?

$(ab)^2$ means that you have $(ab) \times (ab)$,

which is the same as $(a \times b) \times (a \times b)$

or $(a \times a \times b \times b)$,

which equals a^2b^2

$$\text{Power of a Product Rule of Exponents: } (xy)^n = x^n y^n$$

Example 6.1.4

What is the value of $\dfrac{a^5}{a^3}$?

$\dfrac{a^5}{a^3}$ means that you have $\dfrac{a \times a \times a \times a \times a}{a \times a \times a}$, or that you are multiplying a by itself five times and dividing it by itself three times.

Multiplying and dividing by the exact same number is a redundant exercise; multiples can be cancelled out prior to doing any multiplying and/or dividing. The easiest way to do this type of a problem is to subtract the exponents, where the exponents in the denominator are being subtracted from the exponents in the numerator. This has the same effect as cancelling any excess or redundant exponents.

For this example, the subtraction looks like a^{5-3}, leaving a^2.

$$\text{Quotient Rule of Exponents: } \frac{x^m}{x^n} = x^{m-n} \qquad (x \neq 0)$$

Example 6.1.5

What is the value of $\left(\dfrac{a}{b}\right)^3$?

Expanded, this exponent is the same as:

$$\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b}$$

Which is the same as:

$$\frac{a \times a \times a}{b \times b \times b} \text{ or } \frac{a^3}{b^3}$$

One can see that this result is very similar to the power of a product rule of exponents.

$$\text{Power of a Quotient Rule of Exponents: } \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n} \qquad (y \neq 0)$$

Questions

Simplify the following.

1. $4 \cdot 4^4 \cdot 4^4$
2. $4 \cdot 4^4 \cdot 4^2$
3. $2m^4n^2 \cdot 4nm^2$
4. $x^2y^4 \cdot xy^2$
5. $(3^3)^4$
6. $(4^3)^4$

7. $(2u^3v^2)^2$
8. $(xy)^3$
9. $4^5 \div 4^3$
10. $3^7 \div 3^3$
11. $3nm^2 \div 3n$
12. $x^2y^4 \div 4xy$
13. $(x^3y^4 \cdot 2x^2y^3)^2$
14. $[(u^2v^2)(2u^4)]^3$
15. $[(2x)^3 \div x^3]^2$
16. $(2a^2b^2a^7) \div (ba^4)^2$
17. $[(2y^{17}) \div (2x^2y^4)^4]^3$
18. $[(xy^2)(y^4)^2] \div 2y^4$
19. $(2xy^5 \cdot 2x^2y^3) \div (2xy^4 \cdot y^3)$
20. $(2y^3x^2) \div [(x^2y^4)(x^2)]$
21. $[(q^3r^2)(2p^2q^2r^3)^2] \div 2p^3$
22. $(2x^4y^5)(2z^{10}x^2y^7) \div (xy^2z^2)^4$

Answer Key 6.1

44. 6.2 Negative Exponents CHECK line of text near bottom

Consider the following chart that shows the expansion of a for several exponents:

$$\begin{array}{lllllllll} a^4 & = & a & \times & a & \times & a & \times & a \\ a^3 & = & a & \times & a & \times & a \\ a^2 & = & a & \times & a \\ a^1 & = & a \\ a^0 & = & 1 \\ a^{-1} & = & \dfrac{1}{a} \\ a^{-2} & = & \dfrac{1}{(a \times a)} \\ a^{-3} & = & \dfrac{1}{(a \times a \times a)} \\ a^{-4} & = & \dfrac{1}{(a \times a \times a \times a)} \end{array}$$

If zero and negative exponents are expanded to base 2, the result is the following:

$$\begin{array}{llcll} 2^0 & = & 1 \\ 2^{-1} & = & \dfrac{1}{2} \\ 2^{-2} & = & \dfrac{1}{2 \times 2} & \text{or} & \dfrac{1}{4} \\ 2^{-3} & = & \dfrac{1}{2 \times 2 \times 2} & \text{or} & \dfrac{1}{8} \\ 2^{-4} & = & \dfrac{1}{2 \times 2 \times 2 \times 2} & \text{or} & \dfrac{1}{16} \end{array}$$

The most unusual of these is the exponent 0. Any base that is not equal to zero to the zeroth exponent is always 1. The simplest explanation of this is by example.

Example 6.2.1

Simplify $\dfrac{x^3}{x^3}$.

Using the quotient rule of exponents, we know that this simplifies to x^{3-3}, which equals x^0. And we know

$$\frac{2^3}{2^3} = \frac{8}{8} = 1,$$
$$\frac{3^3}{3^3} = \frac{27}{27} = 1,$$
$$\frac{4^3}{4^3} = \frac{64}{64} = 1,$$
$$\frac{5^3}{5^3} = \frac{125}{125} = 1,$$

and so on. A base raised to an exponent divided by that same base raised to that same exponent will always equal 1 unless the base is 0. This leads us to the zero power rule of exponents:

$$\text{Zero Power Rule of Exponents: } x^0 = 1 \qquad (x \neq 0)$$

This zero rule of exponents can make difficult problems elementary simply because whatever the 0 exponent is attached to reduces to 1. Consider the following examples:

Example 6.2.2

Simplify the following expressions.

1. $5x^0y^2$
   Since $x^0 = 1$, this simplifies to $5y^2$.
2. $(5x^0y^2)^0$
   Since the zero exponent is on the outside of the parentheses, everything contained inside the parentheses is cancelled out to 1.
3. $[(15x^3y^2)(25x^2y^2)]^0$
   Since the zero exponent is on the outside of the brackets, everything contained inside the brackets cancels out to 1.

When encountering these types of problems, always remain aware of what the zero power is attached to, since only what it is attached to cancels to 1.

When dealing with negative exponents, the simplest solution is to reciprocate the power. For instance:

Example 6.2.3

Simplify the following expressions.

1. $3x^{-2}y^2$

   Since the only negative exponent is x^{-2}, this simplifies to $\dfrac{3y^2}{x^2}$.
2. $4x^2y^{-3}$

   Since the only negative exponent is y^{-3}, this simplifies to $\dfrac{4x^2}{y^3}$.
3. $(4x^2y^{-3})^{-1}$

   Using the power of a power rule of exponents, we get $4^{-1}x^{-2}y^3$.

   Simplifying the negative exponents of $4^{-1}x^{-2}$, we get $\dfrac{y^3}{4x^2}$.
4. $(2m^{-1}n^{-3})(2m^{-1}n^{-3})^4$

   First using the power of a power rule on $(2m^{-1}n^{-3})^4$ yields $2^4m^{-4}n^{-12}$.
   Now we multiply $2m^{-1}n^{-3}$ by $2^4m^{-4}n^{-12}$, yielding $2^5m^{-5}n^{-15}$.

   We can write this without any negative exponents as $\dfrac{2^5}{m^5n^{15}}$.

Four Rules of Negative Exponents

$$x^{-n} = \frac{1}{x^n} \qquad (x \neq 0)$$
$$\frac{1}{x^{-n}} = x^n \qquad (x \neq 0)$$
$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n \qquad (x, y \neq 0)$$
$$\left(\frac{x^a y^b}{z^c}\right)^{-n} = \frac{z^{cn}}{x^{an}y^{bn}} \qquad (x, y, z \neq 0)$$

Questions

Simplify. Your answer should contain only positive exponents.

1. $(2x^4y^{-2})(2xy^3)^4$

2. $(2a^{-2}b^{-3})(2a^0b^4)^4$
3. $(2x^2y^2)^4x^{-4}$
4. $[(m^0n^3)(2m^{-3}n^{-3})]^0$
5. $(2x^{-3}y^2) \div (3x^{-3}y^3 \cdot 3x^0)$
6. $3y^3 \div [(3yx^3)(2x^4y^{-3})]$
7. $2y \div (x^0y^2)^4$
8. $(a^4)^4 \div 2b$
9. $(2a^2b^3)^4 \div a^{-1}$
10. $(2y^{-4})^{-2} \div x^2$
11. $(2mn)^4 \div m^0n^{-2}$
12. $2x^{-3} \div (x^4y^{-3})^{-1}$
13. $[(2u^{-2}v^3)(2uv^4)^{-1}] \div 2u^{-4}v^0$
14. $[(2yx^2)(x^{-2})] \div (2x^0y^4)^{-1}$
15. $b^{-1} \div [(2a^4b^0)^0(2a^{-3}b^2)]$
16. $2yzx^2 \div [(2x^4y^4z^{-2})(zy^2)^4]$
17. $[(cb^3)^2(2a^{-3}b^2)] \div (a^3b^{-2}c^3)^3$
18. $2q^4(m^2p^2q) \div (2m \cdot 4p^2)^3$
19. $(yx^{-4}z^2)^{-1} \div z^3 \cdot x^2y^3z^{-1}$
20. $2mpn^{-3} \div [2n^2p^0(m^0n^{-4}p^2)^3]$

Answer Key 6.2

45. 6.3 Scientific Notation (Homework Assignment)

Scientific notation is a convenient notation system used to represent large and small numbers. Examples of these are the mass of the sun or the mass of an electron in kilograms. Simplifying basic operations such as multiplication and division with these numbers requires using exponential properties.

Scientific notation has two parts: a number between one and nine and a power of ten, by which that number is multiplied.

$$\text{Scientific notation: } a \times 10^b, \text{ where } 1 \leq a \leq 9$$

The exponent tells how many times to multiply by 10. Each multiple of 10 shifts the decimal point one place. To decide which direction to move the decimal (left or right), recall that positive exponents means there is big number (larger than ten) and negative exponents means there is a small number (less than one).

Example 6.3.1

Convert 14,200 to scientific notation.

$$\begin{aligned} 1.42 &\quad \text{Put a decimal after the first nonzero number} \\ \times 10^4 &\quad \text{The exponent is how many times the decimal moved} \\ 1.42 \times 10^4 &\quad \text{Combine to yield the solution} \end{aligned}$$

Example 6.3.2

Convert 0.0028 to scientific notation.

$$\begin{aligned} 2.8 &\quad \text{Put a decimal after the first nonzero number} \\ \times 10^{-3} &\quad \text{The exponent is how many times the decimal moved} \\ 2.8 \times 10^{-3} &\quad \text{Combine to yield the solution} \end{aligned}$$

Example 6.3.3

Convert 3.21×10^5 to standard notation.

Starting with 3.21, Shift the decimal 5 places to the right, or multiply 3.21 by 10^5.

321,000 is the solution.

Example 6.3.4

Convert 7.4×10^{-3} to standard notation

Shift the decimal 3 places to the left, or divide 6.4 by 10^{3}.

0.0074 is the solution.

Working with scientific notation is easier than working with other exponential notation, since the base of the exponent is always 10. This means that the exponents can be treated separately from any other numbers. For instance:

Example 6.3.5

Multiply $(2.1 \times 10^{-7})(3.7 \times 10^{5})$.

First, multiply the numbers 2.1 and 3.7, which equals 7.77.

Second, use the product rule of exponents to simplify the expression $10^{-7} \times 10^{5}$, which yields 10^{-2}.

Combine these terms to yield the solution 7.77×10^{-2}.

Example 6.3.6

$(4.96 \times 10^{4}) \div (3.1 \times 10^{-3})$

First, divide: $4.96 \div 3.1 = 1.6$

Second, subtract the exponents (it is a division): $10^{4 - -3} = 10^{4 + 3} = 10^{7}$

Combine these to yield the solution 1.6×10^{7}.

Questions

For questions 1 to 6, write each number in scientific notation.

1. 885.3
2. 0.000744
3. 0.081
4. 1.09
5. 0.039
6. 15,000

For questions 7 to 12, write each number in standard notation.

7. 8.7×10^5
8. 2.56×10^2
9. 9×10^{-4}
10. 5×10^4
11. 2×10^0
12. 6×10^{-5}

For questions 13 to 20, simplify each expression and write each answer in scientific notation.

13. $(7 \times 10^{-1})(2 \times 10^{-3})$
14. $(2 \times 10^{-6})(8.8 \times 10^{-5})$
15. $(5.26 \times 10^{-5})(3.16 \times 10^{-2})$
16. $(5.1 \times 10^6)(9.84 \times 10^{-1})$
17. $\dfrac{(2.6 \times 10^{-2})(6 \times 10^{-2})}{(4.9 \times 10^1)(2.7 \times 10^{-3})}$
18. $\dfrac{(7.4 \times 10^4)(1.7 \times 10^{-4})}{(7.2 \times 10^{-1})(7.32 \times 10^{-1})}$
19. $\dfrac{(5.33 \times 10^{-6})(9.62 \times 10^{-2})}{(5.5 \times 10^{-5})^2}$
20. $\dfrac{(3.2 \times 10^{-3})(5.02 \times 10^0)}{(9.6 \times 10^3)^{-4}}$

Answer Key 6.3

46. 6.4 Basic Operations Using Polynomials

Many applications in mathematics have to do with what are called polynomials. Polynomials are made up of terms. Terms are a product of numbers and/or variables. For example, $5x$, $2y^2$, -5, ab^3c, and x are all terms. Terms are connected to each other by addition or subtraction.

Expressions are often defined by the number of terms they have.

A monomial has one term, such as $x, xy, 3x^2$.

A binomial has two terms connected by a + or -, such as $a^2 - b^2, 3x - y, 4x + 2xy^3$

A trinomial has three terms connected by a + or -, such as $ax^2 + bx + c$

The term polynomial is generic for many terms. Monomials, binomials, trinomials, and expressions with more terms all fall under the umbrella of "polynomials."

Polynomials are classified by the sum of exponents of the term with the highest exponent sum, which is called the degree of the polynomial.

A **first degree polynomial** is a linear polynomial and would not have any terms with a sum of exponents greater than one. Examples include $3x + 2y + 4z, 42x - 56y, 22z$.

A **second degree polynomial** is a quadratic polynomial and would not have any terms with a sum of exponents greater than two. Examples include $3x^2 + 2x + 4y^2, 42xy - 3z, 2zx$.

A **third degree polynomial** is a cubic polynomial and would not have any terms with a sum of exponents greater than three. Examples include $5x^2 + 2xy^2 + 4y^2, 42xyz - 5z^2, 3zx^2$.

A **fourth degree polynomial** is a quartic polynomial and would not have any terms with a sum of exponents greater than four. Examples include $5x^2 + 3xy^2z + 4y^2, 42xyz - 6z^4, 8z^3x$.

A **fifth degree polynomial** is a quintic polynomial.

A **sixth degree polynomial** is a sextic polynomial.

A **seventh degree polynomial** is a septic polynomial.

A **eighth degree polynomial** is a octic polynomial.

A **ninth degree polynomial** is a nonic polynomial.

A **tenth degree polynomial** is a decic polynomial.

The degree of any term is the sum of its exponents:

$x^9, x^7y^2, x^3y^3z^3$ are all ninth degree terms (nonic polynomial)

x^6, x^4y^2, x^3yz^2 are all sixth degree terms (sextic polynomial)

x^4, x^2y^2, xyz^2 are all fourth degree terms (quartic polynomial)

Terms of a polynomial are named in the order of their appearance. For instance, the polynomial $2x^4y + x^3y^2 - x^2y^3 + 5xy^4$ has four terms, each one of the fifth degree. The terms are numbered in order for this polynomial, starting from the first term $(2x^4y)$ and continuing to the second ((x^3y^2)\), third $(-x^2y^3)$, and fourth terms $(5xy^4)$.

If it is known what the variable in a polynomial represents, it is possible to substitute in the value and evaluate the polynomial, as shown in the following example.

Example 6.4.1

Simplify the expression $2x^2 - 4x + 6$ using $x = -4$.

$$\begin{array}{rlllll} \text{When } x=-4, \text{ replace all } x \text{ with } -4: & 2x^2 & - & 4x & + & 6 \\ \text{Reduce the exponents:} & 2(-4)^2 & - & 4(-4) & + & 6 \\ \text{Multiply all coefficients:} & 2(16) & - & 4(-4) & + & 6 \\ \text{Add:} & 32 & + & 16 & + & 6 \\ \text{Solution:} & 54 & & & & \end{array}$$

Remember the exponent only affects the number to which it is physically attached. This means $-3^2 = -9$ because the exponent is only attached to the 3, whereas $(-3)^2 = 9$ because the exponent is attached to the parentheses and everything inside.

Example 6.4.2

Simplify the expression $-x^2 + 2x + 6$ using $x = 3$.

$$\begin{array}{rlllll} \text{When } x=3, \text{ replace all } x \text{ with 3:} & -x^2 & + & 2x & + & 6 \\ \text{Reduce the exponents:} & -(3)^2 & + & 2(3) & + & 6 \\ \text{Multiply:} & -9 & + & 2(3) & + & 6 \\ \text{Add:} & -9 & + & 6 & + & 6 \\ \text{Solution:} & 3 & & & & \end{array}$$

Sometimes, when working with polynomials, the value of the variable is unknown and the polynomial will be simplified rather than ending with some value. The simplest operation with polynomials is addition. When adding polynomials, it is merely combining like terms. Consider the following example.

Example 6.4.3

Simplify the expression $(4x^3 - 2x + 8) + (3x^3 - 9x^2 - 11)$.

The first thing one should do is place the equations one over top of the other, ordering each of the terms into columns so they can simply be added or subtracted.

$$\begin{array}{llllllll} 4x^3 & & & - & 2x & + & 8 & \\ 3x^3 & - & 9x^2 & & & - & 11 & \text{Add each of the columns} \\ 7x^3 & - & 9x^2 & - & 2x & - & 3 & \text{Solution} \end{array}$$

Subtracting polynomials is almost as fast as addition. Subtraction is generally shown by a minus sign in front of the parentheses. When there is a negative sign in front of parentheses, distribute it throughout the expression, changing the signs of everything inside.

Example 6.4.4

Simplify the expression $(5x^2-2x+7)-(3x^2+6x-4)$.

$$\begin{array}{rrrrrrrl} & 5x^2 & - & 2x & + & 7 & & \\ - & (3x^2 & + & 6x & - & 4) & & \text{Subtract each bottom term from each top term} \\ & 2x^2 & - & 8x & + & 11 & & \text{Solution} \end{array}$$

Example 6.4.5

Simplify the expression $(2x^2-4x+3)+(5x^2-6x+1)-(x^2-9x+8)$.

$$\begin{array}{rrrrrrl} & 2x^2 & - & 4x & + & 3 & \\ & 5x^2 & - & 6x & + & 1 & \\ - & (x^2 & - & 9x & + & 8) & \text{Add and subtract} \\ & 6x^2 & - & x & - & 4 & \text{Solution} \end{array}$$

Questions

For questions 1 to 8, simplify each expression using the variables given.

1. $-a^3-a^2+6a-21$ when $a=-4$
2. $n^2+3n-11$ when $n=-6$
3. $-5n^4-11n^3-9n^2-n-5$ when $n=-1$
4. x^4-5x^3-x+13 when $x=5$
5. $x^2+9x+23$ when $x=-3$
6. $-6x^3+41x^2-32x+11$ when $x=6$
7. $x^4-6x^3+x^2-24$ when $x=6$
8. $m^4+8m^3+14m^2+13m+5$ when $m=-6$

For questions 9 to 28, simplify the following expressions.

9. $(5p - 5p^4) - (8p - 8p^4)$
10. $(7m^2 + 5m^3) - (6m^3 - 5m^2)$
11. $(1 + 5p^3) - (1 - 8p^3)$
12. $(6x^3 + 5x) - (8x + 6x^3)$
13. $(5n^4 + 6n^3) + (8 - 3n^3 - 5n^4)$
14. $(8x^2 + 1) - (6 - x^2 - x^4)$
15. $(2a + 2a^4) - (3a^2 - 5a^4 + 4a)$
16. $(6v + 8v^3) + (3 + 4v^3 - 3v)$
17. $(4p^2 - 3 - 2p) - (3p^2 - 6p + 3)$
18. $(7 + 4m + 8m^4) - (5m^4 + 1 + 6m)$
19. $(3 + 2n^2 + 4n^4) + (n^3 - 7n^2 - 4n^4)$
20. $(7x^2 + 2x^4 + 7x^3) + (6x^3 - 8x^4 - 7x^2)$
21. $(8r^4 - 5r^3 + 5r^2) + (2r^2 + 2r^3 - 7r^4 + 1)$
22. $(4x^3 + x - 7x^2) + (x^2 - 8 + 2x + 6x^3)$
23. $(2n^2 + 7n^4 - 2) + (2 + 2n^3 + 4n^2 + 2n^4)$
24. $(7b^3 - 4b + 4b^4) - (8b^3 - 4b^2 + 2b^4 - 8b)$
25. $(8 - b + 7b^3) - (3b^4 + 7b - 8 + 7b^2) + (3 - 3b + 6b^3)$
26. $(1 - 3n^4 - 8n^3) + (7n^4 + 2 - 6n^2 + 3n^3) + (4n^3 + 8n^4 + 7)$
27. $(8x^4 + 2x^3 + 2x) + (2x + 2 - 2x^3 - x^4) - (x^3 + 5x^4 + 8x)$
28. $(6x - 5x^4 - 4x^2) - (2x - 7x^2 - 4x^4 - 8) - (8 - 6x^2 - 4x^4)$

Answer Key 6.4

47. 6.5 Multiplication of Polynomials

Multiplying monomials is done by multiplying the numbers or coefficients and then adding the exponents on like factors. This is shown in the next example.

Example 6.5.1

Find the following product: $(4x^3y^4z)(2x^2y^6z^3)$.

$$\begin{array}{rrl} & 4x^3y^4z & \\ \times & 2x^2y^6z^3 & \text{Multiply coefficients and add exponents} \\ & 8x^5y^{10}z^4 & \text{Solution} \end{array}$$

Some notes: z has an exponent of 1 when no exponent is written. When adding or subtracting, the exponents will stay the same, but when multiplying (or dividing), the exponents will change.

Next, consider multiplying a monomial by a polynomial. We have seen this operation before with distributing throughout parentheses. Here, its the exact same process.

Example 6.5.2

Find the following product: $4x^3(5x^2-2x+5)$.

$$\begin{array}{rrcrcrl} & 5x^2 & - & 2x & + & 5 & \\ \times & 4x^3 & & & & & \text{Multiply} \\ & 20x^5 & - & 8x^4 & + & 20x^3 & \text{Solution} \end{array}$$

Example 6.5.3

Find the following product: $(4x+7y)(3x-2y)$.

$$\begin{array}{rrrrrrl} & 4x & + & 7y & & & \\ \times & 3x & - & 2y & & & \text{Multiply} \\ & 12x^2 & + & 21xy & & & \text{Product of } 3x(4x+7y) \\ & & - & 8xy & - & 14y^2 & \text{Product of } -2y(4x+7y) \\ & 12x^2 & + & 13xy & - & 14y^2 & \text{Solution} \end{array}$$

Example 6.5.4

Find the following product: $(3x^2+2x-5)(4x^2-7x+3)$.

$$\begin{array}{rrrrrrrrrrl} & 3x^2 & + & 2x & - & 5 & & & & & \\ \times & 4x^2 & - & 7x & + & 3 & & & & & \\ & 12x^4 & + & 8x^3 & - & 20x^2 & & & & & \text{Product of } 4x^2(3x^2+2x-5) \\ & & - & 21x^3 & - & 14x^2 & + & 35x & & & \text{Product of } -7x(3x^2+2x-5) \\ & & & & & 9x^2 & + & 6x & - & 15 & \text{Product of } 3(3x^2+2x-5) \\ & 12x^4 & - & 13x^3 & - & 25x^2 & + & 41x & - & 15 & \text{Solution} \end{array}$$

Example 6.5.5

Find the following product: $(x^3+2x^2+x-3)(x^3-3x^2-4x+6)$.

$$\begin{array}{rrrrrrrrrrrrrr} & x^3 & + & 2x^2 & + & x & - & 3 & & & & & & \\ \times & x^3 & - & 3x^2 & - & 4x & + & 6 & & & & & & \\ & x^6 & - & 3x^5 & - & 4x^4 & + & 6x^3 & & & & & & \\ & & & 2x^5 & - & 6x^4 & - & 8x^3 & + & 12x^2 & & & & \\ & & & & & x^4 & - & 3x^3 & - & 4x^2 & + & 6x & & \\ & & & & & & - & 3x^3 & + & 9x^2 & + & 12x & + & 18 \\ & x^6 & - & x^5 & - & 9x^4 & - & 8x^3 & + & 17x^2 & + & 18x & + & 18 \end{array}$$

As seen in the last two examples, the strategy used is that of foiling, with the only difference being that the answers are organized into columns. This eliminates the need to chase terms scattered all over the page, as they are now grouped.

This is the superior strategy to use when multiplying polynomials.

Questions

Find each product.

1. $6(p-7)$
2. $4k(8k+4)$
3. $2(6x+3)$
4. $3n^2(6n+7)$
5. $(4n+6)(8n+8)$
6. $(2x+1)(x-4)$
7. $(8b+3)(7b-5)$
8. $(r+8)(4r+8)$
9. $(3v-4)(5v-2)$
10. $(6a+4)(a-8)$
11. $(5x+y)(6x-4y)$
12. $(2u+3v)(8u-7v)$
13. $(7x+5y)(8x+3y)$
14. $(5a+8b)(a-3b)$
15. $(r-7)(6r^2-r+5)$
16. $(4x+8)(4x^2+3x+5)$
17. $(6n-4)(2n^2-2n+5)$
18. $(2b-3)(4b^2+4b+4)$
19. $(6x+3y)(6x^2-7xy+4y^2)$
20. $(3m-2n)(7m^2+6mn+4n^2)$
21. $(8n^2+4n+6)(6n^2-5n+6)$
22. $(2a^2+6a+3)(7a^2-6a+1)$
23. $(5k^2+3k+3)(3k^2+3k+6)$
24. $(7u^2+8uv-6v^2)(6u^2+4uv+3v^2)$
25. $(2n^3-8n^2+3n+6)(n^3-6n^2-2n+3)$
26. $(a^3+2a^2+3a+3)(a^3+2a^2-4a+1)$
27. $3(3x-4)(2x+1)$
28. $5(x-4)(2x-3)$
29. $3(2x+1)(4x-5)$
30. $2(4x+1)(2x-6)$

Answer Key 6.5

48. 6.6 Special Products

There are a few shortcuts available when multiplying polynomials. When recognized, they help arrive at the solution much quicker.

The first is called a difference of squares. A difference of squares is easily recognized because the numbers and variables in its two factors are exactly the same, but the sign in each factor is different (one plus sign, one minus sign). To illustrate this, consider the following example.

Example 6.6.1

Multiply the following pair of binomials: $(a+b)(a-b)$.

$$\begin{array}{rcrcr} a & + & b & & \\ a & - & b & & \\ a^2 & + & ab & & \\ & - & ab & - & b^2 \\ a^2 & - & b^2 & & \end{array}$$

Notice the middle term cancels out and $(a+b)(a-b)=a^2-b^2$. Cancelling the middle term during multiplication is the same for any difference of squares.

Example 6.6.2

Multiply the following pair of binomials: $(x-5)(x+5)$.

Recognize a difference of squares. The solution is x^2-25.

Example 6.6.3

Multiply the following pair of binomials: $(3x+7)(3x-7)$.

Recognize a difference of squares. The solution is $9x^2-49$.

Example 6.6.4

Multiply the following pair of binomials: $(2x - 6y)(2x + 6y)$.

Recognize a difference of squares. The solution is $4x^2 - 36y^2$.

Another pair of binomial multiplications useful to know are perfect squares. These have the form of $(a + b)^2$ or $(a - b)^2$.

Example 6.6.5

Multiply the following pair of binomials: $(a + b)^2$ and $(a - b)^2$.

$$\begin{array}{rrrrrrr} & (a & + & b) & & \\ \times & (a & + & b) & & \\ & a^2 & + & ab & & \\ & & + & ab & + & b^2 \\ & a^2 & + & 2ab & + & b^2 \end{array} \qquad \begin{array}{rrrrrrr} & (a & - & b) & & \\ \times & (a & - & b) & & \\ & a^2 & - & ab & & \\ & & - & ab & + & b^2 \\ & a^2 & - & 2ab & + & b^2 \end{array}$$

The pattern of multiplication for any perfect square is the same. The first term in the answer is the square of the first term in the problem. The middle term is 2 times the first term times the second term. The last term is the square of the last term.

$$(a + b)^2 = a^2 + 2ab + b^2 \text{ and } (a - b)^2 = a^2 - 2ab + b^2$$

Example 6.6.6

Multiply out the following expression: $(x - 5)^2$.

Recognize a perfect square. Square the first term, subtract twice the product of the first and last terms, and square the last term.

$$(x - 5)^2 = x^2 - 10x + 25$$

Example 6.6.7

Multiply out the following expression: $(3x - 7y)^2$.

Recognize a perfect square. Square the first term, subtract twice the product of the first and last terms, and square the last term.

$$(3x - 7y)^2 = 9x^2 - 42xy + 49y^2$$

Example 6.6.8

Multiply out the following expression: $(5a + 9b)^2$.

Recognize a perfect square. Square the first term, add twice the product of the first and last terms, and square the last term.

$$(5a + 9b)^2 = 25a^2 + 90ab + 81b^2$$

Questions

Find each product.

1. $(x + 8)(x - 8)$
2. $(a - 4)(a + 4)$
3. $(1 + 3p)(1 - 3p)$
4. $(x - 3)(x + 3)$
5. $(1 - 7n)(1 + 7n)$
6. $(8m + 5)(8m - 5)$
7. $(4y - x)(4y + x)$
8. $(7a + 7b)(7a - 7b)$
9. $(4m - 8n)(4m + 8n)$
10. $(3y - 3x)(3y + 3x)$
11. $(6x - 2y)(6x + 2y)$
12. $(1 + 5n)^2$
13. $(a + 5)^2$
14. $(x - 8)^2$
15. $(1 - 6n)^2$

16. $(4x-5)^2$
17. $(5m-8)^2$
18. $(3a+3b)^2$
19. $(5x+7y)^2$
20. $(4m-n)^2$
21. $(5+2r)^2$
22. $(m-7)^2$
23. $(4v-7)(4v+7)$
24. $(b+4)(b-4)$

Answer Key 6.6

49. 6.7 Dividing Polynomials

Dividing polynomials is a process very similar to long division of whole numbers. But before looking at that, first master dividing a polynomial by a monomial. The way to do this is very similar to distributing, but the operation to distribute is the division, dividing each term by the monomial and reducing the resulting expression. This is shown in the following examples.

Example 6.7.1

Divide the following:

1. $(9x^5+6x^4-18x^3-24x^2)\div 3x^2$Breaking this up into fractions, we get: $\dfrac{9x^5}{3x^2}+\dfrac{6x^4}{3x^2}-\dfrac{18x^3}{3x^2}-\dfrac{24x^2}{3x^2}$Which yields:$3x^3+2x^2-6x-8$
2. $(8x^3+4x^2-2x+6)\div 4x^2$Breaking this up into fractions, we get: $\dfrac{8x^3}{4x^2}+\dfrac{4x^2}{4x^2}-\dfrac{2x}{4x^2}+\dfrac{6}{4x^2}$Which yields:$2x+1-\dfrac{1}{2x}+\dfrac{3}{2x^2}$

Long division is required when dividing by more than just a monomial. Long division with polynomials is similar to long division with whole numbers.

Example 6.7.2

Divide the polynomial $3x^3-5x^2-32x+7$ by $x-4$.

$$3x^3-5x^2-32x+7x-4$$

The steps to get this result are as follows:

1. Divide $3x^3$ by x, yielding $3x^2$. Multiply $(x-4)$ by $3x^2$, yielding $3x^3+12x^2$. Subtract and bring down the next term and repeat.
2. Divide $7x^2$ by x, yielding $7x$. Multiply $(x-4)$ by $7x$, yielding $7x^2-28x$. Subtract and bring down the next term and repeat.
3. Divide $-4x$ by x, yielding -4. Multiply $(x-4)$ by -4, yielding $-4x+16$. Subtract.

The solution can be written as either $3x^2+7x-4\ \text{R}\ -9$ or $3x^2+7x-4-\dfrac{9}{x-4}$.

The more formal way of writing this answer is the second option.

Example 6.7.3

Divide the polynomial $6x^3 - 8x^2 + 10x + 100$ by $2x + 4$.

$$6x^3 - 8x^2 + 10x + 1002x + 4$$

The steps to get this result are as follows:

1. Divide $6x^3$ by $2x$, yielding $3x^2$. Multiply $(2x + 4)$ by $3x^2$, yielding $6x^3 + 12x^2$. Subtract and bring down the next term and repeat.
2. Divide $-20x^2$ by $2x$, yielding $-10x$. Multiply $(2x + 4)$ by $-10x$, yielding $-20x^2 - 40x$. Subtract and bring down the next term and repeat.
3. Divide $50x$ by $2x$, yielding 25. Multiply $(2x + 4)$ by 25, yielding $50x + 100$. Subtract.

The solution is $3x^2 - 10x + 25$ with no remainder.

Questions

Solve the following polynomial divisions.

1. $(20x^4 + x^3 + 2x^2) \div (4x^3)$
2. $(5x^4 + 45x^3 + 4x^2) \div (9x)$
3. $(20n^4 + n^3 + 40n^2) \div (10n)$
4. $(3k^3 + 4k^2 + 2k) \div (8k)$
5. $(12x^4 + 24x^3 + 3x^2) \div (6x)$
6. $(5p^4 + 16p^3 + 16p^2) \div (4p)$
7. $(10n^4 + 50n^3 + 2n^2) \div (10n^2)$
8. $(3m^4 + 18m^3 + 27m^2) \div (9m^2)$
9. $(45x^2 + 56x + 16) \div (9x + 4)$
10. $(6x^2 + 16x + 16) \div (6x - 2)$
11. $(10x^2 - 32x + 6) \div (10x - 2)$
12. $(x^2 + 7x + 12) \div (x + 4)$
13. $(4x^2 - 33x + 35) \div (4x - 5)$
14. $(4x^2 - 23x - 35) \div (4x + 5)$
15. $(x^3 + 15x^2 + 49x - 49) \div (x + 7)$
16. $(6x^3 - 12x^2 - 43x - 20) \div (x - 4)$
17. $(x^3 - 6x - 40) \div (x + 4)$
18. $(x^3 - 16x^2 + 512) \div (x - 8)$
19. $(x^3 - x^2 - 8x - 16) \div (x - 4)$
20. $(2x^3 + 6x^2 + 4x + 12) \div (2x + 6)$
21. $(12x^3 + 12x^2 - 15x - 9) \div (2x + 3)$

22. $(6x + 18 - 21x^2 + 4x^3) \div (4x + 3)$

Answer Key 6.7

50. 6.8 Mixture and Solution Word Problems

Solving mixture problems generally involves solving systems of equations. Mixture problems are ones in which two different solutions are mixed together, resulting in a new, final solution. Using a table will help to set up and solve these problems. The basic structure of this table is shown below:

Example Mixture Problem Solution Table

Name	Amount	Value	Equation

The first column in the table (Name) is used to identify the fluids or objects being mixed in the problem. The second column (Amount) identifies the amounts of each of the fluids or objects. The third column (Value) is used for the value of each object or the percentage of concentration of each fluid. The last column (Equation) contains the product of the Amount times the Value or Concentration.

Example 6.8.1

Jasnah has 70 mL of a 50% methane solution. How much of a 80% solution must she add so the final solution is 60% methane? Find the equation.

- The solution names are 50% (S_{50}), 60% (S_{60}), and 80% (S_{80}).
- The amounts are S_{50} = 70 mL, S_{80}, and S_{60} = 70 mL + S_{80}.
- The concentrations are S_{50} = 0.50, S_{60} = 0.60, and S_{80} = 0.80.

Name	Amount	Value	Equation
S_{50}	70 mL	0.50	0.50 (70 mL)
S_{80}	S_{80}	0.80	0.80 (S_{80})
S_{60}	70 mL + S_{80}	0.60	0.60 (70 mL + S_{80})

The equation derived from this data is 0.50 (70 mL) + 0.80 (S_{80}) = 0.60 (70 mL + S_{80}).

Example 6.8.2

Sally and Terry blended a coffee mix that sells for $\$2.50$ by mixing two types of coffee. If they used 40 mL of a coffee that costs $\$3.00,$ how much of another coffee costing $\$1.50$ did they mix with the first?

Name	Amount	Value	Equation
$C_{1.50}$	$C_{1.50}$	$\$1.50$	$\$1.50$ ($C_{1.50}$)
$C_{3.00}$	40 mL	$\$3.00$	$\$3.00$ (40 mL)
$C_{2.50}$	40 mL + $C_{1.50}$	$\$2.50$	$\$2.50$ (40 mL + $C_{1.50}$)

The equation derived from this data is:

$$\begin{array}{rcrcrcr} 1.50(C_{1.50}) & + & 3.00(40) & = & 2.50(40 & + & C_{1.50}) \\ 1.50(C_{1.50}) & + & 120 & = & 100 & + & 2.50(C_{1.50}) \\ -2.50(C_{1.50}) & - & 120 & = & -120 & - & 2.50(C_{1.50}) \\ & & -1.00(C_{1.50}) & = & -20 & & \\ \\ & & (C_{1.50}) & = & \dfrac{-20}{-1} & & \\ \\ & & C_{1.50} & = & 20 & & \end{array}$$

This means 20 mL of the coffee selling for $\$1.50$ is needed for the mix.

Example 6.8.3

Nick and Chloe have two grades of milk from their small dairy herd: one that is 24% butterfat and another that is 18% butterfat. How much of each should they use to end up with 42 litres of 20% butterfat?

Name	Amount	Value	Equation
B_{24}	B_{24}	0.24	0.24 (B_{24})
B_{18}	42 L − B_{24}	0.18	0.18 (42 L − B_{24})
B_{20}	42 L	0.20	0.20 (42 L)

The equation derived from this data is:

$$\begin{array}{rcrcrcr} 0.24(B_{24}) & + & 0.18(42 & - & B_{24}) & = & 0.20(42) \\ 0.24(B_{24}) & + & 7.56 & - & 0.18(B_{24}) & = & 8.4 \\ & - & 7.56 & & & & -7.56 \\ & & & & 0.06(B_{24}) & = & 0.84 \\ & & & & B_{24} & = & \dfrac{0.84}{0.06} \\ & & & & B_{24} & = & 14 \end{array}$$

This means 14 litres of the 24% buttermilk, and 28 litres of the 18% buttermilk is needed.

Example 6.8.4

In Natasha's candy shop, chocolate, which sells for $\$4$ a kilogram, is mixed with nuts, which are sold for $\$2.50$ a kilogram. Chocolate and nuts are combined to form a chocolate-nut candy, which sells for $\$3.50$ a kilogram. How much of each are used to make 30 kilograms of the mixture?

Name	Amount	Value	Equation
Chocolate	C	$\$4.00$	$\$4.00$ (C)
Nuts	30 kg – C	$\$2.50$	$\$2.50$ (30 kg – C)
Mix	30 kg	$\$3.50$	$\$3.50$ (30 kg)

The equation derived from this data is:

$$\begin{array}{rcrcrcl} 4.00(C) & + & 2.50(30 & - & C) & = & 3.50(30) \\ 4.00(C) & + & 75 & - & 2.50(C) & = & 105 \\ & - & 75 & & & & -75 \\ & & & & 1.50(C) & = & 30 \\ & & & & C & = & \dfrac{30}{1.50} \\ & & & & C & = & 20 \end{array}$$

Therefore, 20 kg of chocolate is needed for the mixture.

With mixture problems, there is often mixing with a pure solution or using water, which contains none of the chemical of interest. For pure solutions, the concentration is 100%. For water, the concentration is 0%. This is shown in the following example.

Example 6.8.5

Joey is making a a 65% antifreeze solution using pure antifreeze mixed with water. How much of each should be used to make 70 litres?

Name	Amount	Value	Equation
Antifreeze (A)	A	1.00	1.00 (A)
Water (W)	70 L − A	0.00	0.00 (70 L − A)
65% Solution	70 L	0.65	0.65 (70 L)

The equation derived from this data is:

$$\begin{array}{rrl} 1.00(A) \quad + \quad 0.00(70-A) & = & 0.65(0.70) \\ 1.00A & = & 45.5 \\ A & = & 45.5 \end{array}$$

This means the amount of water added is 70 L − 45.5 L = 24.5 L.

Questions

For questions 1 to 9, write the equations that define the relationship.

1. A tank contains 8000 litres of a solution that is 40% acid. How much water should be added to make a solution that is 30% acid?
2. How much pure antifreeze should be added to 5 litres of a 30% mixture of antifreeze to make a solution that is 50% antifreeze?
3. You have 12 kilograms of 10% saline solution and another solution of 3% strength. How many kilograms of the second should be added to the first in order to get a 5% solution?
4. How much pure alcohol must be added to 24 litres of a 14% solution of alcohol in order to produce a 20% solution?
5. How many litres of a blue dye that costs $\$1.60$ per litre must be mixed with 18 litres of magenta dye that costs $\$2.50$ per litre to make a mixture that costs $\$1.90$ per litre?
6. How many grams of pure acid must be added to 40 grams of a 20% acid solution to make a solution which is 36% acid?
7. A 100-kg bag of animal feed is 40% oats. How many kilograms of pure oats must be added to this feed to produce a blend of 50% oats?
8. A 20-gram alloy of platinum that costs $\$220$ per gram is mixed with an alloy that costs $\$400$ per gram. How many grams of the $\$400$ alloy should be used to make an alloy that costs $\$300$ per gram?
9. How many kilograms of tea that cost $\$4.20$ per kilogram must be mixed with 12 kilograms of tea that cost $\$2.25$ per kilogram to make a mixture that costs $\$3.40$ per kilogram?

Solve questions 10 to 21.

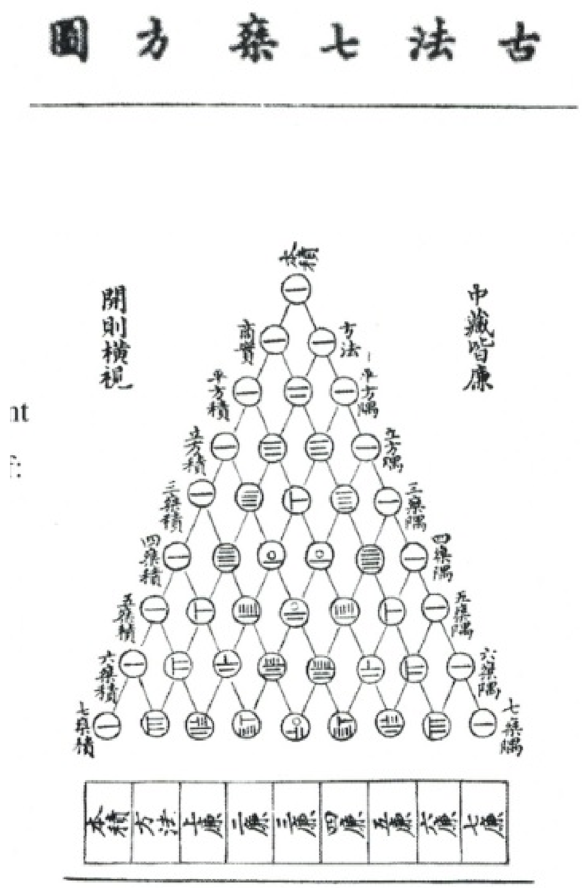
10. How many litres of a solvent that costs $\$80$ per litre must be mixed with 6 litres of a solvent that costs $\$25$ per litre to make a solvent that costs $\$36$ per litre?
11. How many kilograms of hard candy that cost $\$7.50$ per kg must be mixed with 24 kg of jelly beans that cost $\$3.25$ per kg to make a mixture that sells for $\$4.50$ per kg?
12. How many kilograms of soil supplement that costs $\$7.00$ per kg must be mixed with 20 kg of aluminum nitrate that costs $\$3.50$ per kg to make a fertilizer that costs $\$4.50$ per kg?
13. A candy mix sells for $\$2.20$ per kg. It contains chocolates worth $\$1.80$ per kg and other candy worth $\$3.00$ per kg. How much of each are in 15 kg of the mixture?
14. A certain grade of milk contains 10% butterfat and a certain grade of cream 60% butterfat. How many litres of each must be taken so as to obtain a mixture of 100 litres that will be 45% butterfat?
15. Solution A is 50% acid and solution B is 80% acid. How much of each should be used to make 100 cc of a solution that is 68% acid?
16. A paint that contains 21% green dye is mixed with a paint that contains 15% green dye. How many litres of each must be used to make 600 litres of paint that is 19% green dye?
17. How many kilograms of coffee that is 40% java beans must be mixed with coffee that is 30% java beans to make an 80-kg coffee blend that is 32% java beans?
18. A caterer needs to make a slightly alcoholic fruit punch that has a strength of 6% alcohol. How many litres of fruit juice must be added to 3.75 litres of 40% alcohol?
19. A mechanic needs to dilute a 70% antifreeze solution to make 20 litres of 18% strength. How many litres of water must be added?
20. How many millilitres of water must be added to 50 millilitres of 100% acid to make a 40% solution?
21. How many litres of water need to be evaporated from 50 litres of a 12% salt solution to produce a 15% salt solution?

Answer Key 6.8

51. 6.9 Pascal's Triangle and Binomial Expansion

Pascal's triangle (1653) has been found in the works of mathematicians dating back before the 2nd century BC. While Pascal's triangle is useful in many different mathematical settings, it will be applied to the expansion of binomials. In this application, Pascal's triangle will generate the leading coefficient of each term of a binomial expansion in the form of:

$$(a+b)^n$$



Yang Hui's (1238-1298) Triangle (ca. 1303).

For example:

$$\begin{array}{lllllllll} (a+b)^2 & = & a^2 & + & 2ab & + & b^2 & & (1+2+1) \\ (a+b)^3 & = & a^3 & + & 3a^2b & + & b^3 & & (1+3+3+1) \end{array}$$

Pascal's Triangle

$$
\begin{array}{lclcl}
(a+b)^0 & 1 & 2^0 & 1 & (a-b)^0 \\
(a+b)^1 & 1+1 & 2^1 & 1-1 & (a-b)^1 \\
(a+b)^2 & 1+2+1 & 2^2 & 1-2+1 & (a-b)^2 \\
(a+b)^3 & 1+3+3+1 & 2^3 & 1-3+3-1 & (a-b)^3 \\
(a+b)^4 & 1+4+6+4+1 & 2^4 & 1-4+6-4+1 & (a-b)^4 \\
(a+b)^5 & 1+5+10+10+5+1 & 2^5 & 1-5+10-10+5-1 & (a-b)^5 \\
(a+b)^6 & 1+6+15+20+15+6+1 & 2^6 & 1-6+15-20+15-6+1 & (a-b)^6 \\
(a+b)^7 & 1+7+21+35+35+21+7+1 & 2^7 & 1-7+21-35+35-21+7-1 & (a-b)^7
\end{array}
$$

The generation of each row of Pascal's triangle is done by adding the two numbers above it.

$$
\begin{array}{cl}
1 & \text{Start with 1} \\
1+1 & \text{The outside number is always 1} \\
1+2+1 & \text{The two 1's in the last row add to 2} \\
1+3+3+1 & 1+2 \text{ above adds to 3} \\
1+4+6+4+1 & \\
1+5+10+10+5+1 & \\
1+6+15+20+15+6+1 & \\
1+7+21+35+35+21+7+1 & \text{We can extend Pascal's triangle using this} \\
1+8+28+56+70+56+28+8+1 & (a+b)^8 \\
1+9+36+84+126+126+84+36+9+1 & (a+b)^9
\end{array}
$$

Example 6.9.1

Use Pascal's triangle to expand $(a+b)^9$.

The variables will follow a pattern of rising and falling powers:

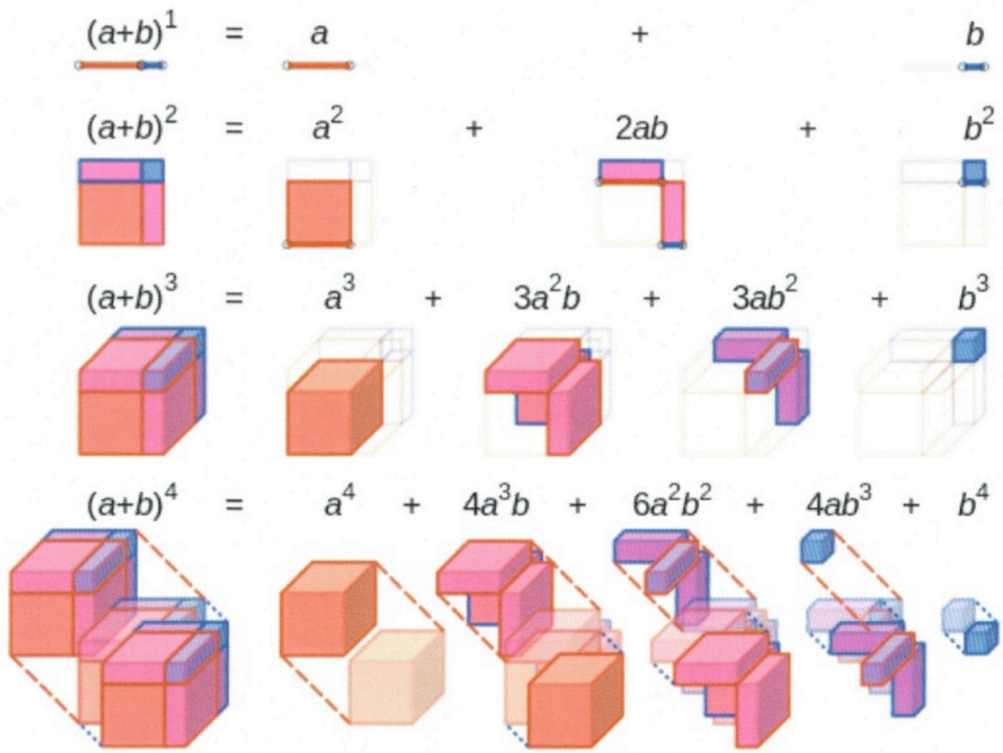
$$a^9+a^8b+a^7b^2+a^6b^3+a^5b^4+a^4b^5+a^3b^6+a^2b^7+ab^8+b^9$$

When we insert the coefficients found from Pascal's triangle, we create:

$$a^9+9a^8b+36a^7b^2+84a^6b^3+126a^5b^4+126a^4b^5+84a^3b^6+36a^2b^7+8ab^8+b^9$$

Problem: Use Pascal's triangle to expand the binomial $(a+b)^{12}$.

A Visual Representation of Binomial Expansion



The fourth expansion of the binomial is generally held to represent time, with the first three expansions being width, length, and height. While we live in a four-dimensional universe (string theory suggests ten dimensions), efforts to represent the fourth dimension of time are challenging. Carl Sagan describes the fourth dimension using an analogy created by Edwin Abbot (Abbot: *Flatland: A Romance of Many Dimensions*). A video clip of Sagan's "Tesseract, 4th Dimension Made Easy" can be found on YouTube.

PART VIII

CHAPTER 7: FACTORING

Learning Objectives

This chapter covers:

- Greatest Common Factor
- Factoring by Grouping
- Factoring Trinomials where a = 1
- Factoring Trinomials where a $\neq 1$
- Factoring Special Products
- Factoring Quadratics of Increasing Difficulty
- Choosing the Correct Factoring Strategy

52. 7.1 Greatest Common Factor

The opposite of multiplying polynomials together is factoring polynomials. Factored polynomials help to solve equations, learn behaviours of graphs, work with fractions and more. Because so many concepts in algebra depend on us being able to factor polynomials, it is important to have very strong factoring skills.

In this section, the focus is on factoring using the greatest common factor or GCF of a polynomial. When you previously multiplied polynomials, you multiplied monomials by polynomials by distributing, solving problems such as $4x^2(2x^2 - 3x + 8)$ to yield $8x^4 - 12x^3 + 32x$. For factoring, you will work the same problem backwards. For instance, you could start with the polynomial $8x^2 - 12x^3 + 32x$ and work backwards to $4x(2x - 3x^2 + 8)$.

To do this, first identify the GCF of a polynomial. Look at finding the GCF of several numbers. To find the GCF of several numbers, look for the largest number that each of the numbers can be divided by.

Example 7.1.1

Find the GCF of 15, 24, 27.

First, break all these numbers into their primes.

$$\begin{array}{rcl}15 & = & 3 \times 5 \\ 24 & = & 2 \times 2 \times 2 \times 3 \text{ or } 2^3 \times 3 \\ 27 & = & 3 \times 3 \times 3 \text{ or } 3^3\end{array}$$

By observation, the only number that each can be divided by is 3. Therefore, the GCF = 3.

Example 7.1.2

Find the GCF of $24x^4y^2z$, $18x^2y^4$, and $12x^3yz^5$.

First, break all these numbers into their primes. (Use • to designate multiplication instead of ×.)

$$\begin{array}{lcl}24x^4y^2z & = & 2^3 \cdot 3 \cdot x^4 \cdot y^2 \cdot z \\ 18x^2y^4 & = & 2 \cdot 3^2 \cdot x^2 \cdot y^4 \\ 12x^3yz^5 & = & 2^2 \cdot 3 \cdot x^3 \cdot y \cdot z^5\end{array}$$

By observation, what is shared between all three monomials is $2 \cdot 3 \cdot x^2 \cdot y$ or $6x^2y$.

Questions

Factor out the common factor in each of the following polynomials.

1. $9+8b^2$
2. $x-5$
3. $45x^2-25$
4. $1+2n^2$
5. $56-35p$
6. $50x-80y$
7. $7ab-35a^2b$
8. $27x^2y^5-72x^3y^2$
9. $-3a^2b+6a^3b^2$
10. $8x^3y^2+4x^3$
11. $-5x^2-5x^3-15x^4$
12. $-32n^9+32n^6+40n^5$
13. $28m^4+40m^3+8$
14. $-10x^4+20x^2+12x$
15. $30b^9+5ab-15a^2$
16. $27y^7+12y^2x+9y^2$
17. $-48a^2b^2-56a^3b-56a^5b$
18. $30m^6+15mn^2-25$
19. $20x^8y^2z^2+15x^5y^2z+35x^3y^3z$
20. $3p+12q-15q^2r^2$
21. $-18n^5+3n^3-21n+3$
22. $30a^8+6a^5+27a^3+21a^2$
23. $-40x^{11}-20x^{12}+50x^{13}-50x^{14}$
24. $-24x^6-4x^4+12x^3+4x^2$
25. $-32mn^8+4m^6n+12mn^4+16mn$
26. $-10y^7+6y^{10}-4y^{10}x-8y^8x$

Answer Key 7.1

53. 7.2 Factoring by Grouping

First thing to do when factoring is to factor out the GCF. This GCF is often a monomial, like in the problem $5xy + 10xz$ where the GCF is the monomial $5x$, so you would have $5x(y + 2z)$. However, a GCF does not have to be a monomial; it could be a binomial. Consider the following two examples.

Example 7.2.1

Find and factor out the GCF for $3ax - 7bx$.

By observation, one can see that both have x in common.

This means that $3ax - 7bx = x(3a - 7b)$.

Example 7.2.2

Find and factor out the GCF for $3a(2a + 5b) - 7b(2a + 5b)$.

Both have $(2a + 5b)$ as a common factor.

This means that if you factor out $(2a + 5b)$, you are left with $3a - 7b$.

The factored polynomial is written as $(2a + 5b)(3a - 7b)$.

In the same way as factoring out a GCF from a binomial, there is a process known as grouping to factor out common binomials from a polynomial containing four terms.

Find and factor out the GCF for $10ab + 15b^2 + 4a + 6b$.

To do this, first split the polynomial into two binomials.

$10ab + 15b^2 + 4a + 6b$ becomes $10ab + 15b^2$ and $4a + 6b$.

Now find the common factor from each binomial.

$10ab + 15b^2$ has a common factor of $5b$ and becomes $5b(2a + 3b)$.

$4a + 6b$ has a common factor of 2 and becomes $2(2a + 3b)$.

This means that $10ab + 15b^2 + 4a + 6b = 5b(2a + 3b) + 2(2a + 3b)$.

$5b(2a + 3b) + 2(2a + 3b)$ can be factored as $(2a + 3b)(5b + 2)$.

Questions

Factor the following polynomials.

1. $40r^3 - 8r^2 - 25r + 5$
2. $35x^3 - 10x^2 - 56x + 16$
3. $3n^3 - 2n^2 - 9n + 6$
4. $14v^3 + 10v^2 - 7v - 5$
5. $15b^3 + 21b^2 - 35b - 49$
6. $6x^3 - 48x^2 + 5x - 40$
7. $35x^3 - 28x^2 - 20x + 16$
8. $7n^3 + 21n^2 - 5n - 15$
9. $7xy - 49x + 5y - 35$
10. $42r^3 - 49r^2 + 18r - 21$
11. $16xy - 56x + 2y - 7$
12. $3mn - 8m + 15n - 40$
13. $2xy - 8x^2 + 7y^3 - 28y^2x$
14. $5mn + 2m - 25n - 10$
15. $40xy + 35x - 8y^2 - 7y$
16. $8xy + 56x - y - 7$
17. $10xy + 30 + 25x + 12y$
18. $24xy + 25y^2 - 20x - 30y^3$
19. $3uv + 14u - 6u^2 - 7v$
20. $56ab + 14 - 49a - 16b$

Answer Key 7.2

54. 7.3 Factoring Trinomials where a = 1

Factoring expressions with three terms, or trinomials, is a very important type of factoring to master, since this kind of expression is often a quadratic and occurs often in real life applications. The strategy to master these is to turn the trinomial into the four-term polynomial problem type solved in the previous section. The tool used to do this is central to the Master Product Method. To better understand this, consider the following example.

Example 7.3.1

Factor the trinomial $x^2 + 2x - 24$.

Start by multiplying the coefficients from the first and the last terms. This is $1 \cdot -24$, which yields −24.

The next task is to find all possible integers that multiply to −24 and their sums.

$$\begin{array}{cc} \text{multiply to } -24 & \text{sum of these integers} \\ -1 \cdot 24 & 23 \\ -2 \cdot 12 & 10 \\ -3 \cdot 8 & 5 \\ -4 \cdot 6 & 2 \\ -6 \cdot 4 & -2 \\ -8 \cdot 3 & -5 \\ -12 \cdot 2 & -10 \\ -24 \cdot 1 & -23 \end{array}$$

Look for the pair of integers that multiplies to −24 and adds to 2, so that it matches the equation that you started with.

For this example, the pair is $-4 \cdot 6$, which adds to 2.

Now take the original trinomial $x^2 + 2x - 24$ and break the $2x$ into $-4x$ and $6x$.

Rewrite the original trinomial as $x^2 - 4x + 6x - 24$.

Now, split this into two binomials as done in the previous section and factor.

$$x^2 - 4x \text{ yields } x(x-4) \text{ and } 6x - 24 \text{ yields } 6(x-4).$$

$$x^2 - 4x + 6x - 24 \text{ becomes } x(x-4) + 6(x-4).$$

$$x(x-4) + 6(x-4) \text{ factors to } (x-4)(x+6).$$

$$x^2 + 2x - 24 = (x-4)(x+6)$$

Example 7.3.2

Factor the trinomial $x^2 + 9x + 18$.

Start by multiplying the coefficients from the first and the last terms. This is $1 \cdot 18$, which yields 18.

The next task is to find all possible integers that multiply to 18 and their sums.

multiply to 18	sum of these integers
$1 \cdot 18$	19
$2 \cdot 9$	11
$3 \cdot 6$	9
$6 \cdot 3$	9
$9 \cdot 2$	11
$18 \cdot 1$	19

Look for the pair of integers that multiplies to 18 and adds to 9, so that it matches the equation that you started with.

For this example, the pair is $3 \cdot 6$, which adds to 9.

Now take the original trinomial $x^2 + 9x + 18$ and break the $9x$ into $3x$ and $6x$.

Rewrite the original trinomial as $x^2 + 3x + 6x + 18$.

Now, split this into two binomials as done in the previous section and factor.

$$x^2 + 3x \text{ yields } x(x+3) \text{ and } 6x + 18 \text{ yields } 6(x+3).$$

$$x^2 + 3x + 6x + 18 \text{ becomes } x(x+3) + 6(x+3).$$

$$x(x+3) + 6(x+3) \text{ factors to } (x+3)(x+6).$$

$$x^2 + 9x + 18 = (x+3)(x+6)$$

Please note the following is also true:

multiply to 18	sum of these integers
$-1 \cdot -18$	-19
$-2 \cdot -9$	-11
$-3 \cdot -6$	-9
$-6 \cdot -3$	-9
$-9 \cdot -2$	-11
$-18 \cdot -1$	-19

This means that solutions can be found where the middle term is $19x$, $11x$, $9x$, $-19x$, $-11x$ or $-9x$.

Questions

Factor each of the following trinomials.

1. $p^2 + 17p + 72$
2. $x^2 + x - 72$

3. $n^2 - 9n + 8$
4. $x^2 + x - 30$
5. $x^2 - 9x - 10$
6. $x^2 + 13x + 40$
7. $b^2 + 12b + 32$
8. $b^2 - 17b + 70$
9. $u^2 - 8uv + 15v^2$
10. $m^2 - 3mn - 40n^2$
11. $m^2 + 2mn - 8n^2$
12. $x^2 + 10xy + 16y^2$
13. $x^2 - 11xy + 18y^2$
14. $u^2 - 9uv + 14v^2$
15. $x^2 + xy - 12y^2$
16. $x^2 + 14xy + 45y^2$

Answer Key 7.3

55. 7.4 Factoring Trinomials where a ≠ 1

Factoring trinomials where the leading term is not 1 is only slightly more difficult than when the leading coefficient is 1. The method used to factor the trinomial is unchanged.

Example 7.4.1

Factor the trinomial $3x^2 + 11x + 6$.

Start by multiplying the coefficients from the first and the last terms. This is $3 \cdot 6$, which yields 18.

The next task is to find all possible integers that multiply to 18 and their sums.

multiply to 18	sum of these integers
$1 \cdot 18$	19
$2 \cdot 9$	11
$3 \cdot 6$	9
$6 \cdot 3$	9
$9 \cdot 2$	11
$18 \cdot 1$	19

Look for the pair of integers that multiplies to 18 and adds to 11, so that it matches the equation that you started with.

For this example, the pair is $2 \cdot 9$, which adds to 11.

Now take the original trinomial $3x^2 + 11x + 6$ and break the $11x$ into $2x$ and $9x$.

Rewrite the original trinomial as $3x^2 + 2x + 9x + 6$.

Now, split this into two binomials as done in the previous section and factor.

$$3x^2 + 2x \text{ yields } x(3x+2) \text{ and } 9x + 6 \text{ yields } 3(3x+2).$$

$$3x^2 + 2x + 9x + 6 \text{ becomes } x(3x+2) + 3(3x+2).$$

$$x(3x+2) + 3(3x+2) \text{ factors to } (3x+2)(x+3).$$

$$3x^2 + 11x + 6 = (3x+2)(x+3)$$

The master product method works for any integer breakup of the polynomial. Slightly more complicated are questions that involve two different variables in the original polynomial.

Example 7.4.2

Factor the trinomial $4x^2 - xy - 5y^2$.

Start by multiplying the coefficients from the first and the last terms. This is $4 \cdot -5$, which yields −20.

The next task is to find all possible integers that multiply to −20 and their sums.

multiply to -20	sum of these integers
$-1 \cdot 20$	19
$-2 \cdot 10$	8
$-4 \cdot 5$	1
$-5 \cdot 4$	-1
$-10 \cdot 2$	-8
$-20 \cdot 1$	-19

Look for the pair of integers that multiplies to −20 and adds to −11, so that it matches the equation that you started with.

For this example, the pair is $-5 \cdot 4$, which adds to −1.

Now take the original trinomial $4x^2 - xy - 5y^2$ and break the $-xy$ into $-5xy$ and $4xy$.

Rewrite the original trinomial as $4x^2 - 5xy + 4xy - 5y^2$.

Now, split this into two binomials as done in the previous section and factor.

$4x^2 - 5xy$ yields $x(4x - 5y)$ and $4xy - 5y^2$ yields $y(4x - 5y)$.

$4x^2 - xy - 5y^2$ becomes $x(4x - 5y) + y(4x - 5y)$.

$x(4x - 5y) + y(4x - 5y)$ factors to $(x + y)(4x - 5y)$.

$$4x^2 - xy - 5y^2 = (x + y)(4x - 5y)$$

There are a number of variations potentially encountered when factoring trinomials. For instance, the original terms might be mixed up. There could be something like $-10x + 3x^2 + 8$ that is not in descending powers of x. This requires reordering in descending powers before beginning to factor.

$$-10x + 3x^2 + 8 \longrightarrow 3x^2 - 10x + 8 \text{ (factorable form)}$$

It might also be necessary to factor out a common factor before starting. The polynomial above can be written as $30x^2 - 100x + 80$, in which a common factor of 10 should be factored out prior to factoring.

$$\text{This turns } 30x^2 - 100x + 80 \text{ into } 10(3x^2 - 10x + 8).$$

There are also slight variations on the common factored binomial that can be illustrated by factoring the trinomial $3x^2 - 10x + 8$.

Example 7.4.3

Factor the trinomial $3x^2 - 10x + 8$.

Start by multiplying the coefficients from the first and the last terms. This is $3 \cdot 8$, which yields 24.

The next task is to find all possible integers that multiply to 24 and their sums (knowing that the middle coefficient must be negative).

multiply to 24	sum of these integers
$-1 \cdot -24$	-25
$-2 \cdot -12$	-14
$-3 \cdot -8$	-11
$-4 \cdot -6$	-10
$-6 \cdot -4$	-10
$-8 \cdot -3$	-11
$-12 \cdot -2$	-14
$-24 \cdot -1$	-25

Look for the pair of integers that multiplies to 24 and adds to −10, so that it matches the equation that you started with.

For this example, the pair is $-4 \cdot -6$, which adds to −10.

Now take the original trinomial $3x^2 - 10x + 8$ and break the $-10x$ into $-4x$ and $-6x$.

Rewrite the original trinomial as $3x^2 - 4x - 6x + 8$.

Now, split this into two binomials as done in the previous section and factor.

$3x^2 - 4x$ yields $x(3x - 4)$, but $-6x + 8$ yields $2(-3x + 4)$.

$x(3x - 4)$ and $2(-3x + 4)$ are a close match, but their signs are different.

The way to deal with this is to factor out a negative, specifically, −2 instead of 2.

$-6x + 8$ can be factored two ways: $2(-3x + 4)$ and $-2(3x - 4)$.

Choose the second factoring, so the common factor matches.

$3x^2 - 10x + 8$ becomes $x(3x - 4) + -2(3x - 4)$.

$x(3x - 4) + -2(3x - 4)$ factors to $(3x - 4)(x - 2)$.

$$3x^2 - 10x + 8 = (3x - 4)(x - 2)$$

Example 7.4.4

Factor the following trinomials, which are both variations of the trinomial seen before in 7.4.3:

1. $3x^2 - 14x + 8$
   The pair of numbers that can be used to break it up is −2 and −12.

$3x^2-14x+8$ breaks into $3x^2-2x-12x+8$

$x(3x-2)-4(3x-2)$ Common factor is $(3x-2)$

$(3x-2)(x-4)$ Left over when factored

2. $3x^2-11x+8$

The pair of numbers that can be used to break it up is −3 and −8.

$3x^2-11x+8$ breaks into $3x^2-3x-8x+8$

$3x(x-1)-8(x-1)$ Common factor is $(x-1)$

$(x-1)(3x-8)$ Left over when factored

Questions

Factor each of the following trinomials.

1. $7x^2-19x-6$
2. $3n^2-2n-8$
3. $7b^2+15b+2$
4. $21v^2-11v-2$
5. $5a^2+13a-6$
6. $5n^2-18n-8$
7. $2x^2-5x+2$
8. $3r^2-4r-4$
9. $2x^2+19x+35$
10. $3x^2+4x-15$
11. $2b^2-b-3$
12. $2k^2+5k-12$
13. $3x^2+17xy+10y^2$
14. $7x^2-2xy-5y^2$
15. $3x^2+11xy-20y^2$
16. $12u^2+16uv-3v^2$
17. $4k^2-17k+4$
18. $4r^2+3r-7$
19. $4m^2-9mn-9n^2$
20. $4x^2-6xy+30y^2$
21. $4x^2+13xy+3y^2$
22. $6u^2+5uv-4v^2$
23. $10x^2+19xy-2y^2$
24. $6x^2-13xy-5y^2$

Answer Key 7.4

56. 7.5 Factoring Special Products

Now transition from multiplying special products to factoring special products. If you can recognize them, you can save a lot of time. The following is a list of these special products (note that $a^2 + b^2$ cannot be factored):

$$\begin{aligned} a^2 - b^2 &= (a+b)(a-b) \\ (a+b)^2 &= a^2 + 2ab + b^2 \\ (a-b)^2 &= a^2 - 2ab + b^2 \\ a^3 - b^3 &= (a-b)(a^2 + ab + b^2) \\ a^3 + b^3 &= (a+b)(a^2 - ab + b^2) \end{aligned}$$

The challenge is therefore in recognizing the special product.

Example 7.5.1

Factor $x^2 - 36$.

This is a difference of squares. $(x-6)(x+6)$ is the solution.

Example 7.5.2

Factor $x^2 - 6x + 9$.

This is a perfect square. $(x-3)(x-3)$ or $(x-3)^2$ is the solution.

Example 7.5.3

Factor $x^2 + 6x + 9$.

This is a perfect square. $(x+3)(x+3)$ or $(x+3)^2$ is the solution.

Example 7.5.4

Factor $4x^2 + 20xy + 25y^2$.

This is a perfect square. $(2x + 5y)(2x + 5y)$ or $(2x + 5y)^2$ is the solution.

Example 7.5.5

Factor $m^3 - 27$.

This is a difference of cubes. $(m - 3)(m^2 + 3m + 9)$ is the solution.

Example 7.5.6

Factor $125p^3 + 8r^3$.

This is a difference of cubes. $(5p + 2r)(25p^2 - 10pr + 4r^2)$ is the solution.

Questions

Factor each of the following polynomials.

1. $r^2 - 16$
2. $x^2 - 9$
3. $v^2 - 25$
4. $x^2 - 1$
5. $p^2 - 4$
6. $4v^2 - 1$
7. $3x^2 - 27$
8. $5n^2 - 20$
9. $16x^2 - 36$
10. $125x^2 + 45y^2$
11. $a^2 - 2a + 1$
12. $k^2 + 4k + 4$
13. $x^2 + 6x + 9$

14. $n^2 - 8n + 16$
15. $25p^2 - 10p + 1$
16. $x^2 + 2x + 1$
17. $25a^2 + 30ab + 9b^2$
18. $x^2 + 8xy + 16y^2$
19. $8x^2 - 24xy + 18y^2$
20. $20x^2 + 20xy + 5y^2$
21. $8 - m^3$
22. $x^3 + 64$
23. $x^3 - 64$
24. $x^3 + 8$
25. $216 - u^3$
26. $125x^3 - 216$
27. $125a^3 - 64$
28. $64x^3 - 27$
29. $64x^3 + 27y^3$
30. $32m^3 - 108n^3$

Answer Key 7.5

57. 7.6 Factoring Quadratics of Increasing Difficulty

Factoring equations that are more difficult involves factoring equations and then checking the answers to see if they can be factored again.

Example 7.6.1

Factor $y^4 - 81x^4$.

This is a standard difference of squares that can be rewritten as $(y^2)^2 - (9x^2)^2$, which factors to $(y^2 - 9x^2)(y^2 + 9x^2)$. This is not completely factored yet, since $(y^2 - 9x^2)$ can be factored once more to give $(y - 3x)(y + 3x)$.

Therefore, $y^4 - 81x^4 = (y^2 + 9x^2)(y - 3x)(y + 3x)$.

This multiple factoring of an equation is also common in mixing differences of squares with differences of cubes.

Example 7.6.2

Factor $x^6 - 64y^6$.This is a standard difference of squares that can be rewritten as $(x^3)^2 + (8x^3)^2$, which factors to $(x^3 - 8y^3)(x^3 + 8x^3)$. This is not completely factored yet, since both $(x^3 - 8y^3)$ and $(x^3 + 8x^3)$ can be factored again.

$(x^3 - 8y^3) = (x - 2y)(x^2 + 2xy + y^2)$ and
$(x^3 + 8y^3) = (x + 2y)(x^2 - 2xy + y^2)$

This means that the complete factorization for this is:

$$x^6 - 64y^6 = (x - 2y)(x^2 + 2xy + y^2)(x + 2y)(x^2 - 2xy + y^2)$$

Example 7.6.3

A more challenging equation to factor looks like $x^6 + 64y^6$. This is not an equation that can be put in the factorable form of a difference of squares. However, it can be put in the form of a sum of cubes.

$$x^6 + 64y^6 = (x^2)^3 + (4y^2)^3$$

In this form, $(x^2)^3 + (4y^2)^3$ factors to $(x^2 + 4y^2)(x^4 + 4x^2y^2 + 64y^4)$.

Therefore, $x^6 + 64y^6 = (x^2 + 4y^2)(x^4 + 4x^2y^2 + 64y^4)$.

Example 7.6.4

Consider encountering a sum and difference of squares question. These can be factored as follows:

$(a+b)^2 - (2a-3b)^2$ factors as a standard difference of squares as shown below:

$$(a+b)^2 - (2a-3b)^2 = [(a+b) - (2a-3b)][(a+b) + (2a-3b)]$$

Simplifying inside the brackets yields:

$$[a + b - 2a + 3b][a + b + 2a - 3b]$$

Which reduces to:

$$[-a + 4b][3a - 2b]$$

Therefore:

$$(a+b)^2 - (2a-3b)^2 = [-a - 4b][3a - 2b]$$

Examples 7.6.5

Consider encountering the following difference of cubes question. This can be factored as follows:

$(a+b)^3 - (2a-3b)^3$ factors as a standard difference of squares as shown below:

$$\begin{gathered}(a+b)^3 - (2a-3b)^3 \\ = [(a+b) - (2a+3b)][(a+b)^2 + (a+b)(2a+3b) + (2a+3b)^2]\end{gathered}$$

Simplifying inside the brackets yields:

$$[a + b - 2a - 3b][a^2 + 2ab + b^2 + 2a^2 + 5ab + 3b^2 + 4a^2 + 12ab + 9b^2]$$

Sorting and combining all similar terms yields:

$$\begin{array}{rrr} & [\quad a + \quad b] & [\ a^2 + \quad 2ab + \quad b^2] \\ & [-2a - 3b] & [2a^2 + \quad 5ab + \ \ 3b^2] \\ + & & [4a^2 + 12ab + \ \ 9b^2] \\ & [-a - 2b] & [7a^2 + 19ab + 13b^2] \end{array}$$

Therefore, the result is:

$$(a+b)^3-(2a-3b)^3=[-a-2b][7a^2+19ab+13b^2]$$

Questions

Completely factor the following equations.

1. x^4-16y^4
2. $16x^4-81y^4$
3. x^4-256y^4
4. $625x^4-81y^4$
5. $81x^4-16y^4$
6. x^4-81y^4
7. $625x^4-256y^4$
8. x^4-81y^4
9. x^6-y^6
10. x^6+y^6
11. x^6-64y^6
12. $64x^6+y^6$
13. $729x^6-y^6$
14. $729x^6+y^6$
15. $729x^6+64y^6$
16. $64x^6-15625y^6$
17. $(a+b)^2-(c-d)^2$
18. $(a+2b)^2-(3a-4b)^2$
19. $(a+3b)^2-(2c-d)^2$
20. $(3a+b)^2-(a-b)^2$
21. $(a+b)^3-(c-d)^3$
22. $(a+3b)^3+(4a-b)^3$

Answer Key 7.6

58. 7.7 Choosing the Correct Factoring Strategy

With so many different tools used to factor, it is prudent to have a section to determine the best strategy to factor.

Factoring Hints

1. Look for any factor to simplify the polynomial before you start!
2. If you have two terms, look for a sum or difference of squares or cubes.
   - $a^2 - b^2 = (a+b)(a-b)a^3 - b^3 = (a-b)(a^2+ab+b^2)$
     $a^3 + b^3 = (a+b)(a^2 - ab + b2)$
3. If you have three terms, see if the master product method works.
4. If you have four terms, see if factoring by grouping works.

If you find that you have trouble factoring the questions in this section, then you might need to redo from the start of Chapter 7. This is not uncommon.

Questions

Factor each completely.

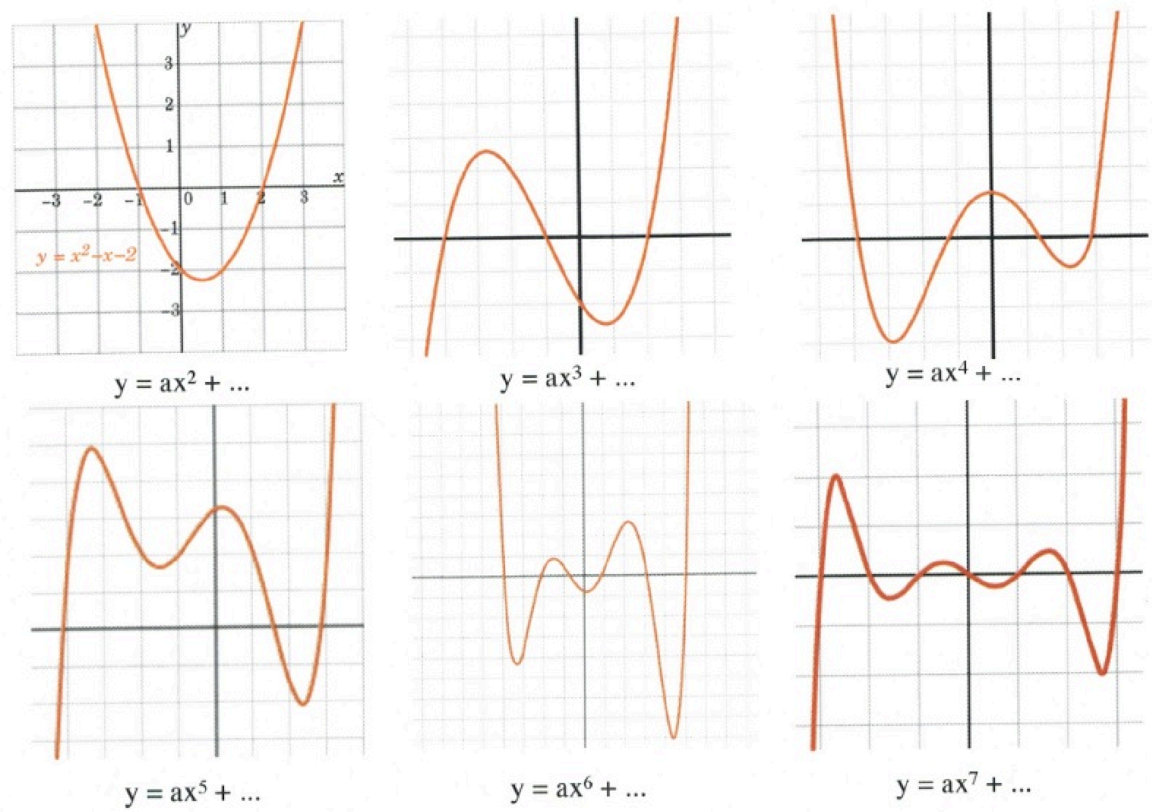
1. $24ac - 18ab + 60dc - 45db$
2. $2x^2 - 11x + 15$
3. $5u^2 - 9uv + 4v^2$
4. $16x^2 + 48xy + 36y^2$
5. $-2x^3 + 128y^3$
6. $20uv - 60u^3 - 5xv + 15xu^2$
7. $54u^3 - 16$
8. $54 - 128x^3$
9. $n^2 - n$
10. $5x^2 - 22x - 15$
11. $x^2 - 4xy + 3y^2$
12. $45u^2 - 150uv + 125v^2$
13. $m^2 - 4n^2$
14. $12ab - 18a + 6nb - 9n$
15. $36b^2c - 16ad - 24b^2d + 24ac$
16. $3m^3 - 6m^2n - 24n^2m$
17. $128 + 54x^3$
18. $64m^3 + 27n^3$
19. $n^3 + 7n^2 + 10n$

20. $64m^3 - n^3$
21. $27x^3 - 64$
22. $16a^2 - 9b^2$
23. $5x^2 + 2x$
24. $2x^2 - 10x + 12$

Answer Key 7.7

59. 7.8 Solving Quadriatic Equations by Factoring

Solving quadratics is an important algebraic tool that finds value in many disciplines. Typically, the quadratic is in the form of $y = ax^2 + bx + c$, which when graphed is a parabola. Of special importance are the x-values that are found when $y = 0$, which show up when graphed as the parabola crossing the x-axis. For a trinomial, there can be as many as three x-axis crossings. The following show the possible number of x-axis intercepts for 2nd degree (quadratic) to 7th degree (septic) functions. Please note that if the fifth degree polynomial were shifted down a few values, it would also show 5 x-axis intercepts. It is these x-axis intercepts that are of interest.



The approach to finding these x-intercepts is elementary: set $y = 0$ in the original equation and factor it. Once the equation is factored, then find the x-values that solve for 0. This is shown in the next few examples.

Example 7.8.1

Solve the following quadratic equation: $(2x - 3)(5x + 1) = 0$.

In this problem there are two separate binomials: $(2x - 3)$ and $(5x + 1)$. Since their product is equal to

0, there will be two solutions: the value for x that makes $2x-3=0$ and the value for x that makes $5x+1=0$.

These are:

$$\begin{array}{rrrrrcrrrrr} 2x & - & 3 & = & 0 & \text{and} & 5x & + & 1 & = & 0 \\ & + & 3 & & +3 & & & - & 1 & & -1 \\ & & 2x & = & 3 & & & & 5x & = & -1 \\ \\ & & x & = & \dfrac{3}{2} & & & & x & = & -\dfrac{1}{5} \end{array}$$

Example 7.8.2

Solve the following polynomial equation: $(x-3)(x+3)(x-1)(x+1)=0$.

For this polynomial, there are four different solutions:

$$(x-3)=0, (x+3)=0, (x-1)=0, (x+1)=0$$

Solving for these four x-values gives us:

$$\begin{array}{rrrrrrrrrrrrrrrrrrrr} x & - & 3 & = & 0 & x & + & 3 & = & 0 & x & - & 1 & = & 0 & x & + & 1 & = & 0 \\ & + & 3 & & +3 & & - & 3 & & -3 & & + & 1 & & +1 & & - & 1 & & -1 \\ & & x & = & 3 & & & x & = & -3 & & & x & = & 1 & & & x & = & -1 \end{array}$$

The solutions are: $x=\pm 3, \pm 1$.

It would be nice if there were only given factored equations to solve, but that is not how it goes. You are generally required to factor the equation first before it can be solved.

Example 7.8.3

Solve the following quadratic equation: $4x^2+x-3=0$.

First, factor $4x^2+x-3$ and get $(4x-3)(x+1)=0$.

Now, solve for $4x-3=0$ and $x+1=0$.

Solving these two binomials yields:

$$\begin{array}{rrrrr} 4x & - & 3 & = & 0 \\ & + & 3 & & +3 \\ & & 4x & = & 3 \\ \\ & & x & = & \dfrac{3}{4} \end{array} \qquad \begin{array}{rrrrr} x & + & 1 & = & 0 \\ & - & 1 & & -1 \\ & & x & = & -1 \\ \\ & & x & = & -1 \end{array}$$

Questions

Solve each of the following polynomials by using factoring.

1. $(k-7)(k+2)=0$
2. $(a+4)(a-3)=0$
3. $(x-1)(x+4)=0$
4. $(2x+5)(x-7)=0$
5. $6x^2-150=0$
6. $p^2+4p-32=0$
7. $2n^2+10n-28=0$
8. $m^2-m-30=0$
9. $7x^2+26x+15=0$
10. $2b^2-3b-2=0$
11. $x^2-4x-8=-8$
12. $v^2-8v-3=-3$
13. $x^2-5x-1=-5$
14. $a^2-6a+6=-2$
15. $7x^2+17x-20=-8$
16. $4n^2-13n+8=5$
17. $x^2-6x=16$
18. $7n^2-28n=0$
19. $4k^2+22k+23=6k+7$
20. $a^2+7a-9=-3+6a$
21. $9x^2-46+7x=7x+8x^2+3$
22. $x^2+10x+30=6$
23. $40p^2+183p-168=p+5p^2$
24. $24x^2+11x-80=3x$

Answer Key 7.8

60. 7.9 Age Word Problems

One application of linear equations is what are termed age problems. When solving age problems, generally the age of two different people (or objects) both now and in the future (or past) are compared. The objective of these problems is usually to find each subject's current age. Since there can be a lot of information in these problems, a chart can be used to help organize and solve. An example of such a table is below.

Person or Object	Current Age	Age Change

Example 7.9.1

Joey is 20 years younger than Becky. In two years, Becky will be twice as old as Joey. Fill in the age problem chart, but do not solve.

- The first sentence tells us that Joey is 20 years younger than Becky (this is the current age)
- The second sentence tells us two things:
  1. The age change for both Joey and Becky is plus two years
  2. In two years, Becky will be twice the age of Joey in two years

Person or Object	Current Age	Age Change (+2)
Joey (J)	$B - 20$	$B - 20 + 2$ $B - 18$
Becky (B)	B	$B = 2$

Using this last statement gives us the equation to solve:

$$B + 2 = 2(B - 18)$$

Example 7.9.2

Carmen is 12 years older than David. Five years ago, the sum of their ages was 28. How old are they now?

- The first sentence tells us that Carmen is 12 years older than David (this is the current age)
- The second sentence tells us the age change for both Carmen and David is five years ago (−5)

Filling in the chart gives us:

Person or Object	Current Age	Age Change (−5)
Carmen (C)	D + 12	D + 12 − 5 D + 7
David (D)	D	D − 5

The last statement gives us the equation to solve:

Five years ago, the sum of their ages was 28

$$\begin{array}{rcl} (D+7)+(D-5) &=& 28 \\ 2D+2 &=& 28 \\ -2 && -2 \\ 2D &=& 26 \\ \\ D &=& \dfrac{26}{2}=13 \end{array}$$

Therefore, Carmen is David's age (13) + 12 years = 25 years old.

Example 7.9.3

The sum of the ages of Nicole and Kristin is 32. In two years, Nicole will be three times as old as Kristin. How old are they now?

- The first sentence tells us that the sum of the ages of Nicole (N) and Kristin (K) is 32. So N + K = 32, which means that N = 32 − K or
  K = 32 − N (we will use these equations to eliminate one variable in our final equation)
- The second sentence tells us that the age change for both Nicole and Kristen is in two years (+2)

Filling in the chart gives us:

Person or Object	Current Age	Age Change (+2)
Nicole (N)	N	N + 2
Kristin (K)	32 − N	(32 − N) + 2 34 − N

The last statement gives us the equation to solve:

In two years, Nicole will be three times as old as Kristin

$$\begin{array}{rcrcrcr} N & + & 2 & = & 3(34 & - & N) \\ N & + & 2 & = & 102 & - & 3N \\ +3N & - & 2 & & -2 & + & 3N \\ & & 4N & = & 100 & & \end{array}$$

$$N = \frac{100}{4} = 25$$

If Nicole is 25 years old, then Kristin is 32 − 25 = 7 years old.

Example 7.9.4

Louise is 26 years old. Her daughter Carmen is 4 years old. In how many years will Louise be double her daughter's age?

- The first sentence tells us that Louise is 26 years old and her daughter is 4 years old
- The second line tells us that the age change for both Carmen and Louise is to be calculated (x)

Filling in the chart gives us:

Person or Object	Current Age	Age Change
Louise (L)	26	$26 = x$
Daughter (D)	4	$D = x$

The last statement gives us the equation to solve:

In how many years will Louise be double her daughter's age?

$$\begin{array}{rcrcrcr} 26 & + & x & = & 2(4 & + & x) \\ 26 & + & x & = & 8 & + & 2x \\ -26 & - & 2x & & -26 & - & 2x \\ & & -x & = & -18 & & \\ & & x & = & 18 & & \end{array}$$

In 18 years, Louise will be twice the age of her daughter.

Questions

For Questions 1 to 8, write the equation(s) that define the relationship.

1. Rick is 10 years older than his brother Jeff. In 4 years, Rick will be twice as old as Jeff.
2. A father is 4 times as old as his son. In 20 years, the father will be twice as old as his son.
3. Pat is 20 years older than his son James. In two years, Pat will be twice as old as James.
4. Diane is 23 years older than her daughter Amy. In 6 years, Diane will be twice as old as Amy.
5. Fred is 4 years older than Barney. Five years ago, the sum of their ages was 48.
6. John is four times as old as Martha. Five years ago, the sum of their ages was 50.
7. Tim is 5 years older than JoAnn. Six years from now, the sum of their ages will be 79.
8. Jack is twice as old as Lacy. In three years, the sum of their ages will be 54.

Solve Questions 9 to 20.

9. The sum of the ages of John and Mary is 32. Four years ago, John was twice as old as Mary.
10. The sum of the ages of a father and son is 56. Four years ago, the father was 3 times as old as the son.
11. The sum of the ages of a wood plaque and a bronze plaque is 20 years. Four years ago, the bronze plaque was one-half the age of the wood plaque.
12. A man is 36 years old and his daughter is 3. In how many years will the man be 4 times as old as his daughter?
13. Bob's age is twice that of Barry's. Five years ago, Bob was three times older than Barry. Find the age of both.
14. A pitcher is 30 years old, and a vase is 22 years old. How many years ago was the pitcher twice as old as the vase?
15. Marge is twice as old as Consuelo. The sum of their ages seven years ago was 13. How old are they now?
16. The sum of Jason and Mandy's ages is 35. Ten years ago, Jason was double Mandy's age. How old are they now?
17. A silver coin is 28 years older than a bronze coin. In 6 years, the silver coin will be twice as old as the bronze coin. Find the present age of each coin.
18. The sum of Clyde and Wendy's ages is 64. In four years, Wendy will be three times as old as Clyde. How old are they now?
19. A sofa is 12 years old and a table is 36 years old. In how many years will the table be twice as old as the sofa?
20. A father is three times as old as his son, and his daughter is 3 years younger than his son. If the sum of all three ages 3 years ago was 63 years, find the present age of the father.

Answer Key 7.9

61. 7.10 The New Committee Member's Age

Every four years, a swim committee has an election for 10 council members. In 2016, nine members stayed, and the oldest council member retired and was replaced by a new, younger member. If the average age of the committee members in 2012 was exactly the same as the average age of the committee members in 2016, how many years younger is the new member than the one who retired?

Answer Key 7.10

PART IX

MIDTERM 2 PREPARATION

The second midterm will be composed of fifteen questions covering chapters 5 to 7.

Twelve questions will be algebra questions, and three questions will be word problems.

- **Questions 1–4 will be drawn from Chapter 5: Systems of Equations**
  - 5.1 Graphed Solutions
  - 5.2 Substitution Solutions
  - 5.3 Addition and Subtraction Solutions
  - 5.4 Solving for Three Variables
- **Questions 5–6 will be drawn from Chapter 6: Polynomials**
  - 6.1 Working with Exponents
  - 6.2 Negative Exponents
  - 6.3 Scientific Notation
  - 6.4 Basic Operations Using Polynomials
  - 6.5 Multiplication of Polynomials
  - 6.6 Special Products
  - 6.7 Dividing Polynomials
- **Questions 9–12 will be drawn from Chapter 7: Factoring**
  - 7.1 Greatest Common Factor
  - 7.2 Factoring by Grouping
  - 7.3 Factoring Trinomials where a = 1
  - 7.4 Factoring Trinomials where a ≠ 1
  - 7.5 Factoring Special Products
  - 7.6 Factoring Quadratics of Increasing Difficulty
  - 7.7 Choosing the Correct Factoring Strategy
  - 7.8 Solving Quadratic Equations by Factoring
- **Questions 13–15 will be drawn from:**
  - 5.5 Monetary Word Problems
  - 6.8 Mixture and Solution Word Problems
  - 7.9 Age Word Problems

Students will be allowed to use MATQ 1099 Data Booklets & Glossaries for both Midterms and the Final Exam.

Midterm Two Review

Chapter 4: Systems of Equations (Exam Type Questions)

Find the solution set of the system of equations below graphically.

1. $\begin{cases} x - 2y + 4 = 3 \\ x + y - 5 = 0 \end{cases}$

For problems 2–4, find the solution set of each system by any convenient method.

2. $\begin{cases} 2x - y = 0 \\ 3x + 4y = -22 \end{cases}$
3. $\begin{cases} 2x - 5y = 15 \\ 3x + 2y = 13 \end{cases}$
4. $\begin{cases} 5x + y + 6z = -2 \\ 2y - 3z = 3 \\ 5x + 6z = -4 \end{cases}$

Chapter 5: Polynomials (Exam Type Questions)

Simplify.

5. $(4a^2 - 9a + 2) - (a^2 - 4a - 5) + (9 - a + 3a^2)$
6. $2x^2(4x^2 - 6y^2) - 3x(5xy^2 + x^3)$
7. $6 - 2[3x - 4(5x - 2) - (1 - 7x)^0]$
8. $(5a^{-5}b^3)^2$
9. $8a^2(a + 5)^2$
10. $4ab^2(a - 2)(a + 2)$
11. $(x^2 - 4x + 7)(x - 3)$
12. $(2x^2 + x - 3)^2$
13. $(x^2 + 5x - 2)(2x^2 - x + 3)$
14. $(x + 4)^3$
15. $\dfrac{(r^{-4}s^9)}{(r^3s^{-9})}$

Divide using long division.

17. $(2x^3 - 7x^2 + 15) \div (x - 2)$

Chapter 6: Factoring (Exam Type Questions)

Questions 18–19 are for review only and will not show up on exams.

18. Find the prime factorization of 88.
19. Find the LCM of 84 and 96.

Questions similar to 20–27 will show up on exams.

Factor each expression completely.

20. $5xy + 6xz - 15y - 18z$
21. $x^2 + x - 12$
22. $x^3 + x^2 - 4x - 4$
23. $x^3 - 27y^3$
24. $x^4 - 35x^2 - 36$

Chapter 4–6 Word Problems (Exam Type Questions)

25. A 70 kg mixture of two different types of nuts costs $\$430.$ If type A costs $\$4$ per kg and type B costs $\$7$ per kg, how many kg of each type were used?
26. A wedding reception bartender is faced with a problem: how many litres of a 5% rum drink must be mixed with 2 litres of a 21% flavoured rum to make a rum punch of 11% strength?
27. The sum of the ages of a boy and a girl is 16 years. Four years ago, the girl was three times the age of the boy. Find the present age of each child.

Mid Term 2 Prep Answer Key

62. Midterm 2: Version A

Find the solution set of the system graphically.

1. $\begin{cases} x + 2y = -5 \\ x - y = -2 \end{cases}$

For problems 2–4, find the solution set of each system by any convenient method.

2. $\begin{cases} 4x - 3y = 13 \\ 5x - 2y = 4 \end{cases}$
3. $\begin{cases} x - 2y = -5 \\ 2x + y = 5 \end{cases}$
4. $\begin{cases} x + y + 2z = 0 \\ 2x + z = 1 \\ 3y + 4z = 0 \end{cases}$

Reduce the following expressions in questions 5–7.

5. $28 - \{5x - [6x - 3(5 - 2x)]^0\} + 5x^2$
6. $4a^2(a - 3)^2$
7. $(x^2 + 2x + 3)^2$

Divide using long division.

8. $(2x^3 - 7x^2 + 15) \div (x - 2)$

For problems 9–12, factor each expression completely.

9. $2ab + 3ac - 4b - 6c$
10. $a^2 - 2ab - 15b^2$
11. $x^3 + x^2 - 9x - 9$
12. $x^3 - 64y^3$

Solve the following word problems.

13. The sum of a brother's and sister's ages is 35. Ten years ago, the brother was twice his sister's age. How old are they now?
14. Kyra gave her brother Mark a logic question to solve: If she has 20 coins in her pocket worth $\$2.75$, and if the coins are only dimes and quarters, how many of each kind of coin does she have?
15. A 50 kg blend of two different grades of tea is sold for $\$191.25$. If grade A sells for $\$3.95$ per kg and grade B sells for $\$3.70$ per kg, how many kg of each grade were used?

Midterm 2: Version A Answer Key

63. Midterm 2: Version B

Find the solution set of the system graphically.

1. $\begin{cases} x + y = 5 \\ 2x - y = 1 \end{cases}$

For problems 2–4, find the solution set of each system by any convenient method.

2. $\begin{cases} 4x + 3y = 8 \\ x = 4y + 2 \end{cases}$
3. $\begin{cases} 5x - 3y = 2 \\ 3x + y = 4 \end{cases}$
4. $\begin{cases} x + y + z = 3 \\ x - 2z = -7 \\ -2y + 4z = 20 \end{cases}$

Reduce the following expressions in questions 5–8.

5. $5 - 3\left[4x - 2(6x-5)^0 - (7-2x)\right]$
6. $3a^2(a+3)^2$
7. $(x^2+x+5)(x^2+x-5)$
8. $\left(\dfrac{x^{4n}x^{-6}}{x^{3n}}\right)^{-1}$

For problems 9–12, factor each expression completely.

9. $14axy - 6az - 7xy + 3z$
10. $a^2 + 2ab - 15b^2$
11. $2x^3 + 8x^2 - x - 4$
12. $27x^3 + 8y^3$

Solve the following word problems.

13. The sum of the ages of a father and his daughter is 38. Six years from now, the father will be four times as old as his daughter. Find the present age of each.
14. A 90 kg mixture of two different types of nuts costs $\$370$. If type A costs $\$3$ per kg and type B costs $\$5$ per kg, how many kg of each type were used?
15. A student lab technician is combining a 10% sulfuric acid solution to 40 ml solution at 25% to dilute it to 15%. How much of the 10% solution does the student need to add?

Midterm 2: Version B Answer Key

64. Midterm 2: Version C

Find the solution set of the system graphically.

1. $\begin{cases} 2x - y - 2 = 0 \\ 2x + 3y + 6 = 0 \end{cases}$

For problems 2–4, find the solution set of each system by any convenient method.

2. $\begin{cases} 3x - 4y = 13 \\ x + y = 2 \end{cases}$
3. $\begin{cases} 4x - 3y = 6 \\ 3y + 4x = 2 \end{cases}$
4. $\begin{cases} x + y + z = 6 \\ 2y + 4z = 10 \\ -2x + z = -3 \end{cases}$

Reduce the following expressions in questions 5–7.

5. $36 + \{-2x - [6x - 3(5 - 2x)]\}^0 + 6x^3$
6. $6a^2b(a - 3)(a + 3)$
7. $(x^2 + 3x + 5)^2$

Divide using long division.

8. $(2x^4 + x^3 + 4x^2 - 4x - 5) \div (2x + 1)$

For problems 9–12, factor each expression completely.

9. $x^2 + 17x - 18$
10. $2a^2 - 4ab - 30b^2$
11. $8x^3 - y^3$
12. $16y^4 - x^4$

Solve the following word problems.

13. The sum of a brother's and sister's ages is 30. Ten years ago, the brother was four times his sister's age. How old are they now?
14. Kyra gave her brother Mark a logic question to solve: If she has 18 coins in her pocket worth $\$1.20$, and if the coins are only dimes and nickels, how many of each type of coin does she have?
15. Tanya needs to make 10 litres of a 25% alcohol solution for the University Green College Founders Social by mixing a 30% alcohol solution with a 5% alcohol solution. How many litres each of the 30% and the 5% alcohol solutions should be used?

Midterm 2: Version C Answer Key

65. Midterm 2: Version D

Find the solution set of the system graphically.

1. $\begin{cases} x - 2y + 6 = 0 \\ x + y - 6 = 0 \end{cases}$

For problems 2–4, find the solution set of each system by any convenient method.

2. $\begin{cases} 3x - 2y = 0 \\ 2x + 5y = 0 \end{cases}$
3. $\begin{cases} 2x - 3y = 8 \\ 3y - 2x = 4 \end{cases}$
4. $\begin{cases} 2x + y - 3z = -7 \\ -2y + 3z = 9 \\ 3x + z = 6 \end{cases}$

Reduce the following expressions in questions 5–7.

5. $36 - \{-2x - [6x - 3(5 - 2x)]\}^0 + 3x^2$
6. $3a^2(a - 2)^2$
7. $(x^2 + 2x - 4)^2$

Divide using long division.

8. $(x^4 + 4x^3 + 4x^2 + 10x + 20) \div (x + 2)$

For problems 9–12, factor each expression completely.

9. $x^2 + 3x - 18$
10. $3x^2 + 25xy + 8y^2$
11. $125x^3 - y^3$
12. $81y^4 - 16x^4$

Solve the following word problems.

13. The sum of the ages of a boy and a girl is 18 years. Four years ago, the girl was four times the age of the boy. Find the present age of each child.
14. A purse contains $\$3.50$ made up of dimes and quarters. If there are 20 coins in all, how many dimes and how many quarters were there?
15. A 60 kg blend of two different grades of tea is sold for $\$218.50$. If grade A sells for $\$3.80$ per kg and grade B sells for $\$3.55$ per kg, how many kg of each grade were used?

Midterm 2: Version D Answer Key

66. Midterm 2: Version E

Find the solution set of the system graphically.

1. $\begin{cases} x - y = -3 \\ x + 2y = 3 \end{cases}$

For problems 2–4, find the solution set of each system by any convenient method.

2. $\begin{cases} 2x - 5y = -2 \\ 3x - 4y = 4 \end{cases}$
3. $\begin{cases} 4x + 3y = -29 \\ 3x + 2y = -21 \end{cases}$
4. $\begin{cases} x + y - 3z = 0 \\ 2y - 2z = -12 \\ 2x - 3y = 16 \end{cases}$

Reduce the following expressions in questions 5–7.

5. $5-4\left[2x-2(6x-5)^0-(7-2x)\right]$
6. $3ab^4(a-5)(a+5)$
7. $(x^2+3x-6)^2$

Divide using long division.

8. $(3x^3+18+7x^2)\div(x+3)$

For problems 9–12, factor each expression completely.

9. $x^2+4x-21$
10. $4x^3+4x^2-9x-9$
11. $8x^3-27y^3$
12. x^4-624x^2-625

Solve the following word problems.

13. The sum of the ages of a boy and a girl is 20 years. Four years ago, the girl was two times the age of the boy. Find the present age of each child.
14. How many ml of a 16% sulfuric acid solution must be added to 20 ml of a 6% solution to create a 12% solution?
15. A 60 kg blend of cereals and raisins is sold for $\$213.$ If the cereal sells for $\$3.40$ per kg and the raisins sells for $\$3.90$ per kg, how many kg of each grade were used?

Midterm 2: Version E Answer Key

PART X

CHAPTER 8: RATIONAL EXPRESSIONS

Learning Objectives

This chapter covers:

- Reducing Rational Expressions
- Multiplication & Division of Rational Expressions
- Least Common Denominators
- Addition & Subtraction of Rational Expressions
- Reducing Complex Fractions
- Solving Complex Fractions
- Sovling Rational Equations
- Rate Word Problems: Speed, Distance & Time

67. 8.1 Reducing Rational Expressions

Definition: A rational expression can be defined as a simple fraction where either the numerator, denominator or both are polynomials. Care must be taken when solving rational expressions in that one must be careful of solutions yielding 0 in the denominator.

Rational expressions are expressions written as a quotient of polynomials. Examples of rational expressions include:

$$\dfrac{x^2-x-12}{x^2-9x+20} \quad \text{or} \quad \dfrac{3}{x-2} \quad \text{or} \quad \dfrac{a-b}{b^2-a^2} \quad \text{or} \quad \dfrac{3}{13}$$

As rational expressions are a special type of fraction, it is important to remember that you cannot have zero in the denominator of a fraction. For this reason, rational expressions have what are called excluded values that make the denominator equal zero if used.

Example 8.1.1

Find the excluded values for the following rational expression:

$$\dfrac{x^2-1}{3x^2+5x}$$

For this expression, the excluded values are found by solving $3x^2+5x\neq 0$.

Factor $3x^2+5x$ first to yield $x(3x+5)\neq 0$.

There are now have two parts of this that can be made to equal 0:

$$x\neq 0 \qquad \text{and} \qquad 3x+5\neq 0$$

Solving the second yields:

$$\begin{array}{rrrrr}
3x & + & 5 & \neq & 0 \\
 & - & 5 & & -5 \\
 & & 3x & \neq & -5 \\
 & & x & \neq & -\dfrac{5}{3}
\end{array}$$

As with our previous polynomials, evaluating rational expressions is easily accomplished by substituting the value for the variable and using order of operations.

Example 8.1.2

Evaluate the following rational expression for $x = -6$:

$$\frac{x^2 - 4}{x^2 + 6x + 8}$$

$$\begin{aligned}
&= \frac{(-6)^2 - 4}{(-6)^2 + 6(-6) + 8} \\
&= \frac{36 - 4}{36 + 6(-6) + 8} \\
&= \frac{32}{36 - 36 + 8} \\
&= \frac{32}{8} \\
&= 4
\end{aligned}$$

Just as the expression was reduced in the previous example, often a rational expression can be reduced, even without knowing the value of the variable. When it is reduced, divide out common factors. This has already been seen with monomials when the properties of exponents was discussed. If the problem only has monomials, you can reduce the coefficients and subtract exponents on the variables.

Example 8.1.3

Reduce the following rational expression:

$$\frac{15x^4y^2}{25x^2y^6}$$

$$= \frac{3 \cdot 5x^{4-2}y^{2-6}}{5 \cdot 5}$$

$$= \frac{3x^2y^{-4}}{5}$$

$$= \frac{3x^2}{5y^4}$$

If there is more than just one term in either the numerator or the denominator, you might need to first factor the numerator and denominator.

Example 8.1.4

Reduce the following rational expressions:

1. $$\frac{28}{8x^2 - 16}$$

$$= \frac{28}{8(x^2 - 2)}$$

$$= \frac{7}{2(x^2 - 2)}$$

2.

$$\frac{9x-3}{18x-6}$$

$$= \frac{3(3x-1)}{6(3x-1)}$$

$$= \frac{1}{2}$$

3.

$$\frac{x^2-25}{x^2+8x+15}$$

$$= \frac{(x+5)(x-5)}{(x+3)(x+5)}$$

$$= \frac{x-5}{x+3}$$

Questions

Evaluate Questions 1 to 6.

1. $\frac{4v+2}{6}$ when $v=4$
2. $\frac{b-3}{3b-9}$ when $b=-2$
3. $\frac{x-3}{x^2-4x+3}$ when $x=-4$
4. $\frac{a+2}{a^2+3a+2}$ when $a=-1$
5. $\frac{b+2}{b^2+4b+4}$ when $b=0$
6. $\frac{n^2-n-6}{n-3}$ when $n=4$

For each of Questions 7 to 14, state the excluded values.

7. $\frac{3k^2+30k}{k+10}$
8. $\frac{27p}{18p^2-36p}$

9. $\dfrac{10m^2+8m}{10m}$

10. $\dfrac{10x+16}{6x+20}$

11. $\dfrac{r^2+3r+2}{5r+10}$

12. $\dfrac{6n^2-21n}{6n^2+3n}$

13. $\dfrac{b^2+12b+32}{b^2+4b-32}$

14. $\dfrac{10v^2+30v}{35v^2-5v}$

For Questions 15 to 32, simplify each expression.

15. $\dfrac{21x^2}{18x}$

16. $\dfrac{12n}{4n^2}$

17. $\dfrac{24a}{40a^2}$

18. $\dfrac{21k}{24k^2}$

19. $\dfrac{18m-24}{60}$

20. $\dfrac{n-9}{9n-81}$

21. $\dfrac{x+1}{x^2+8x+7}$

22. $\dfrac{28m+12}{36}$

23. $\dfrac{n^2+4n-12}{n^2-7n+10}$

24. $\dfrac{b^2+14b+48}{b^2+15b+56}$

25. $\dfrac{9v+54}{v^2-4v-60}$

26. $\dfrac{k^2-12k+32}{k^2-64}$

27. $\dfrac{2n^2+19n-10}{9n+90}$

28. $\dfrac{3x^2-29x+40}{5x^2-30x-80}$

29. $\dfrac{2x^2-10x+8}{3x^2-7x+4}$

30. $\dfrac{7n^2 - 32n + 16}{4n - 16}$

31. $\dfrac{7a^2 - 26a - 45}{6a^2 - 34a + 20}$

32. $\dfrac{4k^3 - 2k^2 - 2k}{k^3 - 18k^2 + 9k}$

Answer Key 8.1

68. 8.2 Multiplication and Division of Rational Expressions

Multiplying and dividing rational expressions is very similar to the process used to multiply and divide fractions.

Example 8.2.1

Reduce and multiply $\frac{15}{49}$ and $\frac{14}{45}$.

$$\frac{15}{49}\cdot\frac{14}{45}\text{ reduces to }\frac{1}{7}\cdot\frac{2}{3},\text{ which equals }\frac{2}{21}$$

(15 and 45 reduce to 1 and 3, and 14 and 49 reduce to 2 and 7)

This process of multiplication is identical to division, except the first step is to reciprocate any fraction that is being divided.

Example 8.2.2

Reduce and divide $\frac{25}{18}$ by $\frac{15}{6}$.

$$\frac{25}{18}\div\frac{15}{6}\text{ reciprocates to }\frac{25}{18}\cdot\frac{6}{15},\text{ which reduces to }\frac{5}{3}\cdot\frac{1}{3},\text{ which equals }\frac{5}{9}$$

(25 and 15 reduce to 5 and 3, and 6 and 18 reduce to 1 and 3)

When multiplying with rational expressions, follow the same process: first, divide out common factors, then multiply straight across.

Example 8.2.3

Reduce and multiply $\frac{25x^2}{9y^8}$ and $\frac{24y^4}{55x^7}$.

$$\frac{25x^2}{9y^8} \cdot \frac{24y^4}{55x^7} \text{ reduces to } \frac{5}{3y^4} \cdot \frac{8}{11x^5}, \text{ which equals } \frac{40}{33x^5y^4}$$

(25 and 55 reduce to 5 and 11, 24 and 9 reduce to 8 and 3, x^2 and x^7 reduce to x^5, y^4 and y^8 reduce to y^4)

Remember: when dividing fractions, reciprocate the dividing fraction.

Example 8.2.4

Reduce and divide $\frac{a^4b^2}{a}$ by $\frac{b^4}{4}$.

$$\frac{a^4b^2}{a} \div \frac{b^4}{4} \text{ reciprocates to } \frac{a^4b^2}{a} \cdot \frac{4}{b^4}, \text{ which reduces to } \frac{a^3}{1} \cdot \frac{4}{b^2}, \text{ which equals } \frac{4a^3}{b^2}$$

(After reciprocating, $4a^4b^2$ and b^4 reduce to $4a^3$ and b^2)

In dividing or multiplying some fractions, the polynomials in the fractions must be factored first.

Example 8.2.5

Reduce, factor and multiply $\frac{x^2-9}{x^2+x-20}$ and $\frac{x^2-8x+16}{3x+9}$.

$$\frac{x^2-9}{x^2+x-20} \cdot \frac{x^2-8x+16}{3x+9} \text{ factors to } \frac{(x+3)(x-3)}{(x-4)(x+5)} \cdot \frac{(x-4)(x-4)}{3(x+3)}$$

Dividing or cancelling out the common factors $(x+3)$ and $(x-4)$ leaves us with $\frac{x-3}{x+5} \cdot \frac{x-4}{3}$, which results in $\frac{(x-3)(x-4)}{3(x+5)}$.

Example 8.2.6

Reduce, factor and multiply or divide the following fractions:

$$\frac{a^2+7a+10}{a^2+6a+5}\cdot\frac{a+1}{a^2+4a+4}\div\frac{a-1}{a+2}$$

Factoring each fraction and reciprocating the last one yields:

$$\frac{(a+5)(a+2)}{(a+5)(a+1)}\cdot\frac{(a+1)}{(a+2)(a+2)}\cdot\frac{(a+2)}{(a-1)}$$

Dividing or cancelling out the common polynomials leaves us with:

$$\frac{1}{a-1}$$

Questions

Simplify each expression.

1. $\dfrac{8x^2}{9}\cdot\dfrac{9}{2}$
2. $\dfrac{8x}{3}\div\dfrac{4x}{7}$
3. $\dfrac{5x^2}{4}\cdot\dfrac{6}{5}$
4. $\dfrac{10p}{5}\div\dfrac{8}{10}$
5. $\dfrac{(m-6)}{7(7m-5)}\cdot\dfrac{5m(7m-5)}{m-6}$
6. $\dfrac{7(n-2)}{10(n+3)}\div\dfrac{n-2}{(n+3)}$
7. $\dfrac{7r}{7r(r+10)}\div\dfrac{r-6}{(r-6)^2}$
8. $\dfrac{6x(x+4)}{(x-3)}\cdot\dfrac{(x-3)(x-6)}{6x(x-6)}$
9. $\dfrac{x-10}{35x+21}\div\dfrac{7}{35x+21}$

10. $\dfrac{v-1}{4} \cdot \dfrac{4}{v^2-11v+10}$

11. $\dfrac{x^2-6x-7}{x+5} \cdot \dfrac{x+5}{x-7}$

12. $\dfrac{1}{a-6} \cdot \dfrac{8a+80}{8}$

13. $\dfrac{4m+36}{m+9} \cdot \dfrac{m-5}{5m^2}$

14. $\dfrac{2r}{r+6} \div \dfrac{2r}{7r+42}$

15. $\dfrac{n-7}{6n-12} \cdot \dfrac{12-6n}{n^2-13n+42}$

16. $\dfrac{x^2+11x+24}{6x^3+18x^2} \cdot \dfrac{6x^3+6x^2}{x^2+5x-24}$

17. $\dfrac{27a+36}{9a+63} \div \dfrac{6a+8}{2}$

18. $\dfrac{k-7}{k^2-k-12} \cdot \dfrac{7k^2-28k}{8k^2-56k}$

19. $\dfrac{x^2-12x+32}{x^2-6x-16} \cdot \dfrac{7x^2+14x}{7x^2+21x}$

20. $\dfrac{9x^3+54x^2}{x^2+5x-14} \cdot \dfrac{x^2+5x-14}{10x^2}$

21. $(10m^2+100m) \cdot \dfrac{18m^3-36m^2}{20m^2-40m}$

22. $\dfrac{n-7}{n^2-2n-35} \div \dfrac{9n+54}{10n+50}$

23. $\dfrac{x^2-1}{2x-4} \cdot \dfrac{x^2-4}{x^2-x-2} \div \dfrac{x^2+x-2}{3x-6}$

24. $\dfrac{a^3+b^3}{a^2+3ab+2b^2} \cdot \dfrac{3a-6b}{3a^2-3ab+3b^2} \div \dfrac{a^2-4b^2}{a+2b}$

Answer Key 8.2

69. 8.3 Least Common Denominators

Finding the least common denominator, or LCD, is very important to working with rational expressions. The process used depends on finding what is common to each rational expression and identifying what is not common. These common and not common factors are then combined to form the LCD.

Example 8.3.1

Find the LCD of the numbers 12, 8, and 6.

First, break these three numbers into primes:

$$\begin{array}{rcl} 12 & = & 2 \cdot 2 \cdot 3 \text{ or } 2^2 \cdot 3 \\ 8 & = & 2 \cdot 2 \cdot 2 \text{ or } 2^3 \\ 6 & = & 2 \cdot 3 \end{array}$$

Then write out the primes for the first number, 12, and set the LCD to $2^2 \cdot 3$.

Notice the factorization of 8 includes 2^3, yet the LCD currently only has 2^2, so you add one 2.

Now the LCD = $2^3 \cdot 3$.

Checking $6 = 2 \cdot 3$, there already is a $2 \cdot 3$ in the LCD, so we need not add any more primes.

The LCD = $2^3 \cdot 3$ or 24.

This process can be duplicated with variables.

Example 8.3.2

Find the LCD of $4x^2y^5$ and $6x^4y^3z^6$.

First, break both terms into primes:

$$\begin{array}{rcl} 4x^2y^5 & = & 2^2 \cdot x^2 \cdot y^5 \\ 6x^4y^3z^6 & = & 2 \cdot 3 \cdot x^4 \cdot y^3 \cdot z^6 \end{array}$$

Then write out the primes for the first term, $4x^2y^5$, and set the LCD to $2^2 \cdot x^2 \cdot y^5$.

The LCD for $6x^4y^3z^6 = 2 \cdot 3 \cdot x^4 \cdot y^3 \cdot z^6$ has an extra 3, x^2, and z^6, which we add to the LCD that we are constructing.

This yields LCD = $2^2 \cdot 3 \cdot x^{2+2} \cdot y^5 \cdot z^6$, or LCD = $12x^4y^5z^6$.

This process can also be duplicated with polynomials.

Example 8.3.3

Find the LCD of $x^2 + 2x - 3$ and $x^2 - x - 12$.

First, we factor both of these polynomials, much like finding the primes of the above terms:

$$\begin{array}{rcl} x^2 + 2x - 3 & = & (x-1)(x+3) \\ x^2 - x - 12 & = & (x-4)(x+3) \end{array}$$

The LCD is constructed as we did before, except this time, we write out the factored terms from the first polynomial, so the LCD = $(x-1)(x+3)$.

Notice that $x^2 - x - 12 = (x-4)(x+3)$, where the $(x+3)$ is already in the LCD, which means that we only need to add $(x-4)$.

The LCD = $(x-1)(x+3)(x-4)$.

Questions

For Questions 1 to 10, find each Least Common Denominator.

1. $2a^3, 6a^4b^2, 4a^3b^5$
2. $5x^2y, 25x^3y^5z$
3. $x^2 - 3x, x - 3, x$
4. $4x - 8, x - 2, 4$
5. $x + 2, x - 4$
6. $x, x - 7, x + 1$
7. $x^2 - 25, x + 5$
8. $x^2 - 9, x^2 - 6x + 9$
9. $x^2 + 3x + 2, x^2 + 5x + 6$
10. $x^2 - 7x + 10, x^2 - 2x - 15, x^2 + x - 6$

For Questions 11 to 20, find the LCD of each fraction and place each expression over the same common denominator.

11. $\dfrac{3a}{5b^2}, \dfrac{2}{10a^3b}$

12. $\frac{3x}{x-4}, \frac{2}{x+2}$

13. $\frac{x+2}{x-3}, \frac{x-3}{x+2}$

14. $\frac{5}{x^2-6x}, \frac{2}{x}, \frac{-3}{x-6}$

15. $\frac{x}{x^2-16}, \frac{3x}{x^2-8x+16}$

16. $\frac{5x+1}{x^2-3x-10}, \frac{4}{x-5}$

17. $\frac{x+1}{x^2-36}, \frac{2x+3}{x^2+12x+36}$

18. $\frac{3x+1}{x^2-x-12}, \frac{2x}{x^2+4x+3}$

19. $\frac{4x}{x^2-x-6}, \frac{x+2}{x-3}$

20. $\frac{3x}{x^2-6x+8}, \frac{x-2}{x^2+x-20}, \frac{5}{x^2+3x-10}$

Answer Key 8.3

70. 8.4 Addition and Subtraction of Rational Expressions

Adding and subtracting rational expressions is identical to adding and subtracting integers. Recall that, when adding fractions with a common denominator, you add the numerators and keep the denominator. This is the same process used with rational expressions. Remember to reduce the final answer if possible.

Example 8.4.1

Add the following rational expressions:

$$\begin{array}{rl}\dfrac{x-4}{x^2-2x-8}+\dfrac{x+8}{x^2-2x-8} & \text{Same denominator, so you add the numerators and combine like terms.}\\[2ex] \dfrac{2x+4}{x^2-2x-8} & \text{Factor the numerator and the denominator.}\\[2ex] \dfrac{2(x+2)}{(x+2)(x-4)} & \text{Divide out } (x+2).\\[2ex] \dfrac{2}{x-4} & \text{Solution.}\end{array}$$

Subtraction of rational expressions with a common denominator follows the same pattern, though the subtraction can cause problems if you are not careful with it. To avoid sign errors, first distribute the subtraction throughout the numerator. Then treat it like an addition problem. This process is the same as "add the opposite," which was seen when subtracting with negatives.

Example 8.4.2

Subtract the following rational expressions:

$$\frac{6x-12}{3x-6}-\frac{15x-6}{3x-6}$$ Add the opposite of the second fraction (distribute the negative).

$$\frac{6x-12}{3x-6}+\frac{-15x+6}{3x-6}$$ Add the numerators and combine like terms.

$$\frac{-9x-6}{3x-6}$$ Factor the numerator and the denominator.

$$\frac{-3(3x+2)}{3(x-2)}$$ Divide out the common factor of 3.

$$\frac{-(3x+2)}{x-2}$$ Solution.

When there is not a common denominator, first find the least common denominator (LCD) and alter each fraction so the denominators match.

Example 8.4.3

Add the following rational expressions:

$$\frac{7a}{3a^2b}+\frac{4b}{6ab^4}$$ The LCD is $6a^2b^4$.

$$\frac{2b^3}{2b^3}\cdot\frac{7a}{3a^2b}+\frac{4b}{6ab^4}\cdot\frac{a}{a}$$ Multiply the first fraction by $2b^3$ and the second by a.

$$\frac{14ab^3}{6a^2b^4}+\frac{4ab}{6a^2b^4}$$ Add the numerators. No like terms to combine.

$$\frac{14ab^3+4ab}{6a^2b^4}$$ Factor the numerator.

$$\frac{2ab(7b^2+2)}{6a^2b^4}$$ Reduce, dividing out factors $2, a,$ and b.

$$\frac{7b^2+2}{3ab^3}$$ Solution.

Example 8.4.4

Subtract the following rational expressions:

$$\frac{x+1}{x-4} - \frac{x+1}{x^2-7x+12} \quad \text{Add the opposite of the second fraction (distribute the negative).}$$

$$\frac{x+1}{x-4} + \frac{-x-1}{x^2-7x+12} \quad \text{Factor the denominators to find the LCD} = (x-4)(x-3).$$

$$\frac{(x-3)(x+1)}{(x-3)(x-4)} + \frac{-x-1}{(x-3)(x-4)} \quad \text{Only the first fraction needs to be multplied by } (x-3).$$

$$\frac{x^2-2x-3}{(x-3)(x-4)} + \frac{-x-1}{(x-3)(x-4)} \quad \text{Add the numerators and combine like terms.}$$

$$\frac{x^2-3x-4}{(x-3)(x-4)} \quad \text{Factor the numerator.}$$

$$\frac{(x-4)(x+1)}{(x-3)(x-4)} \quad \text{Divide out the common factor of } (x-4).$$

$$\frac{x+1}{x-3} \quad \text{Solution.}$$

Questions

Add or subtract the rational expressions. Simplify your answers whenever possible.

1. $\dfrac{2}{a+3} + \dfrac{4}{a+3}$
2. $\dfrac{x^2}{x-2} - \dfrac{6x-8}{x-2}$
3. $\dfrac{t^2+4t}{t-1} + \dfrac{2t-7}{t-1}$
4. $\dfrac{a^2+3a}{a^2+5a-6} - \dfrac{4}{a^2+5a-6}$
5. $\dfrac{5}{6r} - \dfrac{5}{8r}$
6. $\dfrac{7}{xy^2} + \dfrac{3}{x^2y}$
7. $\dfrac{8}{9t^3} + \dfrac{5}{6t^2}$

8. $\frac{x+5}{8}+\frac{x-3}{12}$
9. $\frac{x-1}{4x}-\frac{2x+3}{x}$
10. $\frac{2c-d}{c^2d}-\frac{c+d}{cd^2}$
11. $\frac{5x+3y}{2x^2y}-\frac{3x+4y}{xy^2}$
12. $\frac{2}{x-1}+\frac{2}{x+1}$
13. $\frac{x}{x^2+5x+6}-\frac{2}{x^2+3x+2}$
14. $\frac{2x}{x^2-1}-\frac{3}{x^2+5x+4}$
15. $\frac{x}{x^2+15x+56}-\frac{7}{x^2+13x+42}$
16. $\frac{2x}{x^2-9}+\frac{5}{x^2+x-6}$
17. $\frac{5x}{x^2-x-6}-\frac{18}{x^2-9}$
18. $\frac{4x}{x^2-2x-3}-\frac{3}{x^2-5x+6}$

Answer Key 8.4

71. 8.5 Reducing Complex Fractions

Complex fractions will have fractions in either the numerator, the denominator, or both. These fractions are generally simplified by multiplying the fractions in the numerator and denominator by the LCD. This process allows you to reduce a complex fraction to a simpler one in one step.

Example 8.5.1

Reduce $\dfrac{1-\dfrac{1}{x^2}}{1-\dfrac{1}{x}}$.

For this fraction, the LCD is x^2. To simplify this complex fraction, multiply each term in the numerator and the denominator by the LCD.

$$\frac{1(x^2)-\dfrac{1}{x^2}(x^2)}{1(x^2)-\dfrac{1}{x}(x^2)}$$

This will result in:

$$\frac{x^2-1}{x^2-x}$$

Now, factor both the numerator and denominator, which results in:

$$\frac{(x-1)(x+1)}{x(x-1)},\ \text{which reduces to}\ \frac{x+1}{x}$$

It matters not how complex these fractions are: simply find the LCD to reduce the complex fraction to one that is simpler.

The more fractions there are in a problem, the more times the process is repeated.

Example 8.5.2

Reduce the following complex fraction:

$$\frac{\dfrac{x-3}{x+3}-\dfrac{x+3}{x-3}}{\dfrac{x-3}{x+3}+\dfrac{x+3}{x-3}}$$

For this fraction, the LCD is $(x-3)(x+3)$. To simplify the above complex fraction, multiply both the numerator and denominator by the LCD. This looks like:

$$\frac{(x-3)(x+3)\dfrac{x-3}{x+3}-\dfrac{x+3}{x-3}(x-3)(x+3)}{(x-3)(x+3)\dfrac{x-3}{x+3}+\dfrac{x+3}{x-3}(x-3)(x+3)}$$

Which reduces to:

$$\frac{(x-3)(x-3)-(x+3)(x+3)}{(x-3)(x-3)+(x+3)(x+3)}$$

Now multiply out the numerator and denominator and add like terms:

$$\frac{(x^2-6x+9)-(x^2+6x+9)}{(x^2-6x+9)+(x^2+6x+9)}$$

Dropping the brackets leaves:

$$\frac{x^2-6x+9-x^2-6x-9}{x^2-6x+9+x^2+6x+9}$$

Adding these terms together yields:

$$\frac{-12x}{2x^2+18}$$

You will notice there is a common factor of 2 in each of the terms that can be factored out. This results in:

$$\frac{-6x}{x^2+9}$$

Questions

Simplify each of the following complex fractions.

1. $$\frac{1+\dfrac{1}{x}}{1-\dfrac{1}{x^2}}$$

2. $\dfrac{1-\dfrac{1}{y^2}}{1+\dfrac{1}{y}}$

3. $\dfrac{\dfrac{a}{b}+2}{\dfrac{a^2}{b^2}-4}$

4. $\dfrac{\dfrac{1}{y^2}-9}{\dfrac{1}{y}+3}$

5. $\dfrac{\dfrac{1}{a^2}-\dfrac{1}{a}}{\dfrac{1}{a^2}+\dfrac{1}{a}}$

6. $\dfrac{\dfrac{1}{b}+\dfrac{1}{2}}{\dfrac{4}{b^2-1}}$

7. $\dfrac{x+2-\dfrac{9}{x+2}}{x+1+\dfrac{x-7}{x+2}}$

8. $\dfrac{a-3+\dfrac{a-3}{a+2}}{a+4-\dfrac{4a+5}{a+2}}$

9. $\dfrac{\dfrac{x+y}{y}+\dfrac{y}{x-y}}{\dfrac{y}{x-y}}$

10. $\dfrac{\dfrac{a-b}{a}-\dfrac{a}{a+b}}{\dfrac{b^2}{a+b}}$

11. $\dfrac{\dfrac{x-y}{y}+\dfrac{x+y}{x-y}}{\dfrac{y}{x-y}}$

12. $\dfrac{\dfrac{x-2}{x+2}-\dfrac{x+2}{x-2}}{\dfrac{x-2}{x+2}+\dfrac{x+2}{x-2}}$

Answer Key 8.5

72. 8.6 Solving Complex Fractions

When solving two or more equated fractions, the easiest solution is to first remove all fractions by multiplying both sides of the equations by the LCD. This strategy is shown in the next examples.

Example 8.6.1

Solve $\dfrac{x+3}{4}=\dfrac{2}{3}$.

For these two fractions, the LCD is 3 × 4 = 12. Therefore, we multiply both sides of the equation by 12:

$$12\left(\frac{x+3}{4}\right)=\left(\frac{2}{3}\right)12$$

This reduces the complex fraction to:

$$3(x+3)=2(4)$$

Multiplying this out yields:

$$3x+9=8$$

Now just isolate and solve for x:

$$\begin{array}{rrrrr}3x&+&9&=&8\\&-&9&&-9\\&&3x&=&-1\\ \\&&x&=&-\dfrac{1}{3}\end{array}$$

Example 8.6.2

Solve $\dfrac{2x-3}{3x+4}=\dfrac{2}{5}$.

For these two fractions, the LCD is $5(3x+4)$. Therefore, both sides of the equation are multiplied by $5(3x+4)$:

$$5(3x+4)\left(\frac{2x-3}{3x+4}\right)=\left(\frac{2}{5}\right)5(3x+4)$$

This reduces the complex fraction to:

$$5(2x-3)=2(3x+4)$$

Multiplying this out yields:

$$10x-15=6x+8$$

Now isolate and solve for x:

$$\begin{array}{rcrcrcr} 10x & - & 15 & = & 6x & + & 8 \\ -6x & + & 15 & & -6x & + & 15 \\ & & 4x & = & 23 & & \\ \\ & & x & = & \dfrac{23}{4} & & \end{array}$$

Example 8.6.3

Solve $\dfrac{k+3}{3}=\dfrac{8}{k-2}$.

For these two fractions, the LCD is $3(k-2)$. Therefore, multiply both sides of the equation by $3(k-2)$:

$$3(k-2)\left(\frac{k+3}{3}\right)=\left(\frac{8}{k-2}\right)3(k-2)$$

This reduces the complex fraction to:

$$(k-2)(k+3)=8(3)$$

This multiplies out to:

$$k^2+k-6=24$$

Now subtract 24 from both sides of the equation to turn this into an equation that can be easily factored:

$$\begin{array}{rcrcrcr} k^2 & + & k & - & 6 & = & 24 \\ & & & - & 24 & & -24 \\ k^2 & + & k & - & 30 & = & 0 \end{array}$$

This equation factors to:

$$(k+6)(k-5)=0$$

The solutions are:

$$k=-6 \text{ and } k=5$$

Questions

Solve each of the following complex fractions.

1. $\dfrac{m-1}{5}=\dfrac{8}{2}$
2. $\dfrac{8}{2}=\dfrac{8}{x-8}$
3. $\dfrac{2}{9}=\dfrac{10}{p-4}$
4. $\dfrac{9}{n+2}=\dfrac{3}{9}$
5. $\dfrac{3}{10}=\dfrac{a}{a+2}$
6. $\dfrac{x+1}{3}=\dfrac{x+3}{4}$
7. $\dfrac{2}{p+4}=\dfrac{p+5}{3}$
8. $\dfrac{5}{n+1}=\dfrac{n-4}{10}$
9. $\dfrac{x+5}{5}=\dfrac{6}{x-2}$
10. $\dfrac{4}{x-3}=\dfrac{x+5}{5}$
11. $\dfrac{m+3}{4}=\dfrac{11}{m-4}$
12. $\dfrac{x-5}{8}=\dfrac{4}{x-1}$

Answer Key 8.6

73. 8.7 Solving Rational Equations

When solving equations that are made up of rational expressions, we use the same strategy we used to solve complex fractions, where the easiest solution involved multiplying by the LCD to remove the fractions. Consider the following examples.

Example 8.7.1

Solve the following:

$$\frac{2x}{3} - \frac{5}{6} = \frac{3}{4}$$

For these three fractions, the LCD is 12. Therefore, multiply all parts of the equation by 12:

$$12\left(\frac{2x}{3} - \frac{5}{6}\right) = \left(\frac{3}{4}\right)12$$

This reduces the rational equation to:

$$4(2x) - 2(5) = 3(3)$$

Multiplying this out yields:

$$8x - 10 = 9$$

Now isolate and solve for x:

$$\begin{array}{rrrrr} 8x & - & 10 & = & 9 \\ & + & 10 & & +10 \\ & & 8x & = & 19 \\ \\ & & x & = & \dfrac{19}{8} \end{array}$$

Example 8.7.2

Solve the following:

$$\frac{x}{x+2} + \frac{1}{x+1} = \frac{5}{(x+1)(x+2)}$$

For these three fractions, the LCD is $(x+1)(x+2)$. Therefore, multiply all parts of the equation by $(x+1)(x+2)$:

$$(x+1)(x+2)\left(\frac{x}{x+2}+\frac{1}{x+1}\right)=\left(\frac{5}{(x+1)(x+2)}\right)(x+1)(x+2)$$

This reduces the rational equation to:

$$x(x+1)+1(x+2)=5$$

Multiplying this out yields:

$$x^2+x+x+2=5$$

Which reduces to:

$$x^2+2x+2=5$$

Now subtract 5 from both sides of the equation to turn this into an equation that can be easily factored:

$$\begin{array}{ccccccc} x^2 & + & 2x & + & 2 & = & 5 \\ & & & - & 5 & & -5 \\ x^2 & + & 2x & - & 3 & = & 0 \end{array}$$

This equation factors to:

$$(x+3)(x-1)=0$$

The solutions are:

$$x=-3 \text{ and } 1$$

Questions

Solve each rational equation.

1. $3x-\dfrac{1}{2}-\dfrac{1}{x}=0$
2. $x+1=\dfrac{4}{x+1}$
3. $x+\dfrac{20}{x-4}=\dfrac{5x}{x-4}-2$
4. $\dfrac{x^2+6}{x-1}+\dfrac{x-2}{x-1}=2x$
5. $x+\dfrac{6}{x-3}=\dfrac{2x}{x-3}$
6. $\dfrac{x-4}{x-1}=\dfrac{12}{3-x}+1$
7. $\dfrac{3m}{2m-5}-\dfrac{7}{3m+1}=\dfrac{3}{2}$

8. $\dfrac{4-x}{1-x}=\dfrac{12}{3-x}$
9. $\dfrac{7}{y-3}-\dfrac{1}{2}=\dfrac{y-2}{y-4}$
10. $\dfrac{1}{x+2}+\dfrac{1}{x-2}=\dfrac{3x+8}{x^2-4}$
11. $\dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}=\dfrac{5}{6}$
12. $\dfrac{x-2}{x+3}-\dfrac{1}{x-2}=\dfrac{1}{x^2+x-6}$
13. $\dfrac{x}{x-1}-\dfrac{2}{x+1}=\dfrac{4x^2}{x^2-1}$
14. $\dfrac{2x}{x+2}+\dfrac{2}{x-4}=\dfrac{3x}{x^2-2x-8}$
15. $\dfrac{2x}{x+1}-\dfrac{3}{x+5}=\dfrac{-8x^2}{x^2+6x+5}$

Answer Key 8.7

74. 8.8 Rate Word Problems: Speed, Distance and Time

Distance, rate and time problems are a standard application of linear equations. When solving these problems, use the relationship **rate** (speed or velocity) times **time** equals **distance**.

$$r \cdot t = d$$

For example, suppose a person were to travel 30 km/h for 4 h. To find the total distance, multiply rate times time or (30km/h)(4h) = 120 km.

The problems to be solved here will have a few more steps than described above. So to keep the information in the problem organized, use a table. An example of the basic structure of the table is below:

Example of a Distance, Rate and Time Chart

Who or What	Rate	Time	Distance

The third column, distance, will always be filled in by multiplying the rate and time columns together. If given a total distance of both persons or trips, put this information in the distance column. Now use this table to set up and solve the following examples.

Example 8.8.1

Joey and Natasha start from the same point and walk in opposite directions. Joey walks 2 km/h faster than Natasha. After 3 hours, they are 30 kilometres apart. How fast did each walk?

Who or What	Rate	Time	Distance
Natasha	r	3 h	$3\text{ h}(r)$
Joey	$r+2$	3 h	$3\text{ h}(r+2)$

The distance travelled by both is 30 km. Therefore, the equation to be solved is:

$$\begin{array}{rrrrrrr} 3r & + & 3(r & + & 2) & = & 30 \\ 3r & + & 3r & + & 6 & = & 30 \\ & & & - & 6 & & -6 \\ & & & & \dfrac{6r}{6} & = & \dfrac{24}{6} \\ \\ & & & & r & = & 4\text{ km/h} \end{array}$$

This means that Natasha walks at 4 km/h and Joey walks at 6 km/h.

Example 8.8.2

Nick and Chloe left their campsite by canoe and paddled downstream at an average speed of 12 km/h. They turned around and paddled back upstream at an average rate of 4 km/h. The total trip took 1 hour. After how much time did the campers turn around downstream?

Who or What	Rate	Time	Distance
Downstream	12 km/h	t	$12 \text{ km/h } (t)$
Upstream	4 km/h	$(1-t)$	$4 \text{ km/h } (1-t)$

The distance travelled downstream is the same distance that they travelled upstream. Therefore, the equation to be solved is:

$$\begin{array}{rclcl} 12(t) & = & 4(1 & - & t) \\ 12t & = & 4 & - & 4t \\ +4t & & & + & 4t \\ \dfrac{16t}{16} & = & \dfrac{4}{16} & & \\ & & & & \\ t & = & 0.25 & & \end{array}$$

This means the campers paddled downstream for 0.25 h and spent 0.75 h paddling back.

Example 8.8.3

Terry leaves his house riding a bike at 20 km/h. Sally leaves 6 h later on a scooter to catch up with him travelling at 80 km/h. How long will it take her to catch up with him?

Who or What	Rate	Time	Distance
Terry	20 km/h	t	$20 \text{ km/h } (t)$
Sally	80 km/h	$(t-6 \text{ h})$	$80 \text{ km/h } (t-6 \text{ h})$

The distance travelled by both is the same. Therefore, the equation to be solved is:

$$\begin{array}{rcrcr}
20(t) & = & 80(t & - & 6) \\
20t & = & 80t & - & 480 \\
-80t & & -80t & & \\
\dfrac{-60t}{-60} & = & \dfrac{-480}{-60} & & \\
t & = & 8 & &
\end{array}$$

This means that Terry travels for 8 h and Sally only needs 2 h to catch up to him.

Example 8.8.4

On a 130-kilometre trip, a car travelled at an average speed of 55 km/h and then reduced its speed to 40 km/h for the remainder of the trip. The trip took 2.5 h. For how long did the car travel 40 km/h?

Who or What	Rate	Time	Distance
Fifty-five	55 km/h	t	$55 \text{ km/h } (t)$
Forty	40 km/h	$(2.5 \text{ h} - t)$	$40 \text{ km/h } (2.5 \text{ h} - t)$

The distance travelled by both is 30 km. Therefore, the equation to be solved is:

$$\begin{array}{rcrcrcr}
55(t) & + & 40(2.5 & - & t) & = & 130 \\
55t & + & 100 & - & 40t & = & 130 \\
 & - & 100 & & & & -100 \\
 & & & & \dfrac{15t}{15} & = & \dfrac{30}{15} \\
 & & & & t & = & 2
\end{array}$$

This means that the time spent travelling at 40 km/h was 0.5 h.

Distance, time and rate problems have a few variations that mix the unknowns between distance, rate and time. They generally involve solving a problem that uses the combined distance travelled to equal some distance or a problem in which the distances travelled by both parties is the same. These distance, rate and time problems will be revisited later on in this textbook where quadratic solutions are required to solve them.

Questions

For Questions 1 to 8, find the equations needed to solve the problems. Do not solve.

1. A is 60 kilometres from B. An automobile at A starts for B at the rate of 20 km/h at the same time that an automobile at B starts for A at the rate of 25 km/h. How long will it be before the automobiles meet?
2. Two automobiles are 276 kilometres apart and start to travel toward each other at the same time. They travel at rates differing by 5 km/h. If they meet after 6 h, find the rate of each.
3. Two trains starting at the same station head in opposite directions. They travel at the rates of 25 and 40 km/h, respectively. If they start at the same time, how soon will they be 195 kilometres apart?
4. Two bike messengers, Jerry and Susan, ride in opposite directions. If Jerry rides at the rate of 20 km/h, at what rate must Susan ride if they are 150 kilometres apart in 5 hours?
5. A passenger and a freight train start toward each other at the same time from two points 300 kilometres apart. If the rate of the passenger train exceeds the rate of the freight train by 15 km/h, and they meet after 4 hours, what must the rate of each be?
6. Two automobiles started travelling in opposite directions at the same time from the same point. Their rates were 25 and 35 km/h, respectively. After how many hours were they 180 kilometres apart?
7. A man having ten hours at his disposal made an excursion by bike, riding out at the rate of 10 km/h and returning on foot at the rate of 3 km/h. Find the distance he rode.
8. A man walks at the rate of 4 km/h. How far can he walk into the country and ride back on a trolley that travels at the rate of 20 km/h, if he must be back home 3 hours from the time he started?

Solve Questions 9 to 22.

9. A boy rides away from home in an automobile at the rate of 28 km/h and walks back at the rate of 4 km/h. The round trip requires 2 hours. How far does he ride?
10. A motorboat leaves a harbour and travels at an average speed of 15 km/h toward an island. The average speed on the return trip was 10 km/h. How far was the island from the harbour if the trip took a total of 5 hours?
11. A family drove to a resort at an average speed of 30 km/h and later returned over the same road at an average speed of 50 km/h. Find the distance to the resort if the total driving time was 8 hours.
12. As part of his flight training, a student pilot was required to fly to an airport and then return. The average speed to the airport was 90 km/h, and the average speed returning was 120 km/h. Find the distance between the two airports if the total flying time was 7 hours.
13. Sam starts travelling at 4 km/h from a campsite 2 hours ahead of Sue, who travels 6 km/h in the same direction. How many hours will it take for Sue to catch up to Sam?
14. A man travels 5 km/h. After travelling for 6 hours, another man starts at the same place as the first man did, following at the rate of 8 km/h. When will the second man overtake the first?
15. A motorboat leaves a harbour and travels at an average speed of 8 km/h toward a small island. Two hours later, a cabin cruiser leaves the same harbour and travels at an average speed of 16 km/h toward the same island. How many hours after the cabin cruiser leaves will it be alongside the motorboat?
16. A long distance runner started on a course, running at an average speed of 6 km/h. One hour later, a second runner began the same course at an average speed of 8 km/h. How long after the second runner started will they overtake the first runner?
17. Two men are travelling in opposite directions at the rate of 20 and 30 km/h at the same time and from the same place. In how many hours will they be 300 kilometres apart?
18. Two trains start at the same time from the same place and travel in opposite directions. If the rate of one is 6 km/h more than the rate of the other and they are 168 kilometres apart at the end of 4 hours, what is the rate of

each?

19. Two cyclists start from the same point and ride in opposite directions. One cyclist rides twice as fast as the other. In three hours, they are 72 kilometres apart. Find the rate of each cyclist.
20. Two small planes start from the same point and fly in opposite directions. The first plane is flying 25 km/h slower than the second plane. In two hours, the planes are 430 kilometres apart. Find the rate of each plane.
21. On a 130-kilometre trip, a car travelled at an average speed of 55 km/h and then reduced its speed to 40 km/h for the remainder of the trip. The trip took a total of 2.5 hours. For how long did the car travel at 40 km/h?
22. Running at an average rate of 8 m/s, a sprinter ran to the end of a track and then jogged back to the starting point at an average of 3 m/s. The sprinter took 55 s to run to the end of the track and jog back. Find the length of the track.

Answer Key 8.8

PART XI

CHAPTER 9: RADICALS

Learning Objectives

This chapter covers:

- Reducing Square Roots
- Reducing Higher Power Roots
- Adding & Subtracting Radicals
- Multiplication & Division of Radicals
- Rationalizing Denominators
- Radicals & Rational Exponents
- Rational Exponents (Increased Difficulty)
- Radicals of Mixed Index
- Complex Numbers
- Rate Word Problems: Work & Time

75. 9.1 Reducing Square Roots

Square roots are the most common type of radical. A square will take some number and multiply it by itself. A square root of a number gives the number that, when multiplied by itself, gives the number shown beneath the radical. For example, because $5^2 = 25$, the square root of 25 is 5.

The square root of 25 is written as $\sqrt{25}$ or as $25^{\frac{1}{2}}$.

Example 9.1.1

Solve the following square roots:

$$\begin{array}{lll} \sqrt{1}=1 & \sqrt{121}=11 & \sqrt{4}=2 \\ \sqrt{625}=25 & \sqrt{9}=3 & \sqrt{-81}=\text{Undefined} \end{array}$$

The final example, $\sqrt{-81}$, is classified as being undefined in the real number system since negatives have no square root. This is because if you square a positive or a negative, the answer will be positive. This means that when using the real number system, take only square roots of positive numbers. There are solutions to negative square roots, but they require a new number system to be created that is termed the imaginary number system. For now, simply say they are undefined in the real number system or that they have no real solution

Not all numbers have a nice even square root. For example, if you look up $\sqrt{8}$ on your calculator, the answer would be 2.8284271247461900976033774484l9…, with this number being a rounded approximation of the square root. The standard for radicals that have large, rounded solutions is that the calculator is not used to find decimal approximations of square roots. Instead, express roots in simplest radical form.

There are a number of properties that can be used when working with radicals. One is known as the product rule:

$$\text{Product Rule of Square Roots: } \sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b}$$

Use the product rule to simplify an expression by finding perfect squares that divide evenly into the radicand (the number under the radical). Commonly used perfect squares are:

$$\begin{array}{llllll} 4=2^2 & 9=3^2 & 16=4^2 & 25=5^2 & 36=6^2 & 49=7^2 \\ 64=8^2 & 81=9^2 & 100=10^2 & 121=11^2 & 144=12^2 & 169=13^2 \\ 196=14^2 & 225=15^2 & 400=20^2 & 625=25^2 & 900=30^2 & 1600=40^2 \end{array}$$

The challenge in reducing radicals is often simplified to finding the perfect square to divide into the radicand.

Example 9.1.2

Find the perfect squares that divide evenly into the radicand.

1. $18 = 2 \cdot 9$
2. $75 = 3 \cdot 25$
3. $125 = 5 \cdot 25$
4. $72 = 2 \cdot 36$
5. $98 = 2 \cdot 47$
6. $45 = 5 \cdot 9$

Combining the strategies used in the above two examples makes the simplest strategy to reduce radicals.

Example 9.1.3

Reduce $\sqrt{75}$.

$$\sqrt{75} = \sqrt{25} \cdot \sqrt{3}$$
$$\sqrt{25} \cdot \sqrt{3} \text{ reduces to } 5 \cdot \sqrt{3} \text{ or } 5\sqrt{3}$$
$$\sqrt{75} = 5\sqrt{3}$$

If there is a coefficient in front of the radical to begin with, the problem merely becomes a big multiplication problem.

Example 9.1.4

Reduce $5\sqrt{63}$.

$$\begin{array}{rl} 5\sqrt{63} & \text{63 equals } 9 \times 7, \text{ and 9 is a perfect square} \\ 5\sqrt{9 \cdot 7} & \text{Take the square root of 9} \\ 5 \cdot 3\sqrt{7} & \text{Multiply 5 and 3} \\ 15\sqrt{7} & \end{array}$$

Variables often are part of the radicand as well. When taking the square roots of variables, divide the exponent by 2.

For example, $\sqrt{x^8} = x^4$, because you divide the exponent 8 by 2. This follows from the power of a power rule of exponents, $(x^4)^2 = x^8$. When squaring, multiply the exponent by two, so when taking a square root, divide the exponent by 2. This is shown in the following example.

Example 9.1.5

Reduce $-5\sqrt{18x^4y^6z^{10}}$.

$$-5\sqrt{18x^4y^6z^{10}} \quad \text{18 is divisible by 9, a perfect square}$$

$$-5\sqrt{9\cdot 2x^4y^6z^{10}} \quad \text{Split into factors}$$

$$-5\sqrt{9}\cdot\sqrt{2}\cdot\sqrt{x^4}\cdot\sqrt{y^6}\cdot\sqrt{z^{10}} \quad \text{Divide exponents by 2}$$

$$-5\cdot 3x^2y^3z^5\sqrt{2} \quad \text{Multiply coefficients}$$

$$-15x^2y^3z^5\sqrt{2}$$

Sometimes, you cannot evenly divide the exponent on a variable by 2. Sometimes, there is a remainder. If there is a remainder, this means the remainder is left inside the radical, and the whole number part goes outside the radical. This is shown in the following example.

Example 9.1.6

Reduce $\sqrt{20x^5y^9z^6}$.

$$\sqrt{20x^5y^9z^6}$$

$$\sqrt{4\cdot 5x^4xy^8yz^6} \quad \text{Break into square root factors}$$

$$\sqrt{4}\cdot\sqrt{5}\cdot\sqrt{x^4}\cdot\sqrt{x}\cdot\sqrt{y^8}\cdot\sqrt{y}\cdot\sqrt{z^6}$$

$$2x^2y^4z^3\sqrt{5xy}$$

Example 9.1.7

Reduce $\sqrt{42x^{11}y^{10}z^9}$.

$$\sqrt{42x^{11}y^{10}z^9}$$

$$\sqrt{42x^{10}xy^{10}z^8z} \quad \text{Break into square root factors}$$

$$\sqrt{42}\cdot\sqrt{x^{10}}\cdot\sqrt{x}\cdot\sqrt{y^{10}}\cdot\sqrt{z^8}\cdot\sqrt{z}$$

$$x^5y^5z^4\sqrt{42xz}$$

Questions

Simplify the following radicals.

1. $\sqrt{245}$
2. $\sqrt{125}$
3. $2\sqrt{36}$
4. $5\sqrt{196}$
5. $\sqrt{12}$
6. $\sqrt{72}$
7. $3\sqrt{12}$
8. $5\sqrt{32}$
9. $6\sqrt{128}$
10. $7\sqrt{128}$
11. $-7\sqrt{64x^4}$
12. $-2\sqrt{128n}$
13. $-5\sqrt{36m}$
14. $8\sqrt{112p^2}$
15. $\sqrt{45x^2y^2}$
16. $\sqrt{72a^3b^4}$
17. $\sqrt{16x^3y^3}$
18. $\sqrt{512a^4b^2}$
19. $\sqrt{320x^4y^4}$
20. $\sqrt{512m^4n^3}$

Answer Key 9.1

76. 9.2 Reducing Higher Power Roots

While square roots are the most common type of radical, there are higher roots of numbers as well: cube roots, fourth roots, fifth roots, and so on. The following is a definition of radicals:

$$\sqrt[m]{a} = b \text{ if } b^m = a$$

The small letter m inside the radical is called the index. It dictates which root you are taking. For square roots, the index is 2, which, since it is the most common root, is not usually written.

Example 9.2.1

Here are several higher powers of positive numbers and their roots:

$2^2 = 4$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$	$2^6 = 64$	$2^7 = 128$
$3^2 = 9$	$3^3 = 27$	$3^4 = 81$	$3^5 = 243$	$3^6 = 729$	$3^7 = 2187$
$4^2 = 16$	$4^3 = 64$	$4^4 = 256$	$4^5 = 1024$	$4^6 = 4096$	$4^7 = 16384$
$5^2 = 25$	$5^3 = 125$	$5^4 = 625$	$5^5 = 3125$	$5^6 = 15625$	$5^7 = 78125$
$6^2 = 36$	$6^3 = 216$	$6^4 = 1296$	$6^5 = 7776$	$6^6 = 46656$	$6^7 = 279936$
$7^2 = 49$	$7^3 = 343$	$7^4 = 2401$	$7^5 = 16807$	$7^6 = 117649$	$7^7 = 823543$
$8^2 = 64$	$8^3 = 512$	$8^4 = 4096$	$8^5 = 32768$	$8^6 = 262144$	$8^7 = 2097152$
$9^2 = 81$	$9^3 = 729$	$9^4 = 6561$	$9^5 = 59049$	$9^6 = 531441$	$9^7 = 4782969$
$10^2 = 100$	$10^3 = 1000$	$10^4 = 10000$	$10^5 = 100000$	$10^6 = 1000000$	

$2 = \sqrt{4}$	$2 = \sqrt[3]{8}$	$2 = \sqrt[4]{16}$	$2 = \sqrt[5]{32}$	$2 = \sqrt[6]{64}$	$2 = \sqrt[7]{128}$
$3 = \sqrt{9}$	$3 = \sqrt[3]{27}$	$3 = \sqrt[4]{81}$	$3 = \sqrt[5]{243}$	$3 = \sqrt[6]{729}$	$3 = \sqrt[7]{2187}$
$4 = \sqrt{16}$	$4 = \sqrt[3]{64}$	$4 = \sqrt[4]{256}$	$4 = \sqrt[5]{1024}$	$4 = \sqrt[6]{4096}$	$4 = \sqrt[7]{16384}$
$5 = \sqrt{25}$	$5 = \sqrt[3]{125}$	$5 = \sqrt[4]{625}$	$5 = \sqrt[5]{3125}$	$5 = \sqrt[6]{15625}$	$5 = \sqrt[7]{78125}$
$6 = \sqrt{36}$	$6 = \sqrt[3]{216}$	$6 = \sqrt[4]{1296}$	$6 = \sqrt[5]{7776}$	$6 = \sqrt[6]{46656}$	$6 = \sqrt[7]{279936}$
$7 = \sqrt{49}$	$7 = \sqrt[3]{343}$	$7 = \sqrt[4]{2401}$	$7 = \sqrt[5]{16807}$	$7 = \sqrt[6]{117649}$	$7 = \sqrt[7]{823543}$
$8 = \sqrt{64}$	$8 = \sqrt[3]{512}$	$8 = \sqrt[4]{4096}$	$8 = \sqrt[5]{32768}$	$8 = \sqrt[6]{262144}$	$8 = \sqrt[7]{2097152}$
$9 = \sqrt{81}$	$9 = \sqrt[3]{729}$	$9 = \sqrt[4]{6561}$	$9 = \sqrt[5]{59049}$	$9 = \sqrt[6]{531441}$	$9 = \sqrt[7]{4782969}$
$10 = \sqrt{100}$	$10 = \sqrt[3]{1000}$	$10 = \sqrt[4]{10000}$	$10 = \sqrt[5]{100000}$	$10 = \sqrt[6]{1000000}$	

Note there is a notable distinction between solutions of even roots and of odd roots. For even-powered roots, the solution is always +/− or ±. The reason for this can shown in the following examples.

Example 9.2.2

Find the solutions to √4.

There are two ways to multiple identical numbers to equal 4:

$$(2)(2) = 4 \text{ and } (-2)(-2) = 4$$

This means that the √4 is either +2 or −2, which is often written as ±2.

The ± solution occurs for all even roots and can be seen in:

$$\sqrt[4]{16} = \pm 2 \text{ and } \sqrt[6]{64} = \pm 2 \text{ and } \sqrt[8]{256} = \pm 2$$

All roots that have an even index will always have ± solutions.

Odd-powered roots do not share this feature and will only maintain the sign of the number that you are taking the root of.

Example 9.2.3

Find the solutions to $\sqrt[3]{8}$ and $\sqrt[3]{-8}$.

The solution of $\sqrt[3]{8}$ is 2 and $\sqrt[3]{-8}$ is −2.

The reason for this is $(2)^3 = 8$ and $(-2)^3 = -8$.

All negative-indexed roots will keep the sign of the number being rooted.

Higher roots can be simplified in much the same way one simplifies square roots: through using the product property of radicals.

$$\text{Product Property of Radicals: } m\sqrt{ab} = m(\sqrt{a} \cdot m\sqrt{b})$$

Examples 9.2.4

Use the product property of radicals to simplify the following.

1. $\sqrt[3]{32}$32 can be broken down into 2^5. Since you are taking the cube root of this number, you can only take out numbers that have a cube root. This means that 32 is broken into 8 × 4, with the number 8 being the only number that you can take the cube root of.

   $$\sqrt[3]{32} = \sqrt[3]{8} \cdot \sqrt[3]{4}$$

   This simplifies to:

$$\sqrt[3]{32} = 2\sqrt[3]{4}$$

2. $\sqrt[5]{96}$

$$\sqrt[5]{96} = \sqrt[5]{32} \cdot \sqrt[5]{3}$$

96 can be broken down into $2^5 \times 3$. Since you are taking the fifth root of this number, you can only take out numbers that have a fifth root. This means that 96 is broken into 32 × 3, with the number 32 being the only number that you can take the fifth root of.This simplifies to:

$$\sqrt[5]{96} = 2\sqrt[5]{3}$$

This strategy is used to take the higher roots of variables. In this case, only take out whole number multiples of the root index. This is shown in the following examples.

Example 9.2.5

Reduce $\sqrt[4]{x^{25}y^{16}z^4}$.

For this root, you will break the exponent into multiples of the index 4.

This means that $x^{25}y^{16}z^4$ will be broken up into $x^{24}xy^{16}z^4$.

The fourth roots of $x^{24}y^{16}z^4$ are x^6y^4z and the solitary x remains under the fourth root radical. This looks like:

$$\sqrt[4]{x^{25}y^{16}z^4} = \sqrt[4]{x^{24}} \cdot \sqrt[4]{x} \cdot \sqrt[4]{y^{16}} \cdot \sqrt[4]{z^4}$$

Which simplifies to:

$$x^6y^4z\sqrt[4]{x}$$

Example 9.2.6

Reduce $\sqrt[5]{64x^{25}y^{16}z^4}$.

For this root, you will break the exponent into multiples of the index 5.

This means that $x^{25}y^{16}z^4$ will be broken up into $x^{25}y^{15}yz^4$ and 64 broken up into 32 × 2.

The fifth roots of $32x^{25}y^{15}$ are $2x^5y^3$ and the remainder $2yz^4$ remains under the fifth root radical.

This looks like:

$$\sqrt[5]{64x^{25}y^{16}z^4} = \sqrt[5]{32} \cdot \sqrt[5]{2} \cdot \sqrt[5]{x^{25}} \cdot \sqrt[5]{y^{15}} \cdot \sqrt[5]{y} \cdot \sqrt[5]{z^4}$$

Which simplifies to:

$$2x^5y^3\sqrt[5]{2yz^4}$$

Questions

Simplify the following radicals.

1. $\sqrt[3]{64}$
2. $\sqrt[3]{-125}$
3. $\sqrt[3]{625}$
4. $\sqrt[3]{250}$
5. $\sqrt[3]{192}$
6. $\sqrt[3]{-24}$
7. $-4\sqrt[4]{96}$
8. $-8\sqrt[4]{48}$
9. $6\sqrt[4]{112}$
10. $5\sqrt[4]{243}$
11. $6\sqrt[4]{648x^5y^7z^2}$
12. $-6\sqrt[4]{405a^5b^8c}$
13. $\sqrt[5]{224n^3p^7q^5}$
14. $\sqrt[5]{-96x^3y^6z^5}$
15. $\sqrt[5]{224p^5q^{10}r^{15}}$
16. $\sqrt[6]{256x^6y^6z^7}$
17. $-3\sqrt[7]{896rs^7t^{14}}$
18. $-8\sqrt[7]{384b^8c^7d^6}$

Answer Key 9.2

77. 9.3 Adding and Subtracting Radicals

Adding and subtracting radicals is similar to adding and subtracting variables. The condition is that the variables, like the radicals, must be identical before they can be added or subtracted. Recall the addition and subtraction of like variables:

Example 9.3.1

Simplify $4x^2+5x-6x^2+3x-2x$.

First, we sort out like variables and reorder them to be combined.

$$\begin{array}{rl} & 4x^2+5x-6x^2+3x-2x \\ \text{becomes} & 4x^2-6x^2 \text{ and } 5x+3x-2x \end{array}$$

Combining like variables yields:

$$-2x^2+6x$$

When adding and subtracting radicals, follow the same logic. Radicals must be the same before they can be combined.

Example 9.3.2

Simplify $5\sqrt{11}+5\sqrt{13}-2\sqrt{13}+6\sqrt{11}-2\sqrt{11}$.

First, we sort out like variables and reorder them to be combined.

$$\begin{array}{rl} & 5\sqrt{11}+5\sqrt{13}-2\sqrt{13}+6\sqrt{11}-2\sqrt{11} \\ \text{becomes} & 5\sqrt{13}-2\sqrt{13} \text{ and } 5\sqrt{11}+6\sqrt{11}-2\sqrt{11} \end{array}$$

Combining like radicals yields:

$$3\sqrt{13}+9\sqrt{11}$$

Generally, it is required to simplify radicals before combining them. For example:

Example 9.3.3

Simplify $4\sqrt{45}+3\sqrt{18}-\sqrt{98}+2\sqrt{20}$.

All of these radicals need to be simplified before they can be combined.

$$\begin{array}{ll} & 4\sqrt{45}+3\sqrt{18}-\sqrt{98}+2\sqrt{20} \\ \text{becomes} & 4\sqrt{9\cdot 5}+3\sqrt{9\cdot 2}-\sqrt{49\cdot 2}+2\sqrt{5\cdot 4} \\ \text{simplifying to} & 4\cdot 3\sqrt{5}+3\cdot 3\sqrt{2}-7\sqrt{2}+2\cdot 2\sqrt{5} \\ \text{and reduces to} & 12\sqrt{5}+9\sqrt{2}-7\sqrt{2}+4\sqrt{5} \end{array}$$

Recombining these so they can be added and subtracted yields:

$$12\sqrt{5}+4\sqrt{5} \text{ and } 9\sqrt{2}-7\sqrt{2}$$

Combining like radicals yields:

$$16\sqrt{5}+2\sqrt{2}$$

Higher order radicals are treated in the same fashion as square roots. For example:

Example 9.3.4

Simplify $4\sqrt[3]{54}-9\sqrt[3]{16}+5\sqrt[3]{9}$.

Like example 9.3.3, these radicals need to be simplified before they can be combined.

$$\begin{array}{ll} & 4\sqrt[3]{54}-9\sqrt[3]{16}+5\sqrt[3]{9} \\ \text{becomes} & 4\sqrt[3]{27\cdot 2}-9\sqrt[3]{8\cdot 2}+5\sqrt[3]{9} \\ \text{simplifying to} & 4\cdot 3\sqrt[3]{2}-9\cdot 2\sqrt[3]{2}+5\sqrt[3]{9} \\ \text{and reduces to} & 12\sqrt[3]{2}-18\sqrt[3]{2}+5\sqrt[3]{9} \end{array}$$

Combining like radicals yields:

$$5\sqrt[3]{9}-6\sqrt[3]{2}$$

Questions

Simplify.

1. $2\sqrt{5}+2\sqrt{5}+2\sqrt{5}$
2. $-3\sqrt{6}-3\sqrt{3}-2\sqrt{3}$
3. $-3\sqrt{2}+3\sqrt{5}+3\sqrt{5}$
4. $-2\sqrt{6}-\sqrt{3}-3\sqrt{6}$

5. $2\sqrt{2}-3\sqrt{18}-\sqrt{2}$
6. $-\sqrt{54}-3\sqrt{6}+3\sqrt{27}$
7. $-3\sqrt{6}-\sqrt{12}+3\sqrt{3}$
8. $-\sqrt{5}-\sqrt{5}-2\sqrt{54}$
9. $3\sqrt{2}+2\sqrt{8}-3\sqrt{18}$
10. $2\sqrt{20}+2\sqrt{20}-\sqrt{3}$
11. $3\sqrt{18}-\sqrt{2}-3\sqrt{2}$
12. $-3\sqrt{27}+2\sqrt{3}-\sqrt{12}$
13. $-3\sqrt{6}-3\sqrt{6}-\sqrt{3}+3\sqrt{6}$
14. $-2\sqrt{2}-\sqrt{2}+3\sqrt{8}+3\sqrt{6}$
15. $-2\sqrt{18}-3\sqrt{8}-\sqrt{20}+2\sqrt{20}$
16. $-3\sqrt{18}-\sqrt{8}+2\sqrt{8}+2\sqrt{8}$
17. $-2\sqrt{24}-2\sqrt{6}+2\sqrt{6}+2\sqrt{20}$
18. $-3\sqrt{8}-\sqrt{5}-3\sqrt{6}+2\sqrt{18}$
19. $3\sqrt{24}-3\sqrt{27}+2\sqrt{6}+2\sqrt{8}$
20. $2\sqrt{6}-\sqrt{54}-3\sqrt{27}-\sqrt{3}$

Answer Key 9.3

78. 9.4 Multiplication and Division of Radicals

Multiplying radicals is very simple if the index on all the radicals match. The product rule of radicals, which is already been used, can be generalized as follows:

$$\text{Product Rule of Radicals: } a\sqrt[m]{b}\cdot c\sqrt[m]{d}=ac\sqrt[m]{bd}$$

This means that, if the index on the radicals match, then simply multiply the factors outside the radical and also multiply the factors inside the radicals. An example showing this is as follows.

Example 9.4.1

Multiply $-5\sqrt{14}\cdot 4\sqrt{6}$.

$$\begin{array}{ll} \text{This results in} & -5\cdot 4\sqrt{14\cdot 6} \\ \text{Which simplifies to} & -20\sqrt{84} \\ \text{Reducing inside the radical leaves} & -20\sqrt{4\cdot 21} \\ \text{Yielding} & -20\cdot 2\sqrt{21} \\ \text{Or} & -40\sqrt{21} \end{array}$$

This same process works with any higher root radicals having matching indices.

Example 9.4.2

Multiply $2\sqrt[3]{18}\cdot 6\sqrt[3]{15}$.

$$\begin{array}{ll} \text{This results in} & 2\cdot 6\sqrt[3]{18\cdot 15} \\ \text{Which simplifies to} & 12\sqrt[3]{270} \\ \text{Reducing inside the radical leaves} & 12\sqrt[3]{27\cdot 10} \\ \text{Yielding} & 12\cdot 3\sqrt[3]{10} \\ \text{Or} & 36\sqrt[3]{10} \end{array}$$

This process of multiplying radicals is the same when multiplying monomial radicals by binomial radicals, binomial radicals by binomial radicals, trinomial radicals (although these are not shown here), and so on.

Example 9.4.3

Multiply $7\sqrt{6}(3\sqrt{10}-5\sqrt{15})$.

$$\begin{array}{ll} \text{Foiling the radicals will leave you with} & 21\sqrt{60}-35\sqrt{90} \\ \text{Reducing inside the radical leaves} & 21\sqrt{4\cdot 15}-35\sqrt{9\cdot 10} \\ \text{Yielding} & 21\cdot 2\sqrt{15}-35\cdot 3\sqrt{10} \\ \text{Or} & 42\sqrt{15}-105\sqrt{10} \end{array}$$

Example 9.4.4

Multiply $(\sqrt{5}-2\sqrt{3})(4\sqrt{10}+6\sqrt{6})$.

Multiplying the factors inside and outside the radicals yields:

$$4\sqrt{50}+6\sqrt{30}-8\sqrt{30}-12\sqrt{18}$$

$$\begin{array}{ll} \text{Reducing inside these radicals leaves} & 4\sqrt{25\cdot 2}+6\sqrt{30}-8\sqrt{30}-12\sqrt{9\cdot 2} \\ \text{Yielding} & 4\cdot 5\sqrt{2}+6\sqrt{30}-8\sqrt{30}-12\cdot 3\sqrt{2} \\ \text{Or} & 20\sqrt{2}+6\sqrt{30}-8\sqrt{30}-36\sqrt{2} \\ \text{Which simplifies to} & -16\sqrt{2}-2\sqrt{30} \end{array}$$

Division with radicals is very similar to multiplication. If you think about division as reducing fractions, you can reduce the coefficients outside the radicals and reduce the values inside the radicals to get our final solution. There is one catch to dividing with radicals: it is considered bad practice to have a radical in the denominator of a final answer, so if there is a radical in the denominator, it should be rationalized by cancelling or multiplying the radicals.

$$\text{Quotient Rule of Radicals: } \frac{a\sqrt[m]{b}}{c\sqrt[m]{d}}=\left(\frac{a}{c}\right)\sqrt[m]{\frac{b}{d}}$$

The quotient rule means that factors outside the radical are divided by each other and the factors inside the radical are also divided by each other. To see this illustrated, consider the following:

Example 9.4.5

Reduce $\dfrac{15\sqrt[3]{108}}{20\sqrt[3]{2}}$.

Using the quotient rule of radicals, this problem is separated into factors inside and outside the radicals. This results in the following:

$$\left(\frac{15}{20}\right)\sqrt[3]{\frac{108}{2}}$$

Simplifying the two resulting divisions leaves us with $\left(\frac{3}{4}\right)\sqrt[3]{54}$

Which we can further reduce to $\left(\frac{3}{4}\right)\sqrt[3]{27\cdot 2}$

Taking the cube root of 27 leaves us with $\left(\frac{3}{4}\right)3\sqrt[3]{2}$

Or $\left(\frac{9}{4}\right)\sqrt[3]{2}$

Which can also be written as $\frac{9\sqrt[3]{2}}{4}$

Removing radicals from the denominator that cannot be divided out by using the numerator is often simply done by multiplying the numerator and denominator by a common radical. This is easily done and is shown by the following examples.

Example 9.4.6

Rationalize the denominator of $\frac{\sqrt{6}}{\sqrt{5}}$.

For this pair of radicals, the denominator $\sqrt{5}$ cannot be cancelled by the $\sqrt{6}$, so the solution requires that $\sqrt{5}$ be rationalized through multiplication. This is done as follows:

$$\frac{\sqrt{6\cdot 5}}{\sqrt{5\cdot 5}}$$

This now simplifies to:

$$\frac{\sqrt{30}}{5}$$

This process is similar for radicals in which the index is greater than 2.

Example 9.4.7

Rationalize the denominator of $\dfrac{4\sqrt[3]{6}}{5\sqrt[3]{25}}$.

To rationalize the denominator, we need to get a cube root of 125, which will leave us with a denominator of 5 × 5. This requires that both the numerator and the denominator to be multiplied by the cube root of 5. This looks like:

$$\frac{4\sqrt[3]{6\cdot 5}}{5\sqrt[3]{25\cdot 5}}=\frac{4\sqrt[3]{30}}{5\sqrt[3]{125}}$$

This simplifies to:

$$\frac{4\sqrt[3]{30}}{5\cdot 5}$$

Or:

$$\frac{4\sqrt[3]{30}}{25}$$

The last example to be considered involves rationalizing denominators that have variables. Remeber to always reduce any fractions (inside and outside of the radical) before rationalizing.

Example 9.4.8

Rationalize the denominator of $\dfrac{18\sqrt[4]{6x^3y^4z}}{8\sqrt[4]{10xy^6z^3}}$.

The first thing to do is cancel all common factors both inside and outside the radicals. This leaves:

$$\frac{9\sqrt[4]{3x^2}}{4\sqrt[4]{5y^2z^2}}$$

The next step is to multiply both the numerator and denominator to rationalize the denominator:

$$\frac{9\sqrt[4]{3x^2}}{4\sqrt[4]{5y^2z^2}}\cdot\frac{\sqrt[4]{125y^2z^2}}{\sqrt[4]{125y^2z^2}}$$

Multiplying these yields:

$$\frac{9\sqrt[4]{375x^2y^2z^2}}{4\sqrt[4]{625x^4y^4z^4}}$$

Taking the fourth root of the denominator leaves:

$$\frac{9\sqrt[4]{375x^2y^2z^2}}{4\cdot 5xyz}$$

Or:

$$\frac{9\sqrt[4]{375x^2y^2z^2}}{20xyz}$$

Questions

Simplify.

1. $3\sqrt{5}\cdot 4\sqrt{16}$
2. $-5\sqrt{10}\cdot\sqrt{15}$
3. $\sqrt{12m}\cdot\sqrt{15m}$
4. $\sqrt{5r^3}-5\sqrt{10r^2}$
5. $\sqrt[3]{4x^3}\cdot\sqrt[3]{2x^4}$
6. $3\sqrt[3]{4a^4}\cdot\sqrt[3]{10a^3}$
7. $\sqrt{6}(\sqrt{2}+2)$
8. $\sqrt{10}(\sqrt{5}+\sqrt{2})$
9. $-5\sqrt{15}(3\sqrt{3}+2)$
10. $5\sqrt{15}(3\sqrt{3}+2)$
11. $5\sqrt{10}(5n+\sqrt{2})$
12. $\sqrt{15}(\sqrt{5}-3\sqrt{3v})$
13. $(2+2\sqrt{2})(-3+\sqrt{2})$
14. $(-2+\sqrt{3})(-5+2\sqrt{3})$
15. $(\sqrt{5}-5)(2\sqrt{5}-1)$
16. $(2\sqrt{3}+\sqrt{5})(5\sqrt{3}+2\sqrt{4})$
17. $(\sqrt{2a}+2\sqrt{3a})(3\sqrt{2a}+\sqrt{5a})$
18. $(-2\sqrt{2p}+5\sqrt{5})(\sqrt{5p}+\sqrt{5p})$
19. $(-5-4\sqrt{3})(-3-4\sqrt{3})$
20. $(5\sqrt{2}-1)(-\sqrt{2m}+5)$
21. $\dfrac{\sqrt{12}}{5\sqrt{100}}$
22. $\dfrac{\sqrt{15}}{2\sqrt{4}}$

23. $\dfrac{\sqrt{5}}{4\sqrt{125}}$
24. $\dfrac{\sqrt{12}}{\sqrt{3}}$
25. $\dfrac{\sqrt{10}}{\sqrt{6}}$
26. $\dfrac{\sqrt{2}}{3\sqrt{5}}$
27. $\dfrac{5x^2}{4\sqrt{3x^3y^3}}$
28. $\dfrac{4}{5\sqrt{3xy^4}}$
29. $\dfrac{\sqrt{2p^2}}{\sqrt{3p}}$
30. $\dfrac{\sqrt{8n^2}}{\sqrt{10n}}$

Answer Key 9.4

79. 9.5 Rationalizing Denominators

It is considered non-conventional to have a radical in the denominator. When this happens, generally the numerator and denominator are multiplied by the same factors to remove the radical denominator. The problems in the previous section dealt with removing a monomial radical. In this section, the previous strategy is expanded to include binomial radicals.

Example 9.5.1

Rationalize $\dfrac{\sqrt{3}-9}{2\sqrt{6}}$.

To rationalize the denominator, multiply out the $\sqrt{6}$.

This will look like:

$$\frac{(\sqrt{3}-9)(\sqrt{6})}{2\sqrt{6}(\sqrt{6})}$$

Multiplying the $\sqrt{6}$ throughout yields:

$$\frac{(\sqrt{3})(\sqrt{6})-(9)(\sqrt{6})}{2\sqrt{36}}$$

Which reduces to:

$$\frac{3\sqrt{2}-9\sqrt{6}}{2\cdot 6}$$

And simplifies to:

$$\frac{\sqrt{2}-3\sqrt{6}}{4}$$

Please note that, in reducing the numerator and denominator by the factor 3, reduce each term in the numerator by 3.

Quite often, there will be a denominator binomial that contains radicals. For these problems, it is easiest to use a feature from the sum and difference of squares: $a^2-b^2=(a+b)(a-b)$.

$(a+b)(a-b)$ are termed conjugates of each other. They are identical binomials, except that their signs are opposite. When encountering radical binomials, simply multiply by the conjugates to square out the radical.

Example 9.5.2

Square out the radical of the binomial $(\sqrt{3}-\sqrt{5})$ using its conjugate.

The conjugate of $(\sqrt{3}-\sqrt{5})$ is $(\sqrt{3}+\sqrt{5})$.

When multiplied, these conjugates yield $(\sqrt{3}-\sqrt{5})(\sqrt{3}+\sqrt{5})$ or $(\sqrt{3})^2-(\sqrt{5})^2$.

This yields 3 − 5 = −2.

When encountering a radicalized binomial denominator, the best solution is to multiply both the numerator and denominator by the conjugate of the denominator.

Example 9.5.3

Rationalize the denominator of $\dfrac{\sqrt{6}}{\sqrt{6}+\sqrt{13}}$.

Multiplying the numerator and denominator by the denominator's conjugate yields:

$$\frac{\sqrt{6}}{\sqrt{6}+\sqrt{13}}\cdot\frac{(\sqrt{6}-\sqrt{13})}{(\sqrt{6}-\sqrt{13})}$$

When multiplied out, this yields:

$$\frac{(\sqrt{6})^2-\sqrt{6}\sqrt{13}}{(\sqrt{6})^2-(\sqrt{13})^2}$$

Which reduces to:

$$\frac{6-\sqrt{78}}{6-13}$$

Or:

$$\frac{6-\sqrt{78}}{-7}$$

Questions

Rationalize the following radical fractions.

1. $\dfrac{4+2\sqrt{3}}{\sqrt{3}}$

2. $\dfrac{-4+\sqrt{3}}{4\sqrt{3}}$
3. $\dfrac{4+2\sqrt{3}}{5\sqrt{6}}$
4. $\dfrac{2\sqrt{3}-2}{2\sqrt{3}}$
5. $\dfrac{2-5\sqrt{5}}{4\sqrt{3}}$
6. $\dfrac{\sqrt{5}+4}{4\sqrt{5}}$
7. $\dfrac{\sqrt{2}-3\sqrt{3}}{\sqrt{3}}$
8. $\dfrac{\sqrt{5}-\sqrt{2}}{3\sqrt{6}}$
9. $\dfrac{5}{3\sqrt{5}+\sqrt{2}}$
10. $\dfrac{5}{\sqrt{3}+4\sqrt{5}}$
11. $\dfrac{2}{5+\sqrt{2}}$
12. $\dfrac{5}{2\sqrt{3}-\sqrt{2}}$
13. $\dfrac{3}{4-\sqrt{3}}$
14. $\dfrac{4}{\sqrt{2}-2}$
15. $\dfrac{4}{3+\sqrt{5}}$
16. $\dfrac{2}{\sqrt{5}+2\sqrt{3}}$
17. $\dfrac{-3+2\sqrt{3}}{\sqrt{3}+2}$
18. $\dfrac{4+\sqrt{5}}{2+2\sqrt{5}}$
19. $\dfrac{2-\sqrt{3}}{1+\sqrt{2}}$
20. $\dfrac{-1+\sqrt{3}}{\sqrt{3}-1}$

Answer Key 9.5

80. 9.6 Radicals and Rational Exponents

When simplifying radicals that use fractional exponents, the numerator on the exponent is divided by the denominator. All radicals can be shown as having an equivalent fractional exponent. For example:

$$\sqrt{x}=x^{\frac{1}{2}}\qquad\sqrt[3]{x}=x^{\frac{1}{3}}\qquad\sqrt[4]{x}=x^{\frac{1}{4}}\qquad\sqrt[5]{x}=x^{\frac{1}{5}}$$

Radicals having some exponent value inside the radical can also be written as a fractional exponent. For example:

$$\sqrt{x^3}=x^{\frac{3}{2}}\qquad\sqrt[3]{x^2}=x^{\frac{2}{3}}\qquad\sqrt[4]{x^5}=x^{\frac{5}{4}}\qquad\sqrt[5]{x^9}=x^{\frac{9}{5}}$$

The general form that radicals having exponents take is:

$$x^{\frac{b}{a}}\;=\;\sqrt[a]{x^b}\text{ or }(\sqrt[a]{x})^b$$

Should the reciprocal of a radical having an exponent, it would look as follows:

$$x^{-\frac{b}{a}}\;=\;\frac{1}{\sqrt[a]{x^b}}\text{ or }\frac{1}{(\sqrt[a]{x})^b}$$

In both cases shown above, the power of the radical is b and the root of the radical is a. These are the two forms that a radical having an exponent is commonly written in. It is convenient to work with a radical containing an exponent in one of these two forms.

Example 9.6.1

Evaluate $27^{-\frac{4}{3}}$.

Converting to a radical form:

$$\frac{1}{\sqrt[3]{27^4}}\text{ or }\frac{1}{(\sqrt[3]{27})^4}$$

First, the cube root of 27 will reduce to 3, which leaves:

$$\frac{1}{3^4}\text{ or }\frac{1}{81}$$

Once the radical having an exponent is converted into a pure fractional exponent, then the following rules can be used.

Properties of Exponents

$$a^m a^n = a^{m+n} \qquad (ab)^m = a^m b^m \qquad a^{-m} = \frac{1}{a^m}$$

$$\frac{a^m}{a^n} = a^{m-n} \qquad \left(\frac{a}{b}\right) = \frac{a^m}{b^m} \qquad \frac{1}{a^{-m}} = a^m$$

$$(a^m)^n = a^{mn} \qquad a^0 = 1 \qquad \left(\frac{a}{b}\right)^{-m} = \frac{b^m}{a^m}$$

Example 9.6.2

Simplify $\left(x^2 y^{\frac{4}{3}}\right)\left(x^{-1} y^{\frac{2}{3}}\right)$.

First, you need to separate the different variables:

$$\left(x^2 y^{\frac{4}{3}}\right)\left(x^{-1} y^{\frac{2}{3}}\right) \text{ becomes } x^2 \cdot x^{-1} \cdot y^{\frac{4}{3}} \cdot y^{\frac{2}{3}}$$

Combining the exponents yields:

$$x^{2-1} \cdot y^{\frac{4}{3}+\frac{2}{3}}$$

Which results in:

$$x^1 \cdot y^{\frac{6}{3}}$$

Which simplifies to:

$$xy^2$$

Example 9.6.3

Simplify $\dfrac{ab^{\frac{2}{3}} 3b^{-\frac{5}{3}}}{5a^{-\frac{3}{2}} b^{-\frac{4}{3}}}$.

First, separate the different variables:

$$\frac{ab^{\frac{2}{3}}3b^{-\frac{5}{3}}}{5a^{-\frac{3}{2}}b^{-\frac{4}{3}}} \text{ becomes } 3 \cdot 5^{-1} \cdot a \cdot a^{\frac{3}{2}} \cdot b^{\frac{2}{3}} \cdot b^{-\frac{5}{3}} \cdot b^{\frac{4}{3}}$$ [1]

Combining the exponents yields:

$$3 \cdot 5^{-1} \cdot a^{1+\frac{3}{2}} \cdot b^{\frac{2}{3}-\frac{5}{3}+\frac{4}{3}}$$

Which gives:

$$3 \cdot 5^{-1} \cdot a^{\frac{5}{2}} \cdot b^{\frac{1}{3}}$$

Which simplifies to:

$$\frac{3 \cdot a^{\frac{5}{2}} \cdot b^{\frac{1}{3}}}{5}$$

Questions

Write each of the following fractional exponents in radical form.

1. $m^{\frac{3}{5}}$
2. $(10r)^{-\frac{3}{4}}$
3. $(7x)^{\frac{3}{2}}$
4. $(6b)^{-\frac{4}{3}}$
5. $(2x+3)^{-\frac{3}{2}}$
6. $(x-3y)^{\frac{3}{4}}$

Write each of the following radicals in exponential form.

7. $\sqrt[3]{5}$
8. $\sqrt[5]{2^3}$
9. $\sqrt[3]{ab^5}$
10. $\sqrt[5]{x^3}$
11. $\sqrt[3]{(a+5)^2}$
12. $\sqrt[5]{(a-2)^3}$

Evaluate the following.

13. $8^{\frac{2}{3}}$
14. $16^{\frac{1}{4}}$

1. When we divide by an exponent, we subtract powers.

15. $\sqrt[3]{4^6}$
16. $\sqrt[5]{32^2}$

Simplify. Your answer should only contain positive exponents.

17. $(xy^{\frac{1}{3}})(xy^{\frac{2}{3}})$
18. $(4v^{\frac{2}{3}})(v^{-1})$
19. $(a^{\frac{1}{2}}b^{\frac{1}{2}})^{-1}$
20. $(x^{\frac{5}{3}}y^{-2})^0$
21. $\dfrac{a^2b^0}{3a^4}$
22. $\dfrac{2x^{\frac{1}{2}}y^{\frac{1}{3}}}{2x^{\frac{4}{3}}y^{\frac{7}{4}}}$
23. $\dfrac{a^{\frac{3}{4}}b^{-1}b^{\frac{7}{4}}}{3b^{-1}}$
24. $\dfrac{2x^{-2}y^{\frac{5}{3}}}{x^{-\frac{5}{4}}y^{-\frac{5}{3}}xy^{\frac{1}{2}}}$
25. $\dfrac{3y^{-\frac{5}{4}}}{y^{-1}2y^{-\frac{1}{3}}}$
26. $\dfrac{ab^{\frac{1}{3}}2b^{-\frac{5}{4}}}{4a^{-\frac{1}{2}}b^{-\frac{2}{3}}}$

Answer Key 9.6

81. 9.7 Rational Exponents (Increased Difficulty)

Simplifying rational exponents equations that are more difficult generally involves two steps. First, reduce inside the brackets. Second, multiplu the power outside the brackets for all terms inside.

Example 9.7.1

Simplify the following rational exponent expression:

$$\left(\frac{x^{-2}y^{-3}}{x^{-2}y^{4}}\right)^{2}$$

First, simplifying inside the brackets gives:

$$x^{-2--2}y^{-3-4}$$

Or:

$$x^{0}y^{-7}$$

Which simplifies to:

$$y^{-7}$$

Second, taking the exponent 2 outside the brackets and applying it to the reduced expression gives:

$$y^{-7\cdot 2} \text{ or } y^{-14}$$

Therefore:

$$\left(\frac{x^{-2}y^{-3}}{x^{-2}y^{4}}\right)^{2} = y^{-14}$$

Example 9.7.2

Simplify the following rational exponent expression:

$$\left(\frac{x^{-4}y^{-6}}{x^{-5}y^{10}}\right)^{-3}$$

First, simplifying inside the brackets gives:

$$x^{-4--5}y^{-6-10}$$

Or:

$$x^1y^{-16}$$

Which simplifies to:

$$xy^{-16}$$

Second, taking the exponent −3 outside the brackets and applying it to the reduced expression gives:

$$(xy^{-16})^{-3} \text{ or } x^{-3}y^{48}$$

Therefore:

$$\left(\frac{x^{-4}y^{-6}}{x^{-5}y^{10}}\right)^{-3} = x^{-3}y^{48} = \frac{y^{48}}{x^3}$$

Example 9.7.3

Simplify the following rational exponent expression:

$$\left(\frac{a^0b^3}{c^6d^{-12}}\right)^{\frac{1}{3}}$$

First, simplifying inside the brackets gives:

$$\frac{b^3}{c^6d^{-12}}$$

Second, taking the exponent $\frac{1}{3}$ outside the brackets and applying it to the reduced expression gives:

$$\frac{b^{3\cdot\frac{1}{3}}}{c^{6\cdot\frac{1}{3}}d^{-12\cdot\frac{1}{3}}}$$

Or:

$$\frac{b}{c^2d^{-4}}$$

Which simplifies to:

$$\frac{bd^4}{c^2}$$

Questions

Simplify the following rational exponents.

1. $\left(\dfrac{x^{-2}y^{-6}}{x^{-2}y^{4}}\right)^{2}$
2. $\left(\dfrac{x^{-3}y^{-3}}{x^{-1}y^{6}}\right)^{3}$
3. $\left(\dfrac{x^{-2}y^{-4}}{x^{2}y^{-4}}\right)^{2}$
4. $\left(\dfrac{x^{-5}y^{-3}}{x^{-4}y^{2}}\right)^{4}$
5. $\left(\dfrac{x^{-2}y^{-2}}{x^{-3}y^{3}}\right)^{8}$
6. $\left(\dfrac{x^{-4}y^{-3}}{x^{-3}y^{2}}\right)^{5}$
7. $\left(\dfrac{x^{-2}y^{-4}}{x^{-2}y^{4}}\right)^{-2}$
8. $\left(\dfrac{x^{-2}y^{-3}}{x^{-5}y^{3}}\right)^{-3}$
9. $\left(\dfrac{x^{-2}y^{-3}}{x^{-2}y^{-3}}\right)^{-1}$
10. $\left(\dfrac{x^{-2}y^{-3}}{x^{-2}y^{4}}\right)^{-2}$
11. $\left(\dfrac{x^{0}y^{-3}}{x^{-2}y^{0}}\right)^{-5}$
12. $\left(\dfrac{x^{-22}y^{-36}}{x^{-24}y^{12}}\right)^{0}$
13. $\left(\dfrac{a^{0}b^{3}}{a^{6}b^{-12}}\right)^{-\frac{1}{3}}$
14. $\left(\dfrac{a^{12}b^{4}}{a^{8}c^{-12}}\right)^{\frac{1}{4}}$
15. $\left(\dfrac{a^{5}c^{10}}{b^{5}d^{-15}}\right)^{\frac{2}{5}}$
16. $\left(\dfrac{a^{2}b^{8}}{a^{6}b^{-12}}\right)^{-\frac{3}{4}}$
17. $\left(\dfrac{a^{0}b^{3}}{c^{6}d^{-12}}\right)^{\frac{0}{3}}$
18. $\left(\dfrac{a^{0}b^{3}}{c^{6}d^{-12}}\right)^{\frac{1}{10}}$

Answer Key 9.7

82. 9.8 Radicals of Mixed Index

Knowing that a radical has the same properties as exponents allows conversion of radicals to exponential form and then reduce according to the various rules of exponents is possible. This is shown in the following examples.

Example 9.8.1

Simplify $\sqrt[8]{x^6y^2}$.

$$\begin{array}{ll} \text{First rewrite the radical as a fractional exponent} & (x^6y^2)^{\frac{1}{8}} \\ \text{Multiply all exponents} & x^{6\cdot\frac{1}{8}}y^{2\cdot\frac{1}{8}} \\ \text{This yields} & x^{\frac{6}{8}}y^{\frac{2}{8}} \\ \text{Reducing this gives} & x^{\frac{3}{4}}y^{\frac{1}{4}} \\ \text{Rewrite as} & \sqrt[4]{x^3y} \end{array}$$

Note: In Example 9.8.1, all exponents are reduced by the common factor 2. If there is a common factor in all exponents, reduce by dividing that common factor without having to convert to a different form.

Example 9.8.2

Simplify $\sqrt[24]{a^6b^9c^{15}}$.

For this radical, notice that each exponent has the common factor 3.

The solution is to divide each exponent by 3, which yields $\sqrt[8]{a^2b^3c^5}$.

When encountering problems where the index of the radicals do not match,convert each radical to individual exponents and use the properties of exponents to combine and then reduce the radicals.

Example 9.8.3

Simplify $\sqrt[3]{4x^2y} \cdot \sqrt[4]{8xy^3}$.

First, convert each radical to a complete exponential form.

This looks like	$(4x^2y)^{\frac{1}{3}}(8xy^3)^{\frac{1}{4}}$
Multiply all exponents	$4^{\frac{1}{3}}x^{2\cdot\frac{1}{3}}y^{\frac{1}{3}}8^{\frac{1}{4}}x^{\frac{1}{4}}y^{3\cdot\frac{1}{4}}$
This yields	$4^{\frac{1}{3}}x^{\frac{2}{3}}y^{\frac{1}{3}}8^{\frac{1}{4}}x^{\frac{1}{4}}y^{\frac{3}{4}}$
Combining like variables leaves	$4^{\frac{1}{3}}8^{\frac{1}{4}}x^{\frac{2}{3}}x^{\frac{1}{4}}y^{\frac{1}{3}}y^{\frac{3}{4}}$
(Note:	$4^{\frac{1}{3}}8^{\frac{1}{4}} = 2^{2\cdot\frac{1}{3}}2^{3\cdot\frac{1}{4}} = 2^{\frac{2}{3}}2^{\frac{3}{4}}$)
Accounting for this yields	$2^{\frac{2}{3}}2^{\frac{3}{4}}x^{\frac{2}{3}}x^{\frac{1}{4}}y^{\frac{1}{3}}y^{\frac{3}{4}}$
Reducing this yields	$2^{\frac{2}{3}+\frac{3}{4}}x^{\frac{2}{3}+\frac{1}{4}}y^{\frac{1}{3}+\frac{3}{4}}$
Which further reduces to	$2^{\frac{17}{12}}x^{\frac{11}{12}}y^{\frac{13}{12}}$
Reduce this	$2y \cdot 2^{\frac{5}{12}}x^{\frac{11}{12}}y^{\frac{1}{12}}$
Convert this back into a radical	$2y(2^5x^{11}y)^{\frac{1}{12}}$
Which leaves	$2y\sqrt[12]{2^5x^{11}y}$

The strategy of converting all radicals to exponents works for increasingly complex radicals.

Example 9.8.4

Simplify $\sqrt{3x(y+z)} \cdot \sqrt[3]{9x(y+z)^2}$.

First, convert each radical to a complete exponential form.

This looks like	$3^{\frac{1}{2}}x^{\frac{1}{2}}(y+z)^{\frac{1}{2}}9^{\frac{1}{3}}x^{\frac{1}{3}}(y+z)^{\frac{2}{3}}$
(Note:	$9^{\frac{1}{3}} = 3^{\frac{2}{3}}$)
Combining like variables leaves	$3^{\frac{1}{2}}3^{\frac{2}{3}}x^{\frac{1}{2}}x^{\frac{1}{3}}(y+z)^{\frac{1}{2}}(y+z)^{\frac{2}{3}}$
Reducing this yields	$3^{\frac{1}{2}+\frac{2}{3}}x^{\frac{1}{2}+\frac{1}{3}}(y+z)^{\frac{1}{2}+\frac{2}{3}}$
Which further reduces to	$3^{\frac{7}{6}}x^{\frac{5}{6}}(y+z)^{\frac{7}{6}}$
Reduce this	$3(y+z)3^{\frac{1}{6}}x^{\frac{5}{6}}(y+z)^{\frac{1}{6}}$
Convert this back into a radical	$3(y+z)[3x^5(y+z)]^{\frac{1}{6}}$
Which leaves	$3(y+z)\sqrt[6]{3x^5(y+z)}$

Questions

Reduce the following radicals. Leave as fractional exponents.

1. $\sqrt[8]{16x^4y^6}$
2. $\sqrt[4]{9x^2y^6}$
3. $\sqrt[12]{64x^4y^6z^8}$
4. $\sqrt[8]{\dfrac{25x^3}{16x^5}}$
5. $\sqrt[6]{\dfrac{16x}{9y^4}}$
6. $\sqrt[15]{x^9y^{12}z^6}$
7. $\sqrt[12]{x^6y^9}$
8. $\sqrt[10]{64x^8y^4}$
9. $\sqrt[8]{x^6y^4z^2}$
10. $\sqrt[4]{25y^2}$
11. $\sqrt[9]{8x^3y^6}$
12. $\sqrt[16]{81x^8y^{12}}$

Combine the following radicals. Leave as fractional exponents.

13. $\sqrt[3]{5}\sqrt{5}$
14. $\sqrt[3]{7}\sqrt[4]{7}$
15. $\sqrt{x}\sqrt[3]{7x}$
16. $\sqrt[3]{y}\sqrt[5]{3y}$
17. $\sqrt{x}\sqrt[3]{x^2}$
18. $\sqrt[4]{3x}\sqrt{x^4}$
19. $\sqrt[5]{x^2y}\sqrt{x^2}$
20. $\sqrt{ab}\sqrt[5]{2a^2b^2}$
21. $\sqrt[4]{xy^2}\sqrt[3]{x^2y}$
22. $\sqrt[5]{3a^2b^3}\sqrt[4]{9a^2b}$
23. $\sqrt[4]{a^2bc^2}\sqrt[5]{a^2b^3c}$
24. $\sqrt[6]{x^2yz^3}\sqrt[5]{x^2yz^2}$

Answer Key 9.8

83. 9.9 Complex Numbers (Optional)

Throughout history, there has been the need for a number that has a zero value (0), numbers smaller than zero (negatives), numbers between integers (fractions), and numbers between fractions (irrational numbers). This need arises when trying to take an even root, such as the square root of a negative number. To accomplish this, mathematicians have created what are termed imaginary and complex numbers.

$$\text{Definition of Imaginary Numbers: } i^2 = -1, \text{ thus } i = (-1)^{\frac{1}{2}} \text{ or } \sqrt{-1}$$

From this, notice that:

$$\begin{aligned}
[(-1)^{\frac{1}{2}}]^2 &= -1 \\
[(-1)^{\frac{1}{2}}]^3 &= (-1)^{\frac{3}{2}} = 1(-1)^{\frac{1}{2}} \text{ or } 1\sqrt{-1} \\
[(-1)^{\frac{1}{2}}]^4 &= (-1)^2 = 1
\end{aligned}$$

The nature of imaginary numbers generates the following repeating pattern:

$$[(-1)^{\frac{1}{2}}]^1 \text{ or } [(-1)^{\frac{1}{2}}]^5 \text{ or } [(-1)^{\frac{1}{2}}]^9 \text{ or } [(-1)^{\frac{1}{2}}]^{13} \text{ or } [(-1)^{\frac{1}{2}}]^{17} = (-1)^{\frac{1}{2}} \text{ or } \sqrt{-1}$$

$$[(-1)^{\frac{1}{2}}]^2 \text{ or } [(-1)^{\frac{1}{2}}]^6 \text{ or } [(-1)^{\frac{1}{2}}]^{10} \text{ or } [(-1)^{\frac{1}{2}}]^{14} \text{ or } [(-1)^{\frac{1}{2}}]^{18} = -1$$

$$[(-1)^{\frac{1}{2}}]^3 \text{ or } [(-1)^{\frac{1}{2}}]^7 \text{ or } [(-1)^{\frac{1}{2}}]^{11} \text{ or } [(-1)^{\frac{1}{2}}]^{15} \text{ or } [(-1)^{\frac{1}{2}}]^{19} = (-1)^{\frac{3}{2}} = 1(-1)^{\frac{1}{2}} \text{ or } 1\sqrt{-1}$$

$$[(-1)^{\frac{1}{2}}]^4 \text{ or } [(-1)^{\frac{1}{2}}]^8 \text{ or } [(-1)^{\frac{1}{2}}]^{12} \text{ or } [(-1)^{\frac{1}{2}}]^{16} \text{ or } [(-1)^{\frac{1}{2}}]^{20} = (-1)^2 = 1$$

Using the symbol i to represent $(-1)^{\frac{1}{2}}$, the above pattern can be rewritten as:

$$i^1 \text{ or } i^5 \text{ or } i^9 \text{ or } i^{13} \text{ or } i^{17} \text{ or } i^{21} \text{ or } i^{25} \text{ or } i^{29} \text{ or } \ldots (-1)^{\frac{1}{2}} \text{ or } \sqrt{-1}$$

$$i^2 \text{ or } i^6 \text{ or } i^{10} \text{ or } i^{14} \text{ or } i^{18} \text{ or } i^{22} \text{ or } i^{26} \text{ or } i^{30} \text{ or } \cdots - 1$$

$$i^3 \text{ or } i^7 \text{ or } i^{11} \text{ or } i^{15} \text{ or } i^{19} \text{ or } i^{23} \text{ or } i^{27} \text{ or } i^{31} \text{ or } \ldots (-1)^{\frac{3}{2}} = 1(-1)^{\frac{1}{2}} \text{ or } 1\sqrt{-1}$$

$$i^4 \text{ or } i^8 \text{ or } i^{12} \text{ or } i^{16} \text{ or } i^{20} \text{ or } i^{24} \text{ or } i^{28} \text{ or } i^{32} \text{ or } \ldots (-1)^2 = 1$$

Notice: This is a repeating pattern of the exponent 4.

Examples of imaginary numbers include $3i, -6i, 35i$ and $3i\sqrt{5}$.

Complex numbers are ones that contains both real and imaginary parts, such as $2 + 5i$.

With this algebraic creation, even powered roots of negative numbers are no longer undefined and should now be able to do basic operations with any root having negatives.

First, consider exponents on imaginary numbers, where the easiest way to reduce them is to divide the exponent by 4.

Example 9.91

Reduce the imaginary number i^{35}.

First, take the power 35 and divide it by 4.

35 ÷ 4 yields 8¾

The 8 is irrelevant in this solution, since i^{35} is the same as i^3.

$$[(-1)^{\frac{1}{2}}]^{35} = (-1)^{\frac{3}{2}} = 1(-1)^{\frac{1}{2}} \text{ or } 1\sqrt{-1}$$

Example 9.9.2

Reduce the imaginary number i^{124}.

First, take the power 124 and divide it by 4.

124 ÷ 4 yields 31

The 8 becomes irrelevant in this solution, since i^{124} is the same as i^4 or i^0.

$$[(-1)^{\frac{1}{2}}]^{124} = (-1)^{\frac{4}{2}} = (-1)^2 \text{ or } 1$$

When performing operations such as adding, subtracting, multiplying, and dividing with complex radicals, work with i just like it was handled previous polynomials. This means, when adding and subtracting complex numbers, simply add or combine like terms.

Example 9.9.3

Add $(2+5i)+(4-7i)$.

$$\begin{array}{rrcr} & 2 & + & 5i \\ + & 4 & - & 7i \\ & 6 & - & 2i \end{array}$$

Example 9.9.4

Subtract and add the following complex numbers:

$$\begin{array}{rrcr} & (4 & - & 8i) \\ - & (3 & - & 5i) \\ & 1 & - & 3i \end{array} \qquad \begin{array}{rrcr} & & & 5i \\ & -(3 & + & 8i) \\ + & (-4 & + & 7i) \\ & -7 & + & 4i \end{array}$$

Multiplying with complex numbers is the same as multiplying with polynomials, with one exception: simplify the final answer so that there are no exponents on i.

Example 9.9.5

Multiply the following complex numbers:

$$\begin{array}{rr} & 3i \\ \times & 7i \\ & 21i^2 \end{array} \qquad \begin{array}{rrcr} & 3i & - & 7 \\ \times & & & 5i \\ & 15i^2 & - & 35i \end{array} \qquad \begin{array}{rrcrcr} & 2 & - & 4i & & \\ \times & 3 & + & 5i & & \\ & 6 & + & 10i & & \\ & & - & 12i & - & 20i^2 \\ & 6 & - & 12i & - & 20i^2 \end{array}$$

The last step in these solutions is to simplify all i exponents.

$$\begin{array}{lcl} 21i^2 & = & -21 \\ 15i^2 - 35i & = & -15 - 35i \\ 6 - 2i - 20i^2 & = & 6 - 2i - (-20) = 26 - 2i \end{array}$$

Dividing complex numbers also has one thing to be careful of. If you have i or $(-1)^{\frac{1}{2}}$ in the denominator, then there is a radical in the denominator, which means that it will need to be rationalized. This is done using the same process used to rationalize denominators with square roots.

Example 9.9.6

Divide the following complex numbers:

$\dfrac{7+3i}{-5i}$		$\dfrac{2-6i}{4+8i}$	
$\dfrac{7+3i}{-5i}\cdot\dfrac{i}{i}$	Multiply by i	$\dfrac{2-6i}{4+8i}\cdot\dfrac{4-8i}{4-8i}$	Multiply by $4-8i$
$\dfrac{7i+3i^2}{-5i^2}$		$\dfrac{8-16i-24i+48i^2}{16-64i^2}$	
$\dfrac{7i-3}{5}$		$\dfrac{-40-40i}{80}$	or $\dfrac{-1-i}{2}$

Questions

Simplify.

1. $3-(-8+4i)$
2. $3i-7i$
3. $7i-(3-2i)$
4. $5+(-6-6i)$
5. $-6i-(3+7i)$
6. $-8i-7i-(5-3i)$
7. $(3-3i)+(-7-8i)$
8. $(-4-i)+(1-5i)$
9. $i-(2+3i)-6$
10. $(5-4i)+(8-4i)$
11. $(6i)(-8i)$
12. $(3i)(-8i)$
13. $(-5i)(8i)$
14. $(8i)(-4i)$
15. $(-7i)^2$
16. $(-i)(7i)(4-3i)$
17. $(6+5i)^2$
18. $(8i)(-2i)(-2-8i)$
19. $(-7-4i)(-8+6i)$
20. $(3i)(-3i)(4-4i)$
21. $(-4+5i)(2-7i)$
22. $-8(4-8i)-2(-2-6i)$
23. $(-8-6i)(-4+2i)$
24. $(-6i)(3-2i)-(7i)(4i)$
25. $(1+5i)(2+i)$
26. $(-2+i)(3-5i)$

27. $\dfrac{-9+5i}{i}$
28. $\dfrac{-3+2i}{-3i}$
29. $\dfrac{-10-9i}{6i}$
30. $\dfrac{-4+2i}{3i}$
31. $\dfrac{-3-6i}{4i}$
32. $\dfrac{-5+9i}{9i}$
33. $\dfrac{10-i}{-i}$
34. $\dfrac{10}{5i}$
35. $\dfrac{4i}{-10+i}$
36. $\dfrac{9i}{1-5i}$
37. $\dfrac{8}{7-6i}$
38. $\dfrac{4}{4+6i}$
39. $\dfrac{7}{10-7i}$
40. $\dfrac{9}{-8-6i}$
41. $\dfrac{5i}{-6-i}$
42. $\dfrac{8i}{6-7i}$
43. $\sqrt{-81}$
44. $\sqrt{-45}$
45. $\sqrt{-10}\sqrt{2}$
46. $\sqrt{-12}\sqrt{-2}$
47. $\dfrac{3+\sqrt{-27}}{6}$
48. $\dfrac{-4-\sqrt{-8}}{-4}$
49. $\dfrac{8-\sqrt{-16}}{4}$
50. $\dfrac{6+\sqrt{-32}}{4}$
51. i^{73}
52. i^{251}
53. i^{48}

54. i^{68}
55. i^{62}
56. i^{181}
57. i^{154}
58. i^{51}

Answer Key 9.9

84. 9.10 Rate Word Problems: Work and Time

If it takes Felicia 4 hours to paint a room and her daughter Katy 12 hours to paint the same room, then working together, they could paint the room in 3 hours. The equation used to solve problems of this type is one of reciprocals. It is derived as follows:

$$\text{rate} \times \text{time} = \text{work done}$$

For this problem:

$$\begin{array}{rrcl} \text{Felicia's rate:} & F_{\text{rate}} \times 4 \text{ h} & = & 1 \text{ room} \\ \\ \text{Katy's rate:} & K_{\text{rate}} \times 12 \text{ h} & = & 1 \text{ room} \\ \\ \text{Isolating for their rates:} & F & = & \dfrac{1}{4} \text{ h and } K = \dfrac{1}{12} \text{ h} \end{array}$$

To make this into a solvable equation, find the total time (T) needed for Felicia and Katy to paint the room. This time is the sum of the rates of Felicia and Katy, or:

$$\begin{array}{rrcl} \text{Total time:} & T\left(\dfrac{1}{4} \text{ h} + \dfrac{1}{12} \text{ h}\right) & = & 1 \text{ room} \\ \\ \text{This can also be written as:} & \dfrac{1}{4} \text{ h} + \dfrac{1}{12} \text{ h} & = & \dfrac{1 \text{ room}}{T} \\ \\ \text{Solving this yields:} & 0.25 + 0.083 & = & \dfrac{1 \text{ room}}{T} \\ \\ & 0.333 & = & \dfrac{1 \text{ room}}{T} \\ \\ & t & = & \dfrac{1}{0.333} \text{ or } \dfrac{3 \text{ h}}{\text{room}} \end{array}$$

Example 9.10.1

Karl can clean a room in 3 hours. If his little sister Kyra helps, they can clean it in 2.4 hours. How long would it take Kyra to do the job alone?

The equation to solve is:

$$\begin{aligned} \frac{1}{3}\text{ h} \quad + \quad \frac{1}{K} \quad &= \quad \frac{1}{2.4}\text{ h} \\ \frac{1}{K} \quad &= \quad \frac{1}{2.4}\text{ h} - \frac{1}{3}\text{ h} \\ \frac{1}{K} \quad &= \quad 0.0833 \text{ or } K = 12 \text{ h} \end{aligned}$$

Example 9.10.2

Doug takes twice as long as Becky to complete a project. Together they can complete the project in 10 hours. How long will it take each of them to complete the project alone?

The equation to solve is:

$$\begin{aligned} \frac{1}{R} + \frac{1}{2R} \quad &= \quad \frac{1}{10}\text{ h, where Doug's rate (D)} = 2 \times \text{ Becky's (R) rate} \\ \frac{1}{3R} \quad &= \quad \frac{1}{10}\text{ h or } 3R = 10\text{ h} \end{aligned}$$

This means that Becky's rate is $\frac{10}{3}$ h.

Since Doug's rate is twice Becky's, the time for Doug is $\frac{20}{3}$ h.

Example 9.10.3

Joey can build a large shed in 10 days less than Cosmo can. If they built it together, it would take them 12 days. How long would it take each of them working alone?

$$\text{The equation to solve: } \frac{1}{(C-10)} + \frac{1}{C} = \frac{1}{12}, \text{ where } J = C - 10$$

$$\text{Multiply each term by the LCD: } (C-10)(C)(12)$$

$$\text{This leaves } 12C + 12(C-10) = C(C-10)$$

$$\text{Multiplying this out: } 12C + 12C - 120 = C^2 - 10C$$

$$\text{Which simplifies to } C^2 - 34C + 120 = 0$$

$$\text{Which will factor to } (C-30)(C-4) = 0$$

Cosmo can build the large shed in either 30 days or 4 days. Joey, therefore, can build the shed in 20 days or −6 days (rejected).

The solution is Cosmo takes 30 days to build and Joey takes 20 days.

Example 9.10.4

Clark can complete a job in one hour less than his apprentice. Together, they do the job in 1 hour and 12 minutes. How long would it take each of them working alone?

$$\text{Convert everything to hours:} \quad 1 \text{ h } 12 \text{ min} = \frac{72}{60} \text{ h} = \frac{6}{5} \text{ h}$$

$$\text{The equation to solve is} \quad \frac{1}{A} + \frac{1}{A-1} = \frac{1}{\frac{6}{5}} = \frac{5}{6}$$

$$\text{Therefore the equation is} \quad \frac{1}{A} + \frac{1}{A-1} = \frac{5}{6}$$

$$\text{To remove the fractions, multiply each term by the LCD} \quad (A)(A-1)(6)$$

$$\text{This leaves} \quad 6(A) + 6(A-1) = 5(A)(A-1)$$

$$\text{Multiplying this out gives} \quad 6A - 6 + 6A = 5A^2 - 5A$$

$$\text{Which simplifies to} \quad 5A^2 - 17A + 6 = 0$$

$$\text{This will factor to} \quad (5A-2)(A-3) = 0$$

The apprentice can do the job in either $\frac{2}{5}$ h (reject) or 3 h. Clark takes 2 h.

Example 9.10.5

A sink can be filled by a pipe in 5 minutes, but it takes 7 minutes to drain a full sink. If both the pipe and the drain are open, how long will it take to fill the sink?

The 7 minutes to drain will be subtracted.

$$\text{The equation to solve is} \quad \frac{1}{5} - \frac{1}{7} = \frac{1}{X}$$

$$\text{To remove the fractions, multiply each term by the LCD} \quad (5)(7)(X)$$

$$\text{This leaves} \quad (7)(X) - (5)(X) = (5)(7)$$

$$\text{Multiplying this out gives} \quad 7X - 5X = 35$$

$$\text{Which simplifies to} \quad 2X = 35 \text{ or } X = \frac{35}{2} \text{ or } 17.5$$

17.5 min or 17 min 30 sec is the solution

Questions

For Questions 1 to 8, write the formula defining the relation. Do Not Solve!!

1. Bill's father can paint a room in 2 hours less than it would take Bill to paint it. Working together, they can complete the job in 2 hours and 24 minutes. How much time would each require working alone?
2. Of two inlet pipes, the smaller pipe takes four hours longer than the larger pipe to fill a pool. When both pipes are open, the pool is filled in three hours and forty-five minutes. If only the larger pipe is open, how many hours are required to fill the pool?
3. Jack can wash and wax the family car in one hour less than it would take Bob. The two working together can complete the job in 1.2 hours. How much time would each require if they worked alone?
4. If Yousef can do a piece of work alone in 6 days, and Bridgit can do it alone in 4 days, how long will it take the two to complete the job working together?
5. Working alone, it takes John 8 hours longer than Carlos to do a job. Working together, they can do the job in 3 hours. How long would it take each to do the job working alone?
6. Working alone, Maryam can do a piece of work in 3 days that Noor can do in 4 days and Elana can do in 5 days. How long will it take them to do it working together?
7. Raj can do a piece of work in 4 days and Rubi can do it in half the time. How long would it take them to do the work together?
8. A cistern can be filled by one pipe in 20 minutes and by another in 30 minutes. How long would it take both pipes together to fill the tank?

For Questions 9 to 20, find and solve the equation describing the relationship.

9. If an apprentice can do a piece of work in 24 days, and apprentice and instructor together can do it in 6 days, how long would it take the instructor to do the work alone?
10. A carpenter and his assistant can do a piece of work in 3.75 days. If the carpenter himself could do the work alone in 5 days, how long would the assistant take to do the work alone?
11. If Sam can do a certain job in 3 days, while it would take Fred 6 days to do the same job, how long would it take them, working together, to complete the job?
12. Tim can finish a certain job in 10 hours. It takes his wife JoAnn only 8 hours to do the same job. If they work together, how long will it take them to complete the job?
13. Two people working together can complete a job in 6 hours. If one of them works twice as fast as the other, how long would it take the slower person, working alone, to do the job?
14. If two people working together can do a job in 3 hours, how long would it take the faster person to do the same job if one of them is 3 times as fast as the other?
15. A water tank can be filled by an inlet pipe in 8 hours. It takes twice that long for the outlet pipe to empty the tank. How long would it take to fill the tank if both pipes were open?
16. A sink can be filled from the faucet in 5 minutes. It takes only 3 minutes to empty the sink when the drain is open. If the sink is full and both the faucet and the drain are open, how long will it take to empty the sink?
17. It takes 10 hours to fill a pool with the inlet pipe. It can be emptied in 15 hours with the outlet pipe. If the pool is half full to begin with, how long will it take to fill it from there if both pipes are open?
18. A sink is $\frac{1}{4}$ full when both the faucet and the drain are opened. The faucet alone can fill the sink in 6 minutes, while it takes 8 minutes to empty it with the drain. How long will it take to fill the remaining $\frac{3}{4}$ of the sink?
19. A sink has two faucets: one for hot water and one for cold water. The sink can be filled by a cold-water faucet in 3.5 minutes. If both faucets are open, the sink is filled in 2.1 minutes. How long does it take to fill the sink with just the hot-water faucet open?

20. A water tank is being filled by two inlet pipes. Pipe A can fill the tank in 4.5 hours, while both pipes together can fill the tank in 2 hours. How long does it take to fill the tank using only pipe B?

Answer Key 9.10

85. 9.11 Radical Pattern Puzzle

The centre number of each square is found by using the order of operations applied to the numbers that surround it. The challenge is to solve for the variable x.

$9^{\frac{1}{2}}$ **44** $64^{\frac{1}{2}}$ $16^{\frac{1}{2}}$	$81^{\frac{1}{2}}$ **32** $49^{\frac{1}{2}}$ $4^{\frac{1}{2}}$	$64^{\frac{1}{2}}$ **75** $49^{\frac{1}{2}}$ $x^{\frac{1}{2}}$

Can you solve for x?
Answer Key 9.11

PART XII

CHAPTER 10: QUADRATICS

Learning Objectives

This chapter covers:

- Solving Radical Equations
- Solving Exponential Equations
- Completing the Square
- The Quadratic Formula
- Solve Quadratic Equations – Substitution
- Graphing Quadratic Equations
- Quadratic Word Problems
- Construct a Quadratic Equation form its Roots

86. 10.1 Solving Radical Equations

In this section, radical equations that need to solved are discussed. The strategy is relatively simple: isolate the radical on one side of the equation and all variables will remain on the other side. Once this is done, square, cube, or raise each side to a power that removes the radical. For example:

$$\begin{array}{ll} \text{For } \sqrt{5x+3}=1, \text{ the solution can be found by squaring both sides} & (\sqrt{5x+3})^2=(1)^2 \\ \text{For } \sqrt[3]{5x+3}=1, \text{ the solution can be found by cubing both sides} & (\sqrt[3]{5x+3})^3=(1)^3 \\ \text{For } \sqrt[4]{5x+3}=1, \text{ the solution can be found by using the fourth power} & (\sqrt[4]{5x+3})^4=(1)^4 \\ \text{For } \sqrt[5]{5x+3}=1, \text{ the solution can be found by using the fifth power} & (\sqrt[5]{5x+3})^5=(1)^5 \end{array}$$

Once the radical is removed, then solve the resulting equation. Consider how this strategy is used in the following example.

Example 10.1.1

Solve for x in $\sqrt{2x+4}-6=0$.

First, isolate the radical:

$$\begin{array}{rrrrr} \sqrt{2x+4} & - & 6 & = & 0 \\ & + & 6 & = & +6 \\ \sqrt{2x+4} & & & = & 6 \end{array}$$

This leaves:

$$\sqrt{2x+4}=6$$

Squaring both sides:

$$(\sqrt{2x+4})^2=(6)^2$$

This results in:

$$\begin{array}{rrrrr} 2x & + & 4 & = & 36 \\ & - & 4 & & -4 \\ & & 2x & = & 32 \\ \\ & & x & = & 16 \end{array}$$

This same strategy works for equations having indices larger than 2.

Example 10.1.2

Solve for x in $\sqrt[3]{3-3x}=\sqrt[3]{2x-7}$.

For this problem, there are two radicals equalling each other, and all that is required is to cube each to cancel the radicals out.

Because there is an odd index, there will have two solutions to this equation: one positive and one negative.

Positive Solution

$$\begin{array}{rrrrcrrr} & 3 & - & 3x & = & 2x & - & 7 \\ - & 3 & - & 2x & & -2x & - & 3 \\ & & & -5x & = & -10 & & \\ & & & x & = & 2 & & \end{array}$$

Negative Solution

$$\begin{array}{rrrrcrrr} & 3 & - & 3x & = & -(2x & - & 7) \\ & 3 & - & 3x & = & -2x & + & 7 \\ - & 3 & + & 2x & & +2x & - & 3 \\ & & & -x & = & 4 & & \\ & & & x & = & -4 & & \end{array}$$

The strategy used above to isolate and solve for the radicals works the same for radicals in inequalities, except that you will now have to square, cube or use a larger power on each of the terms. For example:

Example 10.1.3

Solve for x in $3<\sqrt{3x+9}\leq 6$.

For this problem, there are three terms to square out.

This looks like:

$$(3)^2 < (\sqrt{3x+9})^2 \leq (6)^2$$

Which results in:

$$\begin{array}{rcrcrcr} 9 & < & 3x & + & 9 & \leq & 36 \\ -9 & & & - & 9 & & -9 \\ \dfrac{0}{3} & < & & \dfrac{3x}{3} & & \leq & \dfrac{27}{3} \\ \\ 0 & < & & x & & \leq & 9 \end{array}$$

For all cases of radical equations, check answers to see if they work. There may be variations of these radical equations in higher levels of math, but the strategy will always be similar in that you will always work to square the radicals out.

Questions

1. $\sqrt{2x+3}-3=0$
2. $\sqrt{5x+1}-4=0$
3. $\sqrt{6x-5}-x=0$
4. $\sqrt{7x+8}=x$
5. $\sqrt{3+x}=\sqrt{6x+13}$
6. $\sqrt{x-1}=\sqrt{7-x}$
7. $\sqrt[3]{3-3x}=\sqrt[3]{2x-5}$
8. $\sqrt[4]{3x-2}=\sqrt[4]{x+4}$
9. $\sqrt{x+7}\geq 2$
10. $\sqrt{x-2}\leq 4$
11. $3<\sqrt{3x+6}\leq 6$
12. $0<\sqrt{x+5}<5$

Answer Key 10.1

87. 10.2 Solving Exponential Equations

Exponential equations are often reduced by using radicals—similar to using exponents to solve for radical equations. There is one caveat, though: while odd index roots can be solved for either negative or positive values, even-powered roots can only be taken for even values, but have both positive and negative solutions. This is shown below:

$$\text{For odd values of } n, \text{ then } a^n = b \text{ and } a = \sqrt[n]{b}$$
$$\text{For even values of } n, \text{ then } a^n = b \text{ and } a = \pm\sqrt[n]{b}$$

Example 10.2.1

Solve for x in the equation $x^5 = 32$.

The solution for this requires that you take the fifth root of both sides.

$$\begin{array}{rcl}(x^5)^{\frac{1}{5}} & = & (32)^{\frac{1}{5}} \\ x & = & 2\end{array}$$

When taking a positive root, there will be two solutions. For example:

Example 10.2.2

Solve for x in the equation $x^4 = 16$.

The solution for this requires that the fourth root of both sides is taken.

$$\begin{array}{rcl}(x^4)^{\frac{1}{4}} & = & (16)^{\frac{1}{4}} \\ \\ x & = & \pm 2\end{array}$$

The answer is ± 2 because $(2)^4 = 16$ and $(-2)^4 = 16$.

When encountering more complicated problems that require radical solutions, work the problem so that there is a single power to reduce as the starting point of the solution. This strategy makes for an easier solution.

Example 10.2.3

Solve for x in the equation $2(2x+4)^2=72$.

The first step should be to isolate $(2x+4)^2$, which is done by dividing both sides by 2. This results in $(2x+4)^2=36$.

Once isolated, take the square root of both sides of this equation:

$$\begin{array}{rcrcl} [(2x & + & 4)^2]^{\frac{1}{2}} & = & 36^{\frac{1}{2}} \\ 2x & + & 4 & = & \pm 6 \\ & & 2x & = & -4 \pm 6 \\ & & x & = & -2 \pm 3 \\ \\ & & x & = & -5, \quad 1 \end{array}$$

Checking these solutions in the original equation indicates that both work.

Example 10.2.4

Solve for x in the equation $(x+4)^3+6=-119$.

First, isolate $(x+4)^3$ by subtracting 6 from both sides. This results in $(x+4)^3=-125$.

Now, take the cube root of both sides, which leaves:

$$\begin{array}{rcrcl} [(x & + & 4)^3]^{\frac{1}{3}} & = & [-125]^{\frac{1}{3}} \\ x & + & 4 & = & -5 \\ & - & 4 & & -4 \\ & & x & = & -9 \end{array}$$

Checking this solution in the original equation indicates that it is a valid solution.

Since you are solving for an odd root, there is only one solution to the cube root of −125. It is only even-powered roots that have both a positive and a negative solution.

Questions

Solve.

1. $x^2 = 75$
2. $x^3 = -8$
3. $x^2 + 5 = 13$
4. $4x^3 - 2 = 106$
5. $3x^2 + 1 = 73$
6. $(x-4)^2 = 49$
7. $(x+2)^5 = -243$
8. $(5x+1)^4 = 16$
9. $(2x+5)^3 - 6 = 21$
10. $(2x+1)^2 + 3 = 21$
11. $(x-1)^{\frac{2}{3}} = 16$
12. $(x-1)^{\frac{3}{2}} = 8$
13. $(2-x)^{\frac{3}{2}} = 27$
14. $(2x+3)^{\frac{4}{3}} = 16$
15. $(2x-3)^{\frac{2}{3}} = 4$
16. $(3x-2)^{\frac{4}{5}} = 16$

Answer Key 10.2

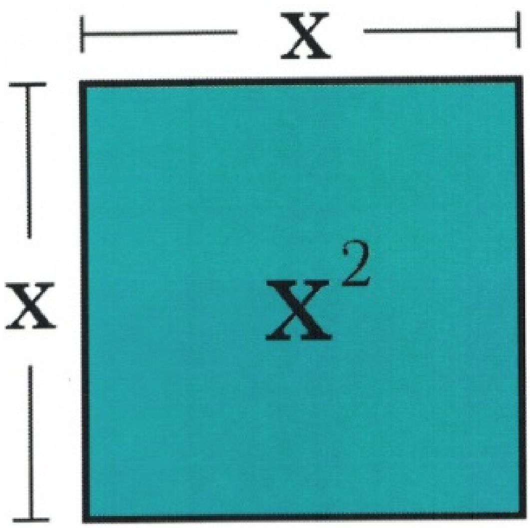
88. 10.3 Completing the Square

How To "Complete the Square" Visually[1]

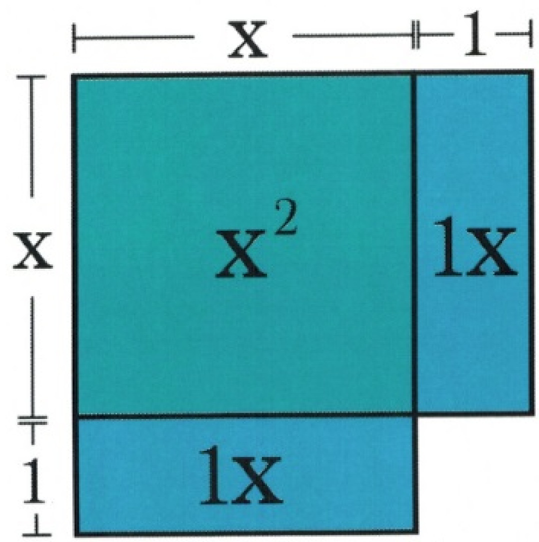
Let's use an area model to visualize how to complete the square of the following equation:

$$y = x^2 + 2x + 12$$

The area model used by Brett Berry is fairly straightforward, having multiple variations and forms that can be found online. The standard explanation begins by representing x^2 as a square whose sides are both x units in length and make an area of x^2.

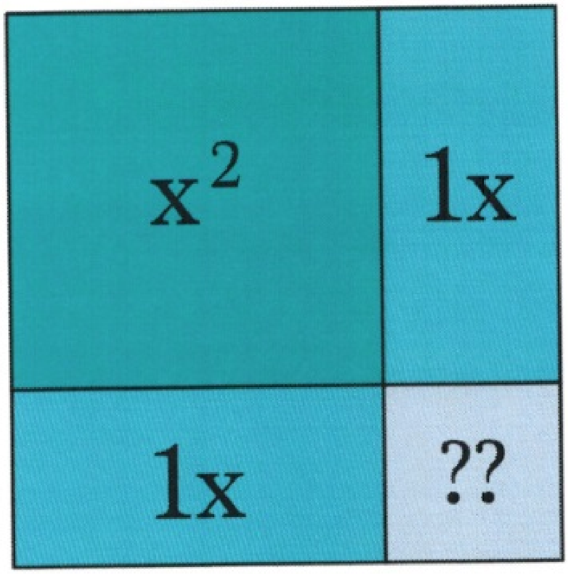


Next, add $2x$ to the block defined as x^2. This is done by taking the $2x$ block and cutting it in half, then add to both sides of your original square x. This acts to continue the sides of x in two directions by $1x$.



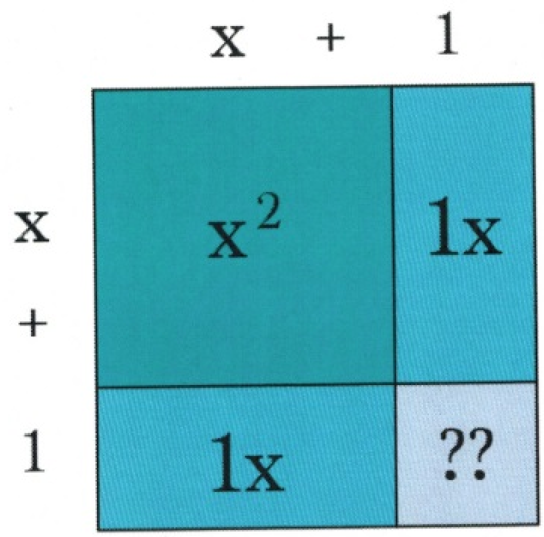
In this example, the square length on each side has increased by 1, but as you can see from the diagram, this larger square is missing the corner piece. To complete the square, add a small piece to complete the visual square. The question is, what is the area of this missing piece?

1. Adapted from Brett Berry

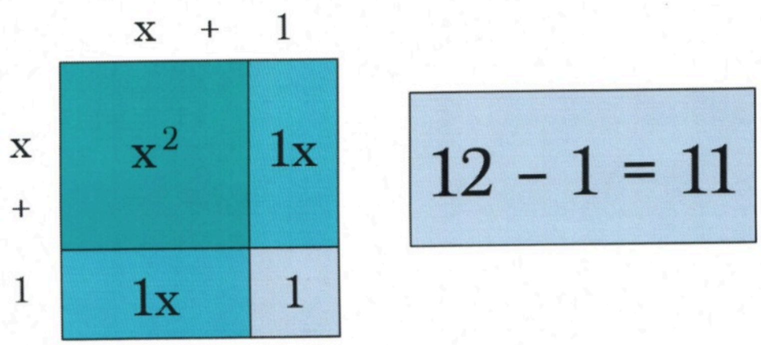


Since the blue blocks adjacent to our missing piece are both 1 unit wide, deduce that the missing block has an area of 1 × 1 = 1.

Also note that, by adding together the outermost units of the square, the area of the square becomes the desired binomial squared $(x+1)^2$.



Now, all that's left to do is literally complete the square and adjust for the extra units. To do this, first, fill in the area of the purple square, which is known to be 1. Since the original equation had a constant of 12, subtract 1 from 12 to account for the 1 added to the square.



The square is now complete! The square is $(x+1)^2$ with 11 leftover. The extra 11 can simply be added to the end of our binomial squared: $y=(x+1)^2+11$.

In the problems most likely be required to solve, $y=0$, so the original equation will not be written as $y=x^2+2x+12$; rather, it will be $0=x^2+2x+12$.

Example 10.3.1

Solve for x in the equation $0 = x^2 + 8x + 12$.

The first step is to complete the square. Rather than drawing out a sketch to show the process of completing the square, simply take half the middle term and rewrite $x^2 + 8x$ as $(x + 4)^2$.

When squared out $(x + 4)^2$, it is $x^2 + 8x + 16$.

Note that this is 4 larger than the original $0 = x^2 + 8x + 12$. This means that $(x + 4)^2 - 4$ is the same as $0 = x^2 + 8x + 12$.

The equation needed to be solved has now become $0 = (x + 4)^2 - 4$. First, add 4 to each side:

$$\begin{array}{rrrrrrr} 0 & = & (x & + & 4)^2 & - & 4 \\ +4 & = & & & & + & 4 \\ 4 & = & (x & + & 4)^2 & & \end{array}$$

Now take the square root from both sides:

$$\begin{array}{rrrrr} (4)^{\frac{1}{2}} & = & [(x & + & 4)^2]^{\frac{1}{2}} \\ \pm 2 & = & x & + & 4 \end{array}$$

Subtracting 4 from both sides leaves $x = -4 \pm 2$, which gives the solutions $x = -6$ and $x = -2$.

It is always wise to check answers in the original equation, which for these two yield:

$$\begin{array}{rrrrrrr} x & = & -6: & & & & \\ & & & & & & \\ 0 & = & x^2 & + & 8x & + & 12 \\ 0 & = & (-6)^2 & + & 8(-6) & + & 12 \\ 0 & = & 36 & - & 48 & + & 12\checkmark \end{array}$$

$$\begin{array}{rrrrrrr} x & = & -2: & & & & \\ & & & & & & \\ 0 & = & x^2 & + & 8x & + & 12 \\ 0 & = & (-2)^2 & + & 8(-2) & + & 12 \\ 0 & = & 4 & - & 16 & + & 12\checkmark \end{array}$$

Sometimes, it is required to complete the square where there is some value ≠ 1 in front of the x^2. For example:

Example 10.3.2

Solve for x in the equation $0 = 2x^2 + 12x - 7$.

The first step is to factor 2 from both terms in $2x^2 + 12x$, which then leaves $0 = 2(x^2 + 6x) - 7$.

Isolating $x^2 + 6x$ yields $x^2 + 6x = \frac{7}{2}$.

As before, complete the square for $x^2 + 6x$, which yields $(x+3)^2$. When squared out $(x+3)^2$, you get $x^2 + 6x + 9$.

Now add 9 to the other side of the equation:

$$x^2 + 6x + 9 = \frac{7}{2} + 9$$

Simplifying this yields:

$$(x+3)^2 = \frac{25}{2}$$

Now take the square root from both sides:

$$[(x+3)^2]^{\frac{1}{2}} = \left(\frac{25}{2}\right)^{\frac{1}{2}}$$

Which leaves:

$$x + 3 = \pm\left(\frac{25}{2}\right)^{\frac{1}{2}}$$

Subtract 3 from both sides:

$$x = -3 \pm \left(\frac{25}{2}\right)^{\frac{1}{2}}$$

Rationalizing the denominator yields:

$$x = -3 + \frac{5\sqrt{2}}{2} \text{ or } x = -3 - \frac{5\sqrt{2}}{2}$$

When checking these answers in the original equation, both solutions are valid.

Questions

Find the value that completes the square and then rewrite as a perfect square.

1. $x^2 - 30x + \_\_$
2. $a^2 - 24a + \_\_$
3. $m^2 - 36m + \_\_$
4. $x^2 - 34x + \_\_$
5. $x^2 - 15x + \_\_$
6. $r^2 - 19r + \_\_$
7. $y^2 - y + \_\_$
8. $p^2 - 17p + \_\_$

Solve each equation by completing the square.

9. $x^2 - 16x + 55 = 0$
10. $n^2 - 4n - 12 = 0$
11. $v^2 - 4v - 21 = 0$
12. $b^2 + 8b + 7 = 0$
13. $x^2 - 8x = -6$
14. $x^2 - 13 = 4x$
15. $3k^2 + 24k = -1$
16. $4a^2 + 36a = -2$

Answer Key 10.3

89. 10.4 The Quadratic Equation

A rule of thumb about factoring: after spending several minutes trying to factor an equation, if its taking to long, use the quadratic equation to generate solutions instead.

Look at the equation $ax^2 + bx + c = 0$, the values of x that make this equation equal to zero can be found by:

$$x = \frac{-b \pm (b^2 - 4ac)^{\frac{1}{2}}}{2a}$$

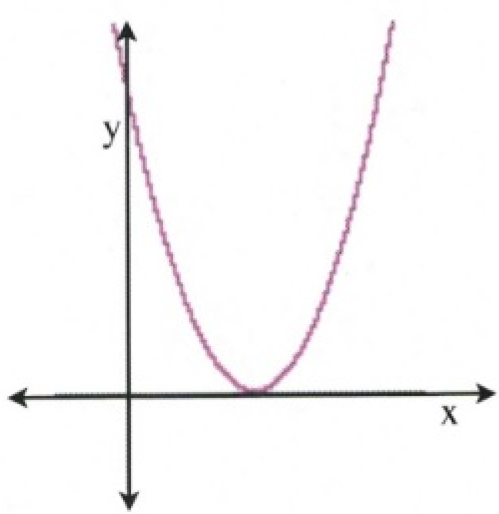
One of the key factors here is the value found from $(b^2 - 4ac)^{\frac{1}{2}}$. The interior of this radical $b^2 - 4ac$ can have three possible values: negative, positive, or zero.

$b^2 - 4ac$ is called the discriminant, and it defines how many solutions of x there will be and what type of solutions they are.

If $b^2 - 4ac = 0$, then there is exactly one solution:

$$x = \frac{-b \pm 0}{2a} = \frac{-b}{2a}$$

The meaning of this is that the parabolic curve that can be drawn from the equation will only touch the x-axis at one spot, and so there is only one solution for that quadratic. This can be seen from the image to the right: the quadratic curve touches the x-axis at only one position, which means that there is only one solution for x.



For example, the equation $4x^2 + 4x + 1 = 0$ has one solution. Check:

$$\begin{array}{llllll} a = 4 & b^2 - 4ac & = & (4)^2 & - & 4(4)(1) \\ b = 4 & & = & 16 & - & 16 \\ c = 1 & & = & 0 & & \end{array}$$

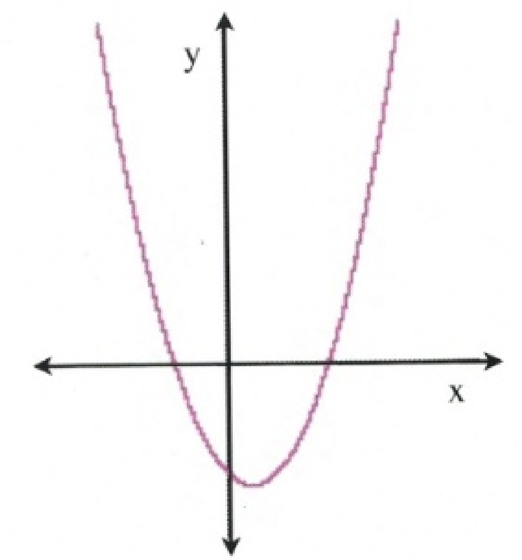
The solution ends up being $x = \frac{-(4)}{2(4)}$ or $x = -\frac{1}{2}$.

If $b^2 - 4ac =$ any positive value, then there are exactly two solutions:

$$x = \frac{-b \pm \text{some positive number}}{2a}$$

$$\frac{-b + \text{some positive number}}{2a} \quad \begin{array}{c}\text{or simply}\\ \text{and}\end{array} \quad \frac{-b - \text{ some positive number}}{2a}$$

The meaning of this is that the parabolic curve that can be drawn from the equation will now touch (and cross) the x-axis at two positions, and so there are now two solutions for the quadratic. This can be seen from the image to the right: the quadratic curve crosses the x-axis at two positions, which means that there are now two solutions for x.



For example, the equation $3x^2 + 4x + 1 = 0$ has two solutions. Check:

$$\begin{array}{lccccccc} a = 3 & b^2 & - & 4ac & = & (4)^2 & - & 4(3)(1) \\ b = 4 & & & & = & 16 & - & 12 \\ c = 1 & & & & = & 4 & & \end{array}$$

When 4 is put back into the quadratic equation and root 4 is taken, the solution now becomes ±2.

For this quadratic:

$$x = \frac{-4 \pm 2}{2(3)} = \frac{-4 \pm 2}{6}$$

The solutions are $x = \dfrac{-6}{6} = -1$ and $x = \dfrac{-2}{6} = -\dfrac{1}{3}$.

There exists one last possible solution for a quadratic, which happens when $b^2 - 4ac =$ any negative value. When this occurs, there are exactly two solutions, which are defined as imaginary roots or solutions or, more properly, complex roots, since the solution involves taking the root of a negative value.

The example provided shows that the quadratic never touches or crosses the x-axis, yet it is possible to generate a solution if using imaginary numbers when solving a negative radical discriminant $b^2 - 4ac$.

For example, the equation $5x^2 + 2x + 1 = 0$ has two complex or imaginary solutions. Check:

$$\begin{array}{lccccccc} a = 5 & b^2 & - & 4ac & = & (2)^2 & - & 4(5)(1) \\ b = 2 & & & & = & 4 & - & 20 \\ c = 1 & & & & = & -16 & & \end{array}$$

When −16 is put back into the quadratic equation and the root of −16 is taken, the solution becomes $\pm 4i$.

For this quadratic:

$$x = \frac{-2 \pm 4i}{2(5)} = \frac{-2 \pm 4i}{10}$$

The solutions are $x = \dfrac{-1+2i}{5}$ and $x = \dfrac{-1-2i}{5}$.

Note: these solutions are complex conjugates of each other.

It is often useful to check the discriminants of a quadratic equation to define the nature of the roots for the quadratic before proceeding to a full solution.

Example 10.4.1

Find the values of x that solve the equation $x^2 + 6x - 7 = 0$.

$$\begin{array}{lrcl} a = 1 & x & = & \dfrac{-6 \pm [6^2 - 4(1)(-7)]^{\frac{1}{2}}}{2(1)} \\ b = 6 & x & = & \dfrac{-6 \pm [36 + 28]^{\frac{1}{2}}}{2} \\ c = -7 & x & = & \dfrac{-6 \pm [64]^{\frac{1}{2}}}{2} \\ \text{Which reduces to} & x & = & \dfrac{-6 \pm 8}{2} \\ \text{And yields} & x & = & -7, 1 \end{array}$$

Example 10.4.2

Find the values of x that solve the equation $9x^2 + 6x + 1 = 0$.

$$
\begin{aligned}
a = 9 \qquad & x = \frac{-(-6) \pm [(-6)^2 - 4(9)(1)]^{\frac{1}{2}}}{2(9)} \\
b = 6 \qquad & x = \frac{6 \pm [36 - 36]^{\frac{1}{2}}}{18} \\
c = 1 \qquad & x = \frac{6 \pm [0]^{\frac{1}{2}}}{18} \\
\text{Which reduces to} \qquad & x = \frac{1}{3}
\end{aligned}
$$

In case you are curious:

How to Derive the Quadratic Formula

$$
\begin{aligned}
ax^2 + bx + c &= 0 && \text{Separate constant from variables} \\
-c - c & && \text{Subtract } c \text{ from both sides} \\
\frac{ax^2}{a} + \frac{bx}{a} &= \frac{-c}{a} && \text{Divide each term by } a \\
x^2 + \frac{b}{a}x &= \frac{-c}{a} && \text{Find the number that completes the square} \\
\left(\frac{1}{2} \cdot \frac{b}{a}\right)^2 = \left(\frac{b}{2a}\right)^2 &= \frac{b^2}{4a^2} && \text{Add to both sides} \\
\frac{b^2}{4a^2} - \frac{c}{a}\left(\frac{4a}{4a}\right) = \frac{b^2}{4a^2} - \frac{4ac}{4a^2} &= \frac{b^2 - 4ac}{4a^2} && \text{Get the common denominator on the right} \\
x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{4ac}{4a^2} &= \frac{b^2 - 4ac}{4a^2} && \text{Factor} \\
\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} && \text{Solve using the even root property} \\
\sqrt{\left(x + \frac{b}{2a}\right)^2} &= \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} && \text{Simplify roots} \\
x + \frac{b}{2a} &= \frac{\pm\sqrt{b^2 - 4ac}}{2a} && \text{Subtract } \frac{b}{2a} \text{ from both sides} \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Our solution}
\end{aligned}
$$

Questions

Use the quadratic discriminant to determine the nature of the roots.

a. $4x^2 + 2x - 5 = 0$
b. $9x^2 - 6x + 1 = 0$
c. $2x^2 + 3x - 5 = 0$
d. $3x^2 + 5x = 3$
e. $3x^2 + 5x = 2$

f. $x^2 - 8x + 16 = 0$
g. $a^2 - 56 = -10a$
h. $x^2 + 4 = 4x$
i. $5x^2 = -26 + 10x$
j. $n^2 = -21 + 10n$

Solve each of the following using the quadratic equation.

1. $4a^2 + 3a - 6 = 0$
2. $3k^2 + 2k - 3 = 0$
3. $2x^2 - 8x - 2 = 0$
4. $6n^2 + 8n - 1 = 0$
5. $2m^2 - 3m + 6 = 0$
6. $5p^2 + 2p + 6 = 0$
7. $3r^2 - 2r - 1 = 0$
8. $2x^2 - 2x - 15 = 0$
9. $4n^2 - 3n + 10 = 0$
10. $b^2 + 6b + 9 = 0$
11. $v^2 - 4v - 5 = -8$
12. $x^2 + 2x + 6 = 4$

Answer Key 10.4

90. 10.5 Solving Quadratic Equations Using Substitution

Factoring trinomials in which the leading term is not 1 is only slightly more difficult than when the leading coefficient is 1. The method used to factor the trinomial is unchanged.

Example 10.5.1

Solve for x in $x^4 - 13x^2 + 36 = 0$.

First start by converting this trinomial into a form that is more common. Here, it would be a lot easier when factoring $x^2 - 13x + 36 = 0$. There is a standard strategy to achieve this through substitution.

First, let $u = x^2$. Now substitute u for every x^2, the equation is transformed into $u^2 - 13u + 36 = 0$.

$u^2 - 13u + 36 = 0$ factors into $(u - 9)(u - 4) = 0$.

Once the equation is factored, replace the substitutions with the original variables, which means that, since $u = x^2$, then $(u - 9)(u - 4) = 0$ becomes $(x^2 - 9)(x^2 - 4) = 0$.

To complete the factorization and find the solutions for x, then $(x^2 - 9)(x^2 - 4) = 0$ must be factored once more. This is done using the difference of squares equation: $a^2 - b^2 = (a + b)(a - b)$.

Factoring $(x^2 - 9)(x^2 - 4) = 0$ thus leaves $(x - 3)(x + 3)(x - 2)(x + 2) = 0$.

Solving each of these terms yields the solutions $x = \pm 3, \pm 2$.

This same strategy can be followed to solve similar large-powered trinomials and binomials.

Example 10.5.2

Factor the binomial $x^6 - 7x^3 - 8 = 0$.

Here, it would be a lot easier if the expression for factoring was $x^2 - 7x - 8 = 0$.

First, let $u = x^3$, which leaves the factor of $u^2 - 7u - 8 = 0$.

$u^2 - 7u - 8 = 0$ easily factors out to $(u - 8)(u + 1) = 0$.

Now that the substituted values are factored out, replace the u with the original x^3. This turns $(u - 8)(u + 1) = 0$ into $(x^3 - 8)(x^3 + 1) = 0$.

The factored $(x^3 - 8)$ and $(x^3 + 1)$ terms can be recognized as the difference of cubes.

These are factored using $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ and $a^3+b^3 = (a+b)(a^2-ab+b^2)$.

And so, (x^3-8) factors out to $(x-2)(x^2+2x+4)$ and (x^3+1) factors out to $(x+1)(x^2-x+1)$.

Combining all of these terms yields:

$$(x-2)(x^2+2x+4)(x+1)(x^2-x+1) = 0$$

The two real solutions are $x=2$ and $x=-1$. Checking for any others by using the discriminant reveals that all other solutions are complex or imaginary solutions.

Questions

Factor each of the following polynomials and solve what you can.

1. $x^4 - 5x^2 + 4 = 0$
2. $y^4 - 9y^2 + 20 = 0$
3. $m^4 - 7m^2 - 8 = 0$
4. $y^4 - 29y^2 + 100 = 0$
5. $a^4 - 50a^2 + 49 = 0$
6. $b^4 - 10b^2 + 9 = 0$
7. $x^4 + 64 = 20x^2$
8. $6z^6 - z^3 = 12$
9. $z^6 - 216 = 19z^3$
10. $x^6 - 35x^3 + 216 = 0$

Answer Key 10.5

91. 10.6 Graphing Quadratic Equations—Vertex and Intercept Method

One useful strategy that is used to get a quick sketch of a quadratic equation is to identify 3 key points of the quadratic: its vertex and the two intercept points. From these 3 points, it's possible to sketch out a rough graph of what the quadratic graph looks like.

The **intercepts** are where the quadratic equation crosses the x-axis and are found when the quadratic is set to equal 0. So instead of the quadratic looking like $y = ax^2 + bx + c$, it is instead factored from the form $0 = ax^2 + bx + c$ to get its x-intercepts (roots). For expedience, you can get these values using the quadratic equation.

$$x = \frac{-b \pm (b^2 - 4ac)^{\frac{1}{2}}}{2a}$$

The **vertex** is found by using the quadratic equation where the discriminant equals zero, which gives us the x-coordinate of $x = \dfrac{-b}{2a}$. The y-coordinate of the vertex is then found by placing the x-coordinate of the vertex $\left(x = \dfrac{-b}{2a}\right)$ back into the original quadratic $(y = ax^2 + bx + c)$ and solving for y.

The vertex then takes the form of $\left[\dfrac{-b}{2a}, a\left(\dfrac{-b}{2a}\right)^2 + \left(\dfrac{-b}{2a}\right)x + c\right]$, or simply as $\left[\dfrac{-b}{2a}, f\left(\dfrac{-b}{2a}\right)\right]$.

What is new here is finding the vertex, so consider the following examples.

Example 10.6.1

Find the vertex of $y = x^2 + 6x - 7$.

For this equation, $a = 1$, $b = 6$ and $c = -7$.

This means that the x-coordinate of the vertex $x = \dfrac{-b}{2a}$ will give us the value $x = \dfrac{-(6)}{2(1)} = -3$.

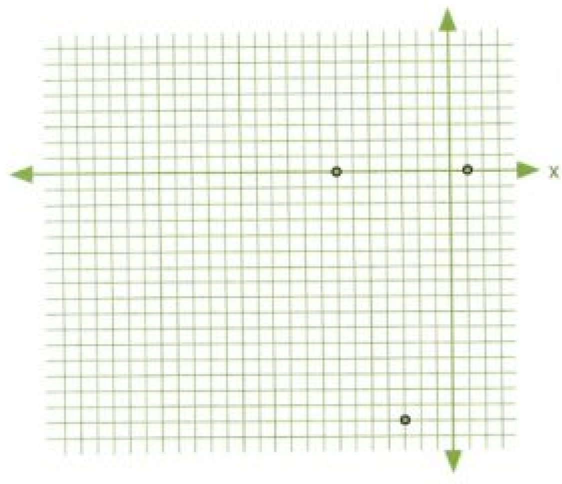
We now use this x-coordinate to find the y-coordinate.

$$\begin{array}{rcccccc} y & = & ax^2 & + & bx & + & c \\ y & = & 1(-3)^2 & + & 6(-3) & - & 7 \\ y & = & 9 & - & 18 & - & 7 \\ y & = & -16 & & & & \end{array}$$

The vertex is at $x = -3$ and $y = -16$ and can be given by the coordinate $(-3, -16)$.

The x-intercepts or roots of the quadratic in Example 10.6.1 are found by factoring $x^2 + 6x - 7 = 0$.

For this problem, the quadratic factors to $(x+7)(x-1) = 0$, which means the roots are $x = -7$ and $x = 1$. Putting all this data together gives us the vertex coordinate $(-3, -16)$ and the two x-intercept coordinates $(-7, 0)$ and $(1, 0)$. These are the values used to create the rough sketch.



Trying to sketch this curve will be somewhat challenging if there is to be any semblance of accuracy.

When this happens, it is quite easy to fill in some of the places where there may have been coordinates by using a data table.

For this graph, choose values from $x = 2$ to $x = -8$.

First, find the value of y when $x = 2$:

$$\begin{array}{rcrcrcr} y & = & x^2 & + & 6x & - & 7 \\ y & = & 1(2)^2 & + & 6(2) & - & 7 \\ y & = & 4 & + & 12 & - & 7 \\ y & = & 9 & & & & \end{array}$$

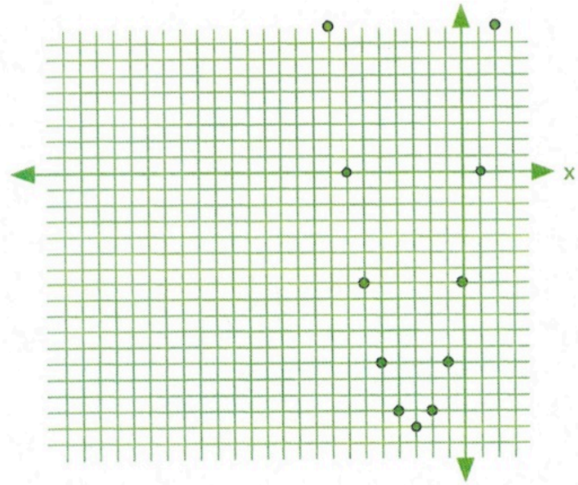
Put this value in the table and then carry on to complete all of it.

x	y
2	9
1	0
0	−7
−1	−12
−2	−15
−3	−16
−4	−15
−5	−12
−6	−7
−7	0
−8	9

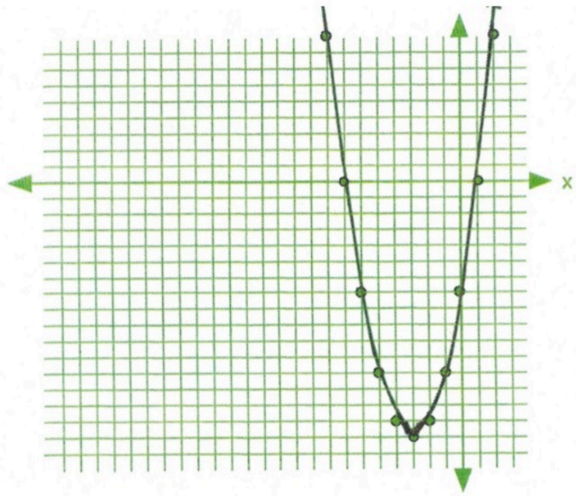
Placing all of these coordinates on the graph will generate a graph showing increased detail, as shown below. All that remains is to draw a curve that connects the points on the graph. The level of detail required to draw the curve only depends on the unique characteristics of the curve itself.

Remember:

For the quadratic equation $y = ax^2 + bx + c$, the x-coordinate of the vertex is $x = \frac{-b}{2a}$ and the y-coordinate of the vertex is $y = a\left(\frac{-b}{2a}\right)^2 + \left(\frac{-b}{2a}\right)x + c$.



The following questions will ask you to sketch the quadratic function using the vertex and the x-intercepts and then later to draw a data table to find the coordinates of data points from which to draw a curve.



Both approaches are quite valuable, the difference is only in the details, which if required can use both techniques to general a curve in increased detail.

Example 10.6.2

Find the vertex of $y = x^2 - 6x - 7$.

In the equation, $a = 1$, $b = -6$, and $c = -7$.

This means that the x-coordinate of the vertex $x = \frac{-b}{2a}$ will give us the value $x = \frac{-(-6)}{2}(1)$ or 3.

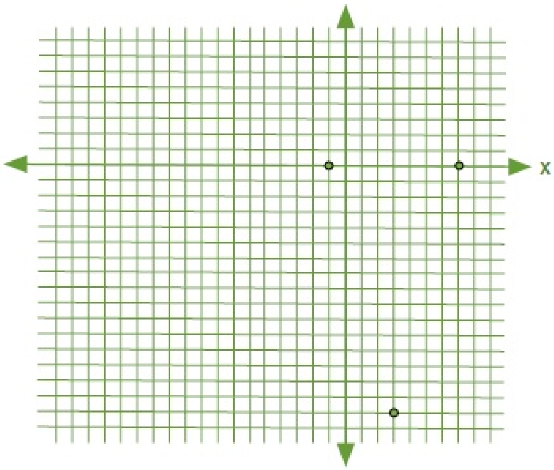
We now use this x-coordinate to find the y-coordinate.

$$\begin{array}{rcrcrcr} y & = & ax^2 & + & bx & + & c \\ y & = & 1(3)^2 & - & 6(3) & - & 7 \\ y & = & 9 & - & 18 & - & 7 \\ y & = & -16 & & & & \end{array}$$

The vertex is at $x = +3$ and $y = -16$ and can be given by the coordinate $(+3, -16)$.

The x-intercepts or roots of this quadratic are found by factoring $x^2 + 6x - 7 = 0$.

For this problem, the quadratic factors to $(x - 7)(x + 1) = 0$, which means the roots are $x = +7$ and $x = -1$. Putting all this data together gives us the vertex coordinate $(-3, -16)$ and the two x-intercept coordinates $(7, 0)$ and $(-1, 0)$.



Trying to sketch this curve will be somewhat challenging if there is to be any semblance of accuracy.

When this happens, it is quite easy to fill in some of the places where there may have been coordinates by using a data table.

For this graph, choose values for $x = 0$ to $x = 6$. First, find the value of y when $x = 0$:

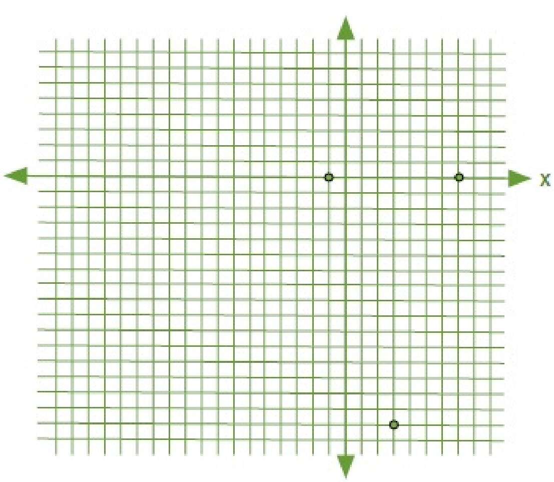
$$\begin{array}{rcrcrcr} y & = & x^2 & - & 6x & - & 7 \\ y & = & (0)^2 & - & 6(0) & - & 7 \\ y & = & 0 & - & 0 & - & 7 \\ y & = & -7 & & & & \end{array}$$

Put this value in the table and then carry on to complete all of it.

x	y
5	7
4	0
3	−5
2	−8
1	−9
0	−8
−1	−5
−2	0
−3	7

Placing all of these coordinates on the graph will generate a graph showing increased detail as shown below. All that remains is to draw a curve that connects the points on the graph. The level of detail you require to draw the curve only depends on the unique characteristics of the curve itself.

Remember:



For the quadratic equation $y = ax^2 + bx + c$, the x-coordinate of the vertex is $x = \dfrac{-b}{2a}$ and the y-coordinate of the vertex is $y = a\left(\dfrac{-b}{2a}\right)^2 + \left(\dfrac{-b}{2a}\right)x + c$.

The following questions will ask you to sketch the quadratic function using the vertex and the x-intercepts, and then later to draw a data table to find the coordinates of data points with which to draw a curve.

Both approaches are quite valuable. The difference is only in the detail. If required, you can use both techniques to generate a curve in increased detail.

Questions

Find the vertex and intercepts of the following quadratics. Use this information to graph the quadratic.

1. $y = x^2 - 2x - 8$
2. $y = x^2 - 2x - 3$
3. $y = 2x^2 - 12x + 10$
4. $y = 2x^2 - 12x + 16$
5. $y = -2x^2 + 12x - 18$
6. $y = -2x^2 + 12x - 10$
7. $y = -3x^2 + 24x - 45$
8. $y = -2(x^2 + 2x) + 6$

First, find the line of symmetry for each of the following equations. Then, construct a data table for each equation. Use this table to graph the equation.

9. $y = 3x^2 - 6x - 5$
10. $y = 2x^2 - 4x - 3$
11. $y = -x^2 + 4x + 2$
12. $y = -3x^2 - 6x + 2$

Answer Key 10.6

92. 10.7 Quadratic Word Problems: Age and Numbers

Quadratic-based word problems are the third type of word problems covered in MATQ 1099, with the first being linear equations of one variable and the second linear equations of two or more variables. Quadratic equations can be used in the same types of word problems as you encountered before, except that, in working through the given data, you will end up constructing a quadratic equation. To find the solution, you will be required to either factor the quadratic equation or use substitution.

Example 10.7.1

The sum of two numbers is 18, and the product of these two numbers is 56. What are the numbers?

First, we know two things:

$$\text{smaller }(S) + \text{larger }(L) = 18 \Rightarrow L = 18 - S$$

$$S \times L = 56$$

Substituting $18 - S$ for L in the second equation gives:

$$S(18 - S) = 56$$

Multiplying this out gives:

$$18S - S^2 = 56$$

Which rearranges to:

$$S^2 - 18S + 56 = 0$$

Second, factor this quadratic to get our solution:

$$\begin{array}{rrrrrcl} S^2 & - & 18S & + & 56 & = & 0 \\ (S & - & 4)(S & - & 14) & = & 0 \\ \\ & & & & S & = & 4, 14 \end{array}$$

Therefore:

$$S = 4, L = 18 - 4 = 14$$

$$S = 14, L = 18 - 14 = 4 \text{ (this solution is rejected)}$$

Example 10.7.2

The difference of the squares of two consecutive even integers is 68. What are these numbers?

The variables used for two consecutive integers (either odd or even) is x and $x + 2$. The equation to use for this problem is $(x+2)^2 - (x)^2 = 68$. Simplifying this yields:

$$\begin{array}{rrrrrrrrr} & & (x & + & 2)^2 & - & (x)^2 & = & 68 \\ x^2 & + & 4x & + & 4 & - & x^2 & = & 68 \\ & & & & 4x & + & 4 & = & 68 \\ & & & & & - & 4 & & -4 \\ & & & & & & \dfrac{4x}{4} & = & \dfrac{64}{4} \\ & & & & & & x & = & 16 \end{array}$$

This means that the two integers are 16 and 18.

Example 10.7.3

The product of the ages of Sally and Joey now is 175 more than the product of their ages 5 years prior. If Sally is 20 years older than Joey, what are their current ages?

The equations are:

$$\begin{aligned} (S)(J) &= 175 + (S - 5)(J - 5) \\ S &= J + 20 \end{aligned}$$

Substituting for S gives us:

$$\begin{array}{rcrcrcrcrcrcr}
(J & + & 20)(J) & = & 175 & + & (J & + & 20-5)(J & - & 5) \\
J^2 & + & 20J & = & 175 & + & (J & + & 15)(J & - & 5) \\
J^2 & + & 20J & = & 175 & + & J^2 & + & 10J & - & 75 \\
-J^2 & - & 10J & & & - & J^2 & - & 10J & & \\
 & & \dfrac{10J}{10} & = & \dfrac{100}{10} & & & & & & \\
 & & J & = & 10 & & & & & &
\end{array}$$

This means that Joey is 10 years old and Sally is 30 years old.

Questions

For Questions 1 to 12, write and solve the equation describing the relationship.

1. The sum of two numbers is 22, and the product of these two numbers is 120. What are the numbers?
2. The difference of two numbers is 4, and the product of these two numbers is 140. What are the numbers?
3. The difference of two numbers is 8, and the sum of the squares of these two numbers are 320. What are the numbers?
4. The sum of the squares of two consecutive even integers is 244. What are these numbers?
5. The difference of the squares of two consecutive even integers is 60. What are these numbers?
6. The sum of the squares of two consecutive even integers is 452. What are these numbers?
7. Find three consecutive even integers such that the product of the first two is 38 more than the third integer.
8. Find three consecutive odd integers such that the product of the first two is 52 more than the third integer.
9. The product of the ages of Alan and Terry is 80 more than the product of their ages 4 years prior. If Alan is 4 years older than Terry, what are their current ages?
10. The product of the ages of Cally and Katy is 130 less than the product of their ages in 5 years. If Cally is 3 years older than Katy, what are their current ages?
11. The product of the ages of James and Susan in 5 years is 230 more than the product of their ages today. What are their ages if James is one year older than Susan?
12. The product of the ages (in days) of two newborn babies Simran and Jessie in two days will be 48 more than the product of their ages today. How old are the babies if Jessie is 2 days older than Simran?

Example 10.7.4

Doug went to a conference in a city 120 km away. On the way back, due to road construction, he had to drive 10 km/h slower, which resulted in the return trip taking 2 hours longer. How fast did he drive on the way to the conference?

The first equation is $r(t) = 120$, which means that $r = \dfrac{120}{t}$ or $t = \dfrac{120}{r}$.

For the second equation, r is 10 km/h slower and t is 2 hours longer. This means the second equation is $(r-10)(t+2) = 120$.

We will eliminate the variable t in the second equation by substitution:

$$(r-10)(\frac{120}{r}+2) = 120$$

Multiply both sides by r to eliminate the fraction, which leaves us with:

$$(r-10)(120+2r) = 120r$$

Multiplying everything out gives us:

$$\begin{array}{rcrcrcrcr} 120r & + & 2r^2 & - & 1200 & - & 20r & = & 120r \\ & & 2r^2 & + & 100r & - & 1200 & = & 120r \\ & & & - & 120r & & & & -120r \\ & & 2r^2 & - & 20r & - & 1200 & = & 0 \end{array}$$

This equation can be reduced by a common factor of 2, which leaves us with:

$$\begin{array}{rcl} r^2 - 10r - 600 & = & 0 \\ (r-30)(r+20) & = & 0 \\ r & = & 30 \text{ km/h or } -20 \text{ km/h (reject)} \end{array}$$

Example 10.7.5

Mark rows downstream for 30 km, then turns around and returns to his original location. The total trip took 8 hr. If the current flows at 2 km/h, how fast would Mark row in still water?

If we let $t =$ the time to row downstream, then the time to return is $8 \text{ h} - t$.

The first equation is $(r+2)t = 30$. The stream speeds up the boat, which means $t = \dfrac{30}{(r+2)}$, and the second equation is $(r-2)(8-t) = 30$ when the stream slows down the boat.

We will eliminate the variable t in the second equation by substituting $t = \dfrac{30}{(r+2)}$:

$$(r-2)\left(8-\frac{30}{(r+2)}\right)=30$$

Multiply both sides by $(r+2)$ to eliminate the fraction, which leaves us with:

$$(r-2)(8(r+2)-30)=30(r+2)$$

Multiplying everything out gives us:

$$\begin{array}{rrrrrrrrrrr}
(r & - & 2)(8r & + & 16 & - & 30) & = & 30r & + & 60 \\
 & & (r & - & 2)(8r & + & (-14)) & = & 30r & + & 60 \\
8r^2 & - & 14r & - & 16r & + & 28 & = & 30r & + & 60 \\
 & & 8r^2 & - & 30r & + & 28 & = & 30r & + & 60 \\
 & & & - & 30r & - & 60 & & -30r & - & 60 \\
 & & 8r^2 & - & 60r & - & 32 & = & 0 & &
\end{array}$$

This equation can be reduced by a common factor of 4, which will leave us:

$$\begin{aligned}
2r^2-15r-8 &= 0 \\
(2r+1)(r-8) &= 0 \\
r &= -\frac{1}{2}\text{ km/h (reject) or } r=8\text{ km/h}
\end{aligned}$$

Questions

For Questions 13 to 20, write and solve the equation describing the relationship.

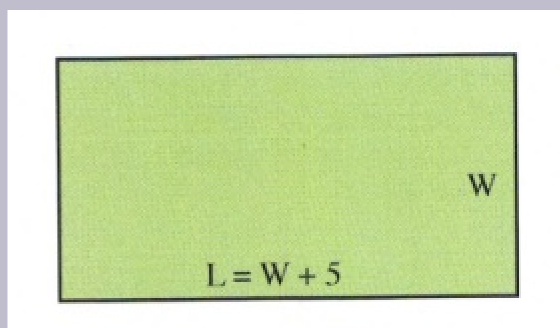
13. A train travelled 240 km at a certain speed. When the engine was replaced by an improved model, the speed was increased by 20 km/hr and the travel time for the trip was decreased by 1 hr. What was the rate of each engine?
14. Mr. Jones visits his grandmother, who lives 100 km away, on a regular basis. Recently, a new freeway has opened up, and although the freeway route is 120 km, he can drive 20 km/h faster on average and takes 30 minutes less time to make the trip. What is Mr. Jones's rate on both the old route and on the freeway?
15. If a cyclist had travelled 5 km/h faster, she would have needed 1.5 hr less time to travel 150 km. Find the speed of the cyclist.
16. By going 15 km per hr faster, a transit bus would have required 1 hr less to travel 180 km. What was the average speed of this bus?
17. A cyclist rides to a cabin 72 km away up the valley and then returns in 9 hr. His speed returning is 12 km/h faster than his speed in going. Find his speed both going and returning.

18. A cyclist made a trip of 120 km and then returned in 7 hr. Returning, the rate increased 10 km/h. Find the speed of this cyclist travelling each way.
19. The distance between two bus stations is 240 km. If the speed of a bus increases by 36 km/h, the trip would take 1.5 hour less. What is the usual speed of the bus?
20. A pilot flew at a constant speed for 600 km. Returning the next day, the pilot flew against a headwind of 50 km/h to return to his starting point. If the plane was in the air for a total of 7 hours, what was the average speed of this plane?

Example 10.7.6

Find the length and width of a rectangle whose length is 5 cm longer than its width and whose area is 50 cm^2.

First, the area of this rectangle is given by $L \times W$, meaning that, for this rectangle, $L \times W = 50$, or $(W+5)W = 50$.



Multiplying this out gives us:

$$W^2 + 5W = 50$$

Which rearranges to:

$$W^2 + 5W - 50 = 0$$

Second, we factor this quadratic to get our solution:

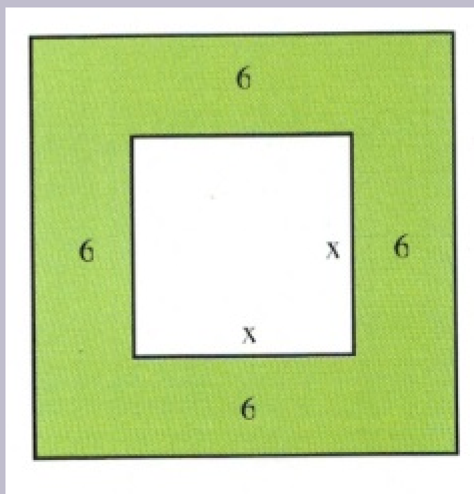
$$\begin{array}{rrrrrrl} W^2 & + & 5W & - & 50 & = & 0 \\ (W & - & 5)(W & + & 10) & = & 0 \\ & & & & W & = & 5, -10 \end{array}$$

We reject the solution $W = -10$.

This means that $L = W + 5 = 5 + 5 = 10$.

Example 10.7.7

If the length of each side of a square is increased by 6, the area is multiplied by 16. Find the length of one side of the original square.



There are two areas to be considered: the area of the smaller square, which is x^2, and the area of the larger square, which is $(x+12)^2$.

The relationship between these two is:

$$\begin{array}{rcl}\text{larger area} &=& 16 \text{ times the smaller area}\\(x+12)^2 &=& 16(x)^2\end{array}$$

Simplifying this yields:

$$\begin{array}{rcrcrcr}x^2 &+& 24x &+& 144 &=& 16x^2\\-16x^2 &&&&&& -16x^2\\-15x^2 &+& 24x &+& 144 &=& 0\end{array}$$

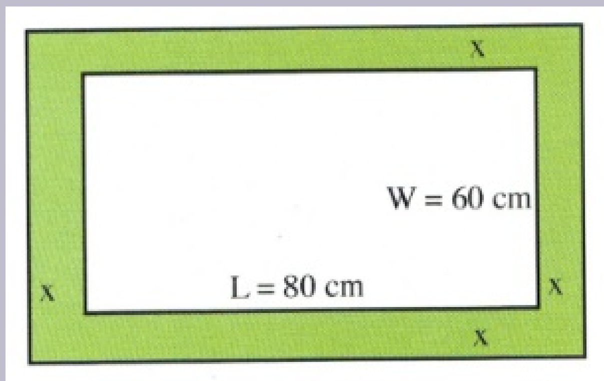
Since this is a problem that requires factoring, it is easiest to use the quadratic equation:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}, \qquad \text{where } a=-15, b=24 \text{ and } c=144$$

Substituting these values in yields $x=4$ or $x=-2.4$ (reject).

Example 10.7.8

Nick and Chloe want to surround their 60 by 80 cm wedding photo with matting of equal width. The resulting photo and matting is to be covered by a 1 m^2 sheet of expensive archival glass. Find the width of the matting.



First, the area of this rectangle is given by $L \times W$, meaning that, for this rectangle:

$$(L + 2x)(W + 2x) = 1\text{ m}^2$$

Or, in cm:

$$(80\text{ cm } + 2x)(60\text{ cm } + 2x) = 10,000\text{ cm}^2$$

Multiplying this out gives us:

$$4800 + 280x + 4x^2 = 10,000$$

Which rearranges to:

$$4x^2 + 280x - 5200 = 0$$

Which reduces to:

$$x^2 + 70x - 1300 = 0$$

Second, we factor this quadratic to get our solution.

It is easiest to use the quadratic equation to find our solutions.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \qquad \text{where } a = 1, b = 70 \text{ and } c = -1300$$

Substituting the values in yields:

$$x = \frac{-70 \pm \sqrt{70^2 - 4(1)(-1300)}}{2(1)} \qquad x = \frac{-70 \pm 10\sqrt{101}}{2}$$

$$x = -35 + 5\sqrt{101} \qquad x = -35 - 5\sqrt{101}\ \text{(rejected)}$$

Questions

For Questions 21 to 28, write and solve the equation describing the relationship.

21. Find the length and width of a rectangle whose length is 4 cm longer than its width and whose area is 60 cm^2.
22. Find the length and width of a rectangle whose width is 10 cm shorter than its length and whose area is 200 cm^2.
23. A large rectangular garden in a park is 120 m wide and 150 m long. A contractor is called in to add a brick walkway to surround this garden. If the area of the walkway is 2800 m^2, how wide is the walkway?
24. A park swimming pool is 10 m wide and 25 m long. A pool cover is purchased to cover the pool, overlapping all 4

sides by the same width. If the covered area outside the pool is 74 m^2, how wide is the overlap area?

25. In a landscape plan, a rectangular flowerbed is designed to be 4 m longer than it is wide. If 60 m^2 are needed for the plants in the bed, what should the dimensions of the rectangular bed be?
26. If the side of a square is increased by 5 units, the area is increased by 4 square units. Find the length of the sides of the original square.
27. A rectangular lot is 20 m longer than it is wide and its area is 2400 m^2. Find the dimensions of the lot.
28. The length of a room is 8 m greater than its width. If both the length and the width are increased by 2 m, the area increases by 60 m^2. Find the dimensions of the room.

Answer Key 10.7

93. 10.8 Construct a Quadratic Equation from its Roots

It is possible to construct an equation from its roots, and the process is surprisingly simple. Consider the following:

Example 10.8.1

Construct a quadratic equation whose roots are $x = 4$ and $x = 6$.

This means that $x = 4$ (or $x - 4 = 0$) and $x = 6$ (or $x - 6 = 0$).

The quadratic equation these roots come from would have as its factored form:

$$(x-4)(x-6)=0$$

All that needs to be done is to multiply these two terms together:

$$(x-4)(x-6)=x^2-10x+24=0$$

This means that the original equation will be equivalent to $x^2 - 10x + 24 = 0$.

This strategy works for even more complicated equations, such as:

Example 10.8.2

Construct a polynomial equation whose roots are $x = \pm 2$ and $x = 5$.

This means that $x = 2$ (or $x - 2 = 0$), $x = -2$ (or $x + 2 = 0$) and $x = 5$ (or $x - 5 = 0$).

These solutions come from the factored polynomial that looks like:

$$(x-2)(x+2)(x-5)=0$$

Multiplying these terms together yields:

$$\begin{array}{rrrrrrrrr} & & (x^2 & - & 4)(x & - & 5) & = & 0 \\ x^3 & - & 5x^2 & - & 4x & + & 20 & = & 0 \end{array}$$

The original equation will be equivalent to $x^3 - 5x^2 - 4x + 20 = 0$.

Caveat: the exact form of the original equation cannot be recreated; only the equivalent. For example, $x^3 - 5x^2 - 4x + 20 = 0$ is the same as $2x^3 - 10x^2 - 8x + 40 = 0$,

$3x^3 - 15x^2 - 12x + 60 = 0, 4x^3 - 20x^2 - 16x + 80 = 0, 5x^3 - 25x^2 - 20x + 100 = 0$, and so on. There simply is not enough information given to recreate the exact original—only an equation that is equivalent.

Questions

Construct a quadratic equation from its solution(s).

1. 2, 5
2. 3, 6
3. 20, 2
4. 13, 1
5. 4, 4
6. 0, 9
7. $\frac{3}{4}, \frac{1}{4}$
8. $\frac{5}{8}, \frac{5}{7}$
9. $\frac{1}{2}, \frac{1}{3}$
10. $\frac{1}{2}, \frac{2}{3}$
11. ± 5
12. ± 1
13. $\pm\frac{1}{5}$
14. $\pm\sqrt{7}$
15. $\pm\sqrt{11}$
16. $\pm2\sqrt{3}$
17. 3, 5, 8
18. −4, 0, 4
19. −9, −6, −2
20. ± 1, 5
21. ± 2, ± 5
22. $\pm2\sqrt{3}, \pm\sqrt{5}$

Answer Key 10.8

PART XIII

MIDTERM 3 PREPARATION AND SAMPLE QUESTIONS

This chapter contains several sections that will help prepare for the midterm exam:

Midterm 3 Composition
Midterm 3 Review
Midterm Sample A
Midterm Sample B
Midterm Sample C
Midterm Sample D
Midterm Sample E

Midterm Three Composition

The third midterm will be composed of fifteen questions covering chapters 7 to 9. Twelve questions will be algebra questions, and three questions will be word problems.

- **Questions 1–4 will be drawn from Chapter 8: Rational Expressions**
  - 8.1 Reducing Rational Expressions
  - 8.2 Multiplication and Division of Rational Expressions
  - 8.3 Least Common Denominators
  - 8.4 Addition and Subtraction of Rational Expressions
  - 8.5 Reducing Complex Fractions
  - 8.6 Solving Complex Fractions
  - 8.7 Solving Rational Equations
- **Questions 5–8 will be drawn from Chapter 9: Radicals**
  - 9.1 Reducing Square Roots
  - 9.2 Reducing Higher Power Roots
  - 9.3 Adding and Subtracting Radicals
  - 9.4 Multiplication and Division of Radicals
  - 9.5 Rationalizing Denominators
  - 9.6 Radicals and Rational Exponents
  - 9.8 Radicals of Mixed Index
  - 9.9 Complex Numbers (Optional)
- **Questions 9–12 will be drawn from Chapter 10: Quadratics**
  - 10.1 Solving Radical Equations
  - 10.2 Solving Exponential Equations
  - 10.3 Completing the Square
  - 10.4 The Quadratic Equation
  - 10.5 Solving Quadratic Equations Using Substitution

- 10.6 Graphing Quadratic Equations—Vertex and Intercept Method
- 10.7 Quadratic Word Problems: Age and Numbers
- 10.8 Construct a Quadratic Equation from its Roots

- Questions 13–15 will be three word problems drawn from:
  - 8.8 Rate Word Problems: Speed, Distance and Time
  - 9.10 Rate Word Problems: Work and Time
  - 10.7 Quadratic Word Problems: Age and Numbers

Students will be allowed to use MATQ 1099 Data Booklets & Glossaries for both Midterms and the Final Exam.

Midterm Three Review

Chapter 7: Rational Expressions (Exam Type Questions)

In the problems below, perform the indicated operations and simplify.

1. $\dfrac{6a-6b}{a^3+b^3} \div \dfrac{a^2-b^2}{a^2-ab+b^2}$
2. $\dfrac{x}{x^2-25} - \dfrac{2}{x^2-6x+5}$
3. $\dfrac{1-\dfrac{6}{x}}{\dfrac{4}{x}-\dfrac{24}{x^2}}$

Solve for x.

4. $\dfrac{4}{x+4} - \dfrac{5}{x-2} = 5$

Chapter 8: Radicals (Exam Type Questions)

Questions 5–6 are for review only and will not show up on exams.

5. $\sqrt[3]{-64} = -4 \text{ (true or false)}$
6. $\sqrt[6]{-64} = -2 \text{ (true or false)}$

Questions similar to 7–13 will show up on exams.
For questions 7–10, simplify each expression.

7. $4\sqrt{36} + 3\sqrt{72} + \sqrt{16}$

8. $\dfrac{\sqrt{300a^5b^2}}{3ab^2}$
9. $\dfrac{12}{3-\sqrt{6}}$
10. $\left(\dfrac{a^0b^3}{c^6d^{-12}}\right)^{\frac{1}{3}}$

For questions 11–13, solve for x.

11. $\sqrt{5x-6}=x$
12. $\sqrt{2x+9}+3=x$
13. $\sqrt{x-3}=\sqrt{2x-5}$

Chapter 9: Quadratics (Exam Type Questions)

Use the quadratic discriminant to determine the nature of the roots of each equation.

14.
    a. $2x^2+4x+3=0$
    b. $3x^2-2x-8=0$

For the questions below, find the solution set by any convenient method.

15.
    a. $3x^2=27$
    b. $2x^2=16x$
16.
    a. $x^2-x-12=0$
    b. $x^2+9x=-8$
17. $\dfrac{x-3}{2}+\dfrac{6}{x+3}=1$
18. $\dfrac{x-2}{x}=\dfrac{x}{x+4}$

Chapter 7–9 Word Problems (Exam Type Questions)

19. The length of a rectangle is 3 cm longer than twice the width. If the area of the rectangle is 65 cm^2, find its length and width.
20. Find three consecutive even integers such that the product of the first and the second is 68 more than the third integer.
21. A boat cruises upriver for 8 hours and returns to its starting point in 6 hours. If the speed of the river is 4 km/h, find the speed of this boat in still water.

22. The base of a right triangle is 8 m longer than its height. If the area of the triangle is 330 m^2, what are its base and height measurements?

Midterm 3: Prep Questions Answer Key

94. Midterm 3: Version A

For problems 1–4, perform the indicated operations and simplify.

1. $\dfrac{15m^3}{4n^2} \div \dfrac{12n}{17m^3} \cdot \dfrac{3m^4}{34n^2}$
2. $\dfrac{8x-8y}{x^3+y^3} \div \dfrac{x^2-y^2}{x^2-xy+y^2}$
3. $\dfrac{5}{6} - 2 - \dfrac{5}{n-3}$
4. $\dfrac{\dfrac{x^2}{y^2}-4}{\dfrac{x+2y}{y^3}}$

Reduce the expressions in questions 5–7.

5. $3\sqrt{25}+2\sqrt{72}-\sqrt{16}$
6. $\dfrac{\sqrt{m^7n^3}}{\sqrt{2n}}$
7. $\dfrac{2-x}{1-\sqrt{3}}$

Solve for x.

8. $\sqrt{7x+8}=x$

For problems 9–12, find the solution set by any convenient method.

9.
   a. $4x^2=64$
   b. $3x^2=12x$
10.
   a. $x^2-6x+5=0$
   b. $x^2+10x=-9$
11. $\dfrac{x+4}{-4}=\dfrac{8}{x}$
12. $x^4-13x^2+36=0$
13. The base of a right triangle is 10 m longer than its height. If the area of this triangle is 300 m^2, what are its base and height measurements?
14. Find three consecutive odd integers such that the product of the first and the third is 38 more than the second.
15. Two airplanes take off from the same airfield, with the first plane leaving at 6 a.m. and the second at 7:30 a.m. The second airplane, travelling at 150 km/h faster than the first, catches up to the first plane by 10:30 a.m. What is the speed of each airplane?

95. Midterm 3: Version B

For problems 1–4, perform the indicated operations and simplify.

1. $\dfrac{5m^3}{4n^2} \div \dfrac{3m^3}{13n^3} \cdot \dfrac{12m^4}{26n^2}$
2. $\dfrac{3x^2+9x}{3x+9} \div \dfrac{x^2+3x-18}{6x^2+18x}$
3. $\dfrac{5x}{x+3} - \dfrac{5x}{x-3} + \dfrac{90}{x^2-9}$
4. $\dfrac{\dfrac{9a^2}{b^2}-25}{\dfrac{3a}{b}+5}$

Reduce the expressions in questions 5–7.

5. $\sqrt{72d^3} + 4\sqrt{18d^3} - 2\sqrt{49d^4}$
6. $\dfrac{\sqrt{a^6b^3}}{\sqrt{5a}}$
7. $\dfrac{\sqrt{5}}{3+\sqrt{5}}$

Solve for x.

8. $\sqrt{4x+12} = x$

For problems 9–12, find the solution set by any convenient method.

9.
   a. $2x^2 = 98$
   b. $4x^2 = 12x$
10.
   a. $x^2 - x - 20 = 0$
   b. $x^2 = 2x + 35$
11. $\dfrac{x-3}{x+2} + \dfrac{6}{x+3} = 1$
12. $x^4 - 5x^2 + 4 = 0$
13. The length of a rectangle is 3 m longer than its width. If it has a perimeter that is 46 m long, then find the length and width of this rectangle.
14. Find three consecutive even integers such that the product of the first two is 16 more than the third.
15. A boat cruises upriver for 4 hours and returns to its starting point in 2 hours. If the speed of the river is 5 km/h, find the speed of this boat in still water.

Midterm 3: Version B Answer Key

96. Midterm 3: Version C

For problems 1–4, perform the indicated operations and simplify.

1. $\dfrac{15m^3}{4n^2} \div \dfrac{30m^3}{17n^3} \cdot \dfrac{3m^4}{34n^2}$
2. $\dfrac{5v^2-25v}{5v+25} \div \dfrac{v^2-11v+30}{10v}$
3. $\dfrac{8}{2x} = \dfrac{2}{x} + 1$
4. $\dfrac{\dfrac{x^2}{y^2}-16}{\dfrac{x+4y}{y^3}}$

Reduce the expressions in questions 5–7.

5. $\sqrt{25y^2} + 2\sqrt{81y^2} + \sqrt{36y^3}$
6. $\dfrac{28}{7-3\sqrt{5}}$
7. $\left(\dfrac{27a^{-\frac{1}{8}}}{a^{\frac{1}{4}}}\right)^{\frac{1}{3}}$

Find the solution set.

8. $\sqrt{3x-2} = \sqrt{5x+4}$

For problems 9–12, find the solution set by any convenient method.

9.
   a. $2x^2 = 72$
   b. $2x^2 = 8x$
10.
   a. $x^2 + 6x + 5 = 0$
   b. $x^2 = 10x - 4$
11. $\dfrac{8}{4x} = \dfrac{2}{x} + 3$
12. $x^4 - 17x^2 + 16 = 0$
13. The width of a rectangle is 6 m less than its length, and its area is 12 units more than its perimeter. What are the dimensions of the rectangle?
14. Find three consecutive odd integers such that the product of the first and the third is 31 more than the second.
15. It took a tugboat 5 hours to travel against an ocean current to get to an isolated outpost 60 km from its home port and 3 hours to return back to port going with the ocean current. What is the speed of the ocean current and what speed can the tug travel on still water?

Midterm 3: Version C Answer Key

97. Midterm 3: Version D

For problems 1–4, perform the indicated operations and simplify.

1. $\dfrac{15m^3}{4n^2} \div \dfrac{45m^6}{13n^3} \cdot \dfrac{3m^4}{39n^2}$
2. $\dfrac{3x^2-9x}{3x+9} \div \dfrac{x^2+2x-15}{12x}$
3. $\dfrac{2}{x+4} - \dfrac{6}{x-3} = 3$
4. $\dfrac{\dfrac{x^2}{y^2}-9}{\dfrac{x+3y}{y^3}}$

Reduce the expressions in questions 5–7.

5. $\sqrt{25y^4} + 2\sqrt{49y^2} + \sqrt{25y^3}$
6. $\dfrac{15}{3-\sqrt{5}}$
7. $\left(\dfrac{a^0b^4}{c^8d^{-12}}\right)^{\frac{1}{4}}$

Find the solution set.

8. $\sqrt{2x+9} - 3 = x$

For problems 9–12, find the solution set by any convenient method.

9.
   a. $8x^2 = 32x$
   b. $3x^2 = 48$
10.
    a. $x^2 = 5x - 4$
    b. $x^2 - 4x + 3 = 0$
11. $\dfrac{2}{x} = \dfrac{x}{x+4}$
12. $x^4 - 48x^2 - 49 = 0$
13. The base of a triangle is 2 cm less than its height. If the area of this triangle is 40 cm^2, find the lengths of its height and base.
14. Find three consecutive odd integers such that the product of the first and the third is 41 more than four times the second integer.
15. Karl paddles downstream in a canoe for 2 hours to reach a store for camp supplies. After getting what he needs, he paddles back upriver for 3 hours before he needs to take a break. If he still has 4 km to go and he can paddle at 6

km/h on still water, what speed is the river flowing at?

Midterm 3: Version D Answer Key

98. Midterm 3: Version E

For problems 1–4, perform the indicated operations and simplify.

1. $\dfrac{12m^3}{5n^2} \div \dfrac{36m^6}{15n^3} \cdot \dfrac{8m^4}{6n^2}$
2. $\dfrac{x^2+2x}{x^2+9x+14} \div \dfrac{2x^3}{2x+14}$
3. $\dfrac{x-3}{7} - \dfrac{x-15}{28} = \dfrac{3}{4}$
4. $\dfrac{\dfrac{x^2}{y^2}-36}{\dfrac{x+6y}{y^3}}$

Reduce the expressions in questions 5–7.

5. $\sqrt{x^7y^5} + 2xy\sqrt{36xy^5} - \sqrt{xy^3}$
6. $\dfrac{\sqrt{7}}{3-\sqrt{7}}$
7. $\left(\dfrac{x^0y^4}{z^{-12}}\right)^{\frac{1}{4}}$

Find the solution set.

8. $\sqrt{4x-5} = \sqrt{2x+3}$

For problems 9–12, find the solution set by any convenient method.

9.
   a. $\dfrac{x^2}{3} = 27$
   b. $27x^2 = -3x$
10.
   a. $x^2 - 11x - 12 = 0$
   b. $x^2 + 13x = -12$
11. $\dfrac{2}{x} = \dfrac{2x}{3x+8}$
12. $x^4 - 63x^2 - 64 = 0$
13. The width of a rectangle is 5 m less than its length, and its area is 20 more units than its perimeter. What are the dimensions of this rectangle?
14. Find three consecutive odd integers such that the product of the first and the third is 35 more than ten times the second integer.
15. Wendy paddles downstream in a canoe for 3 hours to reach a store for camp supplies. After getting what she

needs, she paddles back upstream for 4 hours before she needs to take a break. If she still has 9 km to go and she can paddle at 5 km/h on still water, what speed is the river flowing at?

Midterm 3: Version E Answer Key

PART XIV

CHAPTER 11: FUNCTIONS

Learning Objectives

This chapter covers:

- Function Notation
- Operations on Functions
- Inverse Functions
- Exponential Functions
- Logarithmic Functions
- Compound Interest Word Problems
- Right angle Trigonometry
- Sine and Cosine Laws

99. 11.1 Function Notation

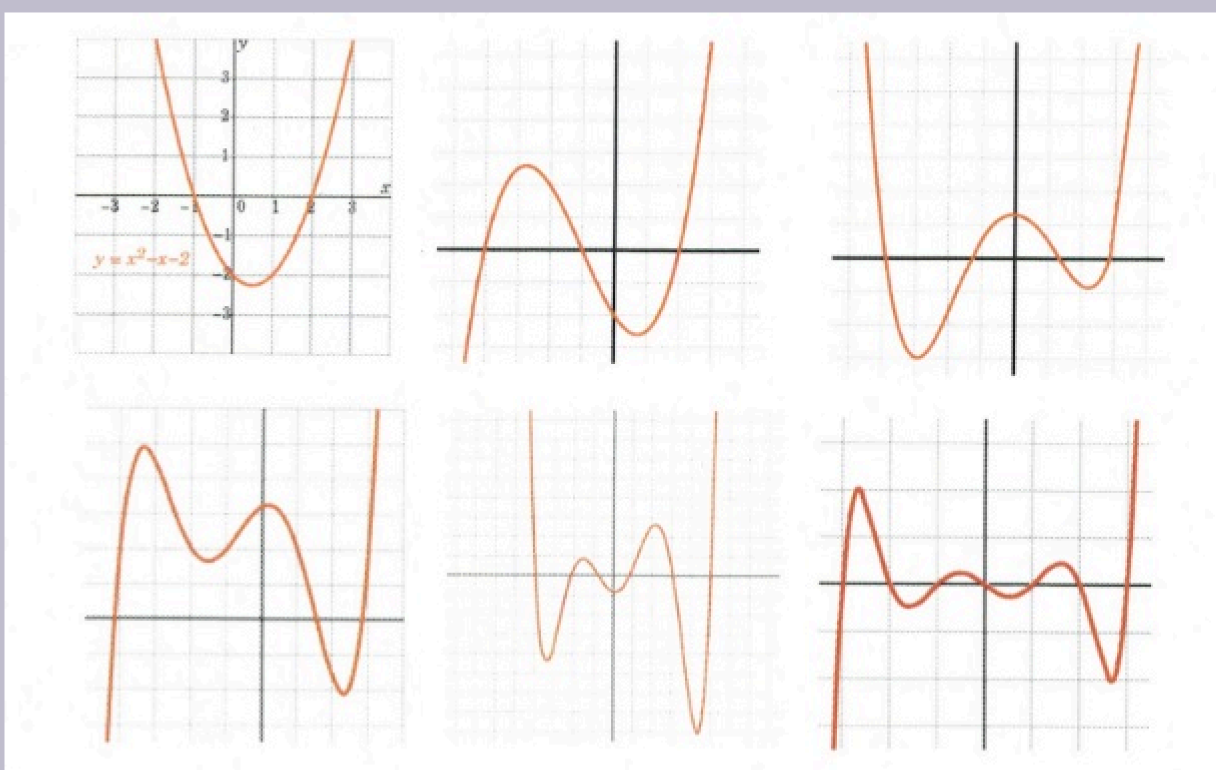
There is a special classification of mathematical relationships known as functions. So far, you will have unknowingly worked with many functions, where the defining characteristic is that functions have at most one output for any input. Properties of addition, subtraction, multiplication or division all bear the needed traits of being functions. For instance, 2 × 3 will always be 6. Formally, functions are defined in equations in terms of x and y, where there will only be one y output for any single x input. An equation is not considered a function if more than one y variable can be found for any x variable.

This means that the definition of a function, in terms of equations in x and y, is that, for any x-value, there is at most one y-value that corresponds with it.

One way to use this definition to see if an equation represents a function is to look at its graph. This is done by looking at any x-value to see if there exists more than one corresponding y-value. The name for this check is the vertical line test. The vertical line test is defined by trying to find if any vertical drawn line will intersect more than one y-value. If you can find any instance of this on the graph, then the equation drawn is not a function. For instance:

Example 11.1.1

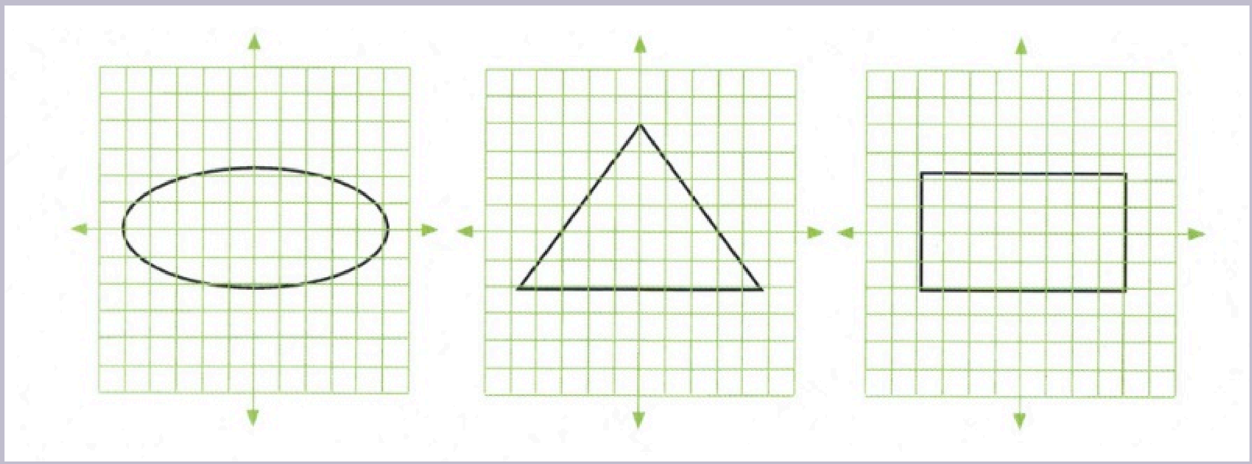
Are all the mathematical relationships shown below functions?



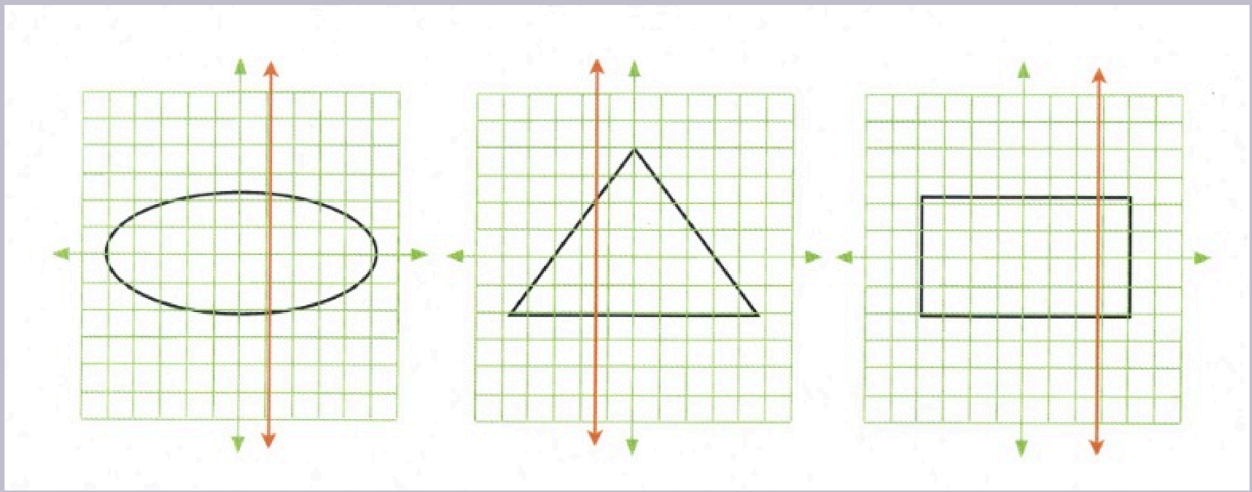
Solution: All of these are functions, since it is impossible to find any vertical line to cross more than one y-value.

Example 11.1.2

Are any of the mathematical relationships shown below functions?



Solution: None of these are functions, since vertical lines can easily be drawn that will have 2 or more y-values for a single x-value.



Deciding if equations are functions requires more effort than using the vertical line test. The easiest method is to isolate the y-variable and see if it results in two potential x-values.

Example 11.1.3

Is the equation $0 = 2x^2 - y - 7$ a function?

First, you need to isolate the y-variable:

$$\begin{array}{rcrcrcr} 0 & = & 2x^2 & - & y & - & 7 \\ +y & & & + & y & & \\ y & = & 2x^2 & - & 7 & & \end{array}$$

There is only one solution for y for any given value of x. Therefore, this equation is a function.

The next example shows an equation that is not a function, since there are two y-values for every given x-value.

Example 11.1.4

Is the equation $0 = y^2 - 5x - 7$ a function?

First, you need to isolate the y-variable:

$$\begin{array}{rcrcrcr} 0 & = & y^2 & - & 5x^2 & - & 7 \\ -y^2 & & -y^2 & & & & \\ -y^2 & = & -5x^2 & - & & 7 & \end{array}$$

Next, we remove the negatives by multiplying the entire equation by −1:

$$y^2 = 5x^2 + 7$$

To reduce the square, take the square root of both sides:

$$y = \pm(5x^2 + 7)^{\frac{1}{2}}$$

We are left with two solutions for any single x-variable. Therefore, this equation is not a function.

Example 11.1.5

Is the equation $x = |y - 5|$ a function?

Solving for y yields $y - 5 = x$ and $y - 5 = -x$.

Isolating for y yields $y = x + 5$ and $y = -x + 5$.

You are left with the same type of solution as you did when taking the square root, except in this case, $y = \pm x + 5$.

We are left with two solutions for any single x variable. Therefore, this equation is not a function

Excluded Values and Domains of a Function

When working with functions, one needs to identify what values of x cannot be used. These x-values are termed the excluded values and are useful in defining the domain of a function. The logic of excluded values is the extension of a property from arithmetic:

$$\text{You cannot divide by zero, or Never divide by zero}$$

Example 11.1.6

Find the excluded values of the following function:

$$y = \frac{2x^2 - 3}{(x-2)(x+3)(x-1)}$$

In this example, there will be 3 excluded values:

$$(x-2) \neq 0 \qquad (x+3) \neq 0 \qquad (x-1) \neq 0$$

Since these terms are all in the denominator of this function, any value that can make one of them equal zero must be excluded.

For these terms, those excluded values are $x \neq 2, x \neq -3$ and $x \neq 1$.

Interpreting this means that the domain of x is any real number except for the excluded values.

You write this as:

domain of x = all real numbers except 2, −3, 1

More formally:

$$\text{domain} = \{x | x \in \mathbb{R}, x \neq 2, -3, 1\}$$

Finding the domains of radicals can lead to an inequality as a solution, since any real solution of an even-valued radical is restricted in that the value inside the radical cannot be negative.

Example 11.1.7

Find the excluded values of the following function: $y = \sqrt{2x-3}$.

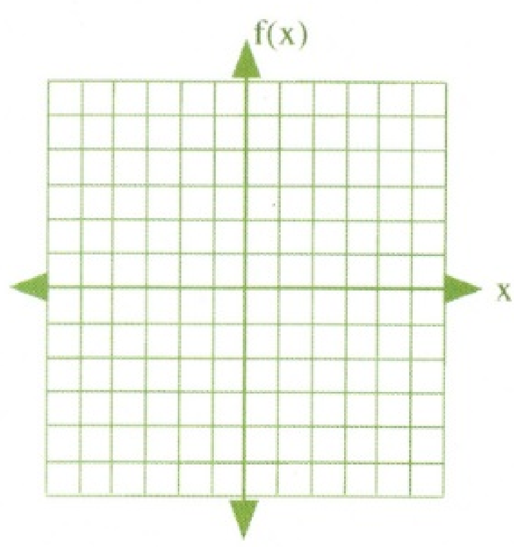
Since its impossible to take any real root of a negative inside a radical, the value inside the radical must always be zero or larger. This means:

$$\begin{array}{rrrcr} 2x & - & 3 & \geq & 0 \\ & + & 3 & & +3 \\ & & \dfrac{2x}{2} & \geq & \dfrac{3}{2} \\ & & x & \geq & \dfrac{3}{2} \end{array}$$

The domain for x is such that x must always be greater than or equal to $\frac{3}{2}$.

Function Notation

The earliest written usage of function notation $f(x)$ appears in the works of Leonhard Euler in the early 1700s. If you have an equation that is found to be a function, such as $y = 2x^2 - 3x + 2$, it can also be written as $f(x) = 2x^2 - 3x + 2$. It can be useful to write a function equation in this form.



You should quickly notice that, in graphing these functions, the y-variable is replaced by the function notation $f(x)$ for the y-axis. That $f(x)$ replaces y is the main change.

When drawing a graph of the function, $f(x)$ is treated as if it is the y-variable.

Evaluating Functions

One of the features of function notation is the way it identifies values of the function for given x inputs. For instance, suppose you are given the function $f(x) = 3x^2 - 5$ and you are asked to find the value of the $f(x)$ when $x = 7$. This would be written as $f(7)$ and you would be asked to evaluate $f(7) = 3x^2 - 5$. The following examples illustrate this process.

Example 11.1.8

Evaluate the function $f(x) = 3x^2 - 2x + 5$ for $f(4)$.

First, you need to replace all values of x with the value 4. This looks like:

$$\begin{aligned} f(4) &= 3(4)^2 - 2(4) + 5 \\ f(4) &= 3(16) - 8 + 5 \\ f(4) &= 48 - 8 + 5 \\ f(4) &= 45 \end{aligned}$$

Functions can be written using other letters outside of the standard f. In fact, just about any letter will suffice. For instance, for the equation $y = 3x^4 - 8$, this can be written in function notation as $f(x) = 3x^4 - 8$, $g(x) = 3x^4 - 8$, $h(x) = 3x^4 - 8$, $k(x) = 3x^4 - 8$, $p(x) = 3x^4 - 8$, and so on.

Example 11.1.9

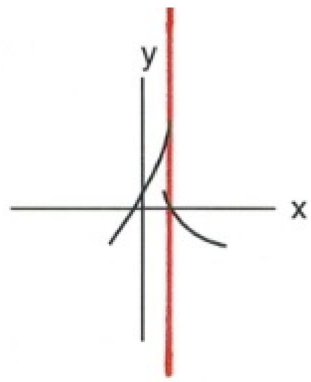
Evaluate the function $h(t) = 3t^2 + 7t + 2$ for $h(-1)$.

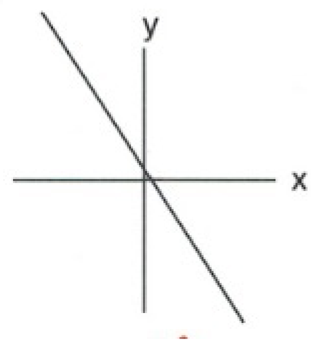
First, you need to replace all values of t with the value −1. This looks like:

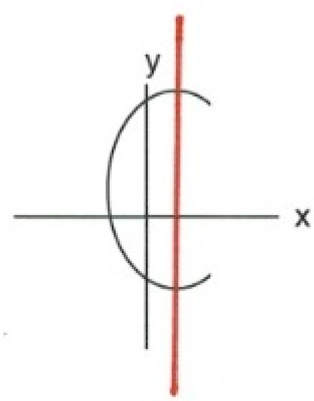
$h(-1) = 3(-1)^2 + 7(-1) + 2$, which simplifies to $h(-1) = -2$

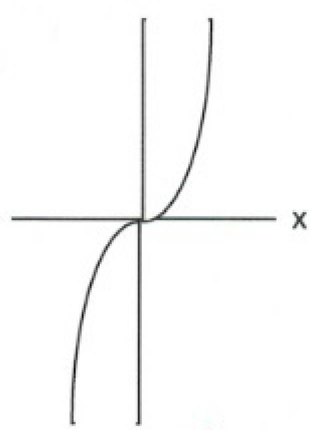
Questions

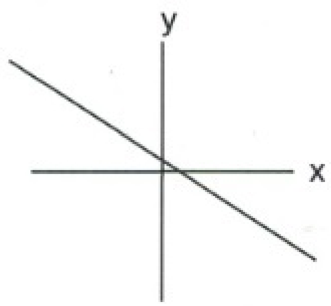
1. Which of the following are functions?

a. 

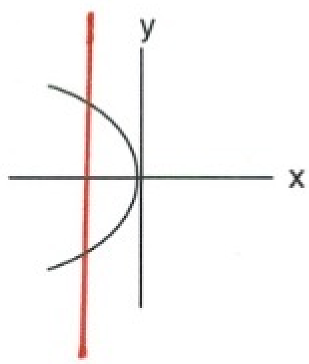
b. 

c. 

d. 



e.



f.

g. $y = 3x - 7$
h. $y^2 - x^2 = 1$
i. $\sqrt{y} + x = 2$
j. $x^2 + y^2 = 1$

Specify the domain of each of the following functions.

2. $f(x) = -5x + 1$
3. $f(x) = \sqrt{5 - 4x}$
4. $s(t) = \dfrac{1}{t^2}$
5. $f(x) = x^2 - 3x - 4$
6. $s(t) = \dfrac{1}{t^2 + 1}$
7. $f(x) = \sqrt{x - 16}$
8. $f(x) = \dfrac{-2}{x^2 - 3x - 4}$
9. $h(x) = \dfrac{\sqrt{3x - 12}}{x^2 - 25}$

Evaluate each of the following functions.

10. $g(x) = 4x - 4 \text{ for } g(0)$
11. $g(n) = -3 \cdot 5^{-n} \text{ for } g(2)$
12. $f(x) = x^2 + 4 \text{ for } f(-9)$

13. $f(n) = n - 3$ for $f(10)$
14. $f(t) = 3^t - 2$ for $f(-2)$
15. $f(a) - 3^{a-1} - 3$ for $f(2)$
16. $k(x) = -2 \cdot 4^{2x-2}$ for $k(2)$
17. $p(t) = -2 \cdot 4^{2t+1} + 1$ for $p(-2)$
18. $h(x) = x^3 + 2$ for $h(-4x)$
19. $h(n) = 4n + 2$ for $h(n+2)$
20. $h(x) = 3x + 2$ for $h(-1+x)$
21. $h(a) = -3 \cdot 2^{a+3}$ for $h\left(\frac{1}{3}\right)$
22. $h(x) = x^2 + 1$ for $h(x^4)$
23. $h(t) = t^2 + t$ for $h(t^2)$
24. $f(x) = |3x + 1| + 1$ for $f(0)$
25. $f(n) = -2|-n-2| + 1$ for $f(-6)$
26. $f(t) = |t + 3|$ for $f(10)$
27. $p(x) = -|x| + 1$ for $p(5)$

Answer Key 11.1

100. 11.2 Operations on Functions

In Chapter 5, you solved systems of linear equations through substitution, addition, subtraction, multiplication, and division. A similar process is employed in this topic, where you will add, subtract, multiply, divide, or substitute functions. The notation used for this looks like the following:

Given two functions $f(x)$ and $g(x)$:

$$\begin{array}{llll} f(x)+g(x) & \text{is the same as} & (f+g)(x) & \text{and means the addition of these two functions} \\ f(x)-g(x) & \text{is the same as} & (f-g)(x) & \text{and means the subtraction of these two functions} \\ f(x)\cdot g(x) & \text{is the same as} & (f\cdot g)(x) & \text{and means the multiplication of these two functions} \\ f(x)\div g(x) & \text{is the same as} & (f\div g)(x) & \text{and means the addition of these two functions} \end{array}$$

When encountering questions about operations on functions, you will generally be asked to do two things: combine the equations in some described fashion and to substitute some value to replace the variable in the original equation. These are illustrated in the following examples.

Example 11.2.1

Perform the following operations on $f(x) = 2x^2 - 4$ and $g(x) = x^2 + 4x - 2$.

a. $f(x) + g(x)$Addition yields $2x^2 - 4 + x^2 + 4x - 2$, which simplifies to $3x^2 + 4x - 6$.
b. $f(x) - g(x)$Subtraction yields $2x^2 - 4 - (x^2 + 4x - 2)$, which simplifies to $x^2 - 4x - 2$
.
c. $f(x) \cdot g(x)$Multiplication yields $(2x^2 - 4)(x^2 + 4x - 2)$, which simplifies to
$2x^4 + 8x^3 - 4x^2 - 16x + 8$.
d. $f(x) \div g(x)$Division yields $(2x^2 - 4) \div (x^2 + 4x - 2)$, which cannot be reduced any further.

Often, you are asked to evaluate operations on functions where you must substitute some given value into the combined functions. Consider the following.

Example 11.2.2

Perform the following operations on $f(x) = x^2 - 3$ and $g(x) = 2x^2 + 3x$ and evaluate for the given values.

a. $f(2) + g(2)$
$[x^2 - 3] + [2x^2 + 3x]$
$[(2)^2 - 3] + [2(2)^2 + 3(2)]$

$4 - 3 + 8 + 6 = 15$
$f(2) + g(2) = 15$

b. $f(1) - g(3)$
$[x^2 - 3] - [2x^2 + 3x]$
$[(1)^2 - 3] - [2(3)^2 + 3(3)]$
$[1 - 3] - [18 + 9] = -29$
$f(1) - g(3) = -29$

c. $f(0) \cdot g(2)$
$[x^2 - 3] \cdot [2x^2 + 3x]$
$[0^2 - 3] \cdot [2(2)^2 + 3(2)]$
$[-3] \cdot [8 + 6] = -42$
$f(0) \cdot g(2) = -42$

d. $f(2) \div g(0)$
$[x^2 - 3] \div [2x^2 + 3x]$
$[2^2 - 3] \div [2(0)^2 + 3(0)]$
$[1] \div [0] = \text{ undefined}$

Composite functions are functions that involve substitution of functions, such as $f(x)$ is substituted for the x-value in the $g(x)$ function or the reverse. Which goes where is outlined by the way the equation is written:

$(f \circ g)(x)$ means that the $g(x)$ function is used to replace the x-values in the $f(x)$ function
$(g \circ f)(x)$ means that the $f(x)$ function is used to replace the x-values in the $g(x)$ function

The more conventional way to write these composite functions is:

$(f \circ g)(x) = f(g(x))$ and $(g \circ f)(x) = g(f(x))$

Consider the following examples of composite functions.

Example 11.2.3

Given the functions $f(x) = 3x - 5$ and $g(x) = x^2 + 2$, evaluate for:

a. $(f \circ g)(2)$
$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ f(g(x)) &= 3(x^2 + 2) - 5 \\ f(g(2)) &= 3(2^2 + 2) - 5 \\ f(g(2)) &= 3(6) - 5 = 13 \end{aligned}$$

b. $(g \circ f)(-1)$
$$\begin{aligned} (g \circ f)(x) &= g(f(x)) \\ g(f(x)) &= [3x - 5]^2 + 2 \\ g(f(-1)) &= [3(-1) - 5]^2 + 2 \\ g(f(-1)) &= [-8]^2 + 2 \\ g(f(-1)) &= 66 \end{aligned}$$

Questions

Perform the indicated operations.

1. $g(a) = a^3 + 5a^2$
   $f(a) = 2a + 4$
   Find $g(3) + f(3)$
2. $f(x) = -3x^2 + 3x$
   $g(x) = 2x + 5$
   Find $\dfrac{f(-4)}{g(-4)}$
3. $g(x) = -4x + 1$
   $h(x) = -2x - 1$
   Find $g(5) + h(5)$
4. $g(x) = 3x + 1$
   $f(x) = x^3 + 3x^2$
   Find $g(2) \cdot f(2)$
5. $g(t) = t - 3$
   $h(t) = -3t^3 + 6t$
   Find $g(1) + h(1)$
6. $g(x) = x^2 - 2$
   $h(x) = 2x + 5$
   Find $g(-6) + h(-6)$
7. $h(n) = 2n - 1$
   $g(n) = 3n - 5$
   Find $\dfrac{h(0)}{g(0)}$
8. $g(a) = 3a - 2$
   $h(a) = 4a - 2$
   Find $(g + h)(10)$
9. $g(a) = 3a + 3$
   $f(a) = 2a - 2$
   Find $(g + f)(9)$
10. $g(x) = 4x + 3$
    $h(x) = x^3 - 2x^2$
    Find $(g - h)(-1)$
11. $g(x) = x + 3$
    $f(x) = -x + 4$
    Find $(g - f)(3)$
12. $g(x) = x^2 + 2$
    $f(x) = 2x + 5$
    Find $(g - f)(0)$
13. $f(n) = n - 5$
    $g(n) = 4n + 2$
    Find $(f + g)(-8)$
14. $h(t) = t + 5$

$g(t) = 3t - 5$
Find $(h \cdot g)(5)$
15. $g(t) = t - 4$
$h(t) = 2t$
Find $(g \cdot h)(3t)$
16. $g(n) = n^2 + 5$
$f(n) = 3n + 5$
Find $\frac{g(n)}{f(n)}$
17. $g(a) = -2a + 5$
$f(a) = 3a + 5$
Find $\left(\frac{g}{f}\right)(a^2)$
18. $h(n) = n^3 + 4n$
$g(n) = 4n + 5$
Find $h(n) + g(n)$
19. $g(n) = n^2 - 4n$
$h(n) = n - 5$
Find $g(n^2) \cdot h(n^2)$
20. $g(n) = n + 5$
$h(n) = 2n - 5$
Find $(g \cdot h)(-3n)$

Solve the following composite functions.

21. $f(x) = -4x + 1$
$g(x) = 4x + 3$
Find $(f \circ g)(9)$
22. $h(a) = 3a + 3$
$g(a) = a + 1$
Find $(h \circ g)(5)$
23. $g(x) = x + 4$
$h(x) = x^2 - 1$
Find $(g \circ h)(10)$
24. $f(n) = -4n + 2$
$g(n) = n + 4$
Find $(f \circ g)(9)$
25. $g(x) = 2x - 4$
$h(x) = 2x^3 + 4x^2$
Find $(g \circ h)(3)$
26. $g(x) = x^2 - 5x$
$h(x) = 4x + 4$
Find $(g \circ h)(x)$
27. $f(a) = -2a + 2$
$g(a) = 4a$
Find $(f \circ g)(a)$
28. $g(x) = 4x + 4$

$f(x) = x^3 - 1$
Find $(g \circ f)(x)$

29. $g(x) = -x + 5$
$f(x) = 2x - 3$
Find $(g \circ f)(x)$

30. $f(t) = 4t + 3$
$g(t) = -4t - 2$
Find $(f \circ g)(t)$

Answer Key 11.2

101. 11.3 Inverse Functions

When working with mathematical functions, it sometimes becomes useful to undo what the original function does. To do this, you need to find the inverse of the function. This feature is commonly used in exponents and logarithms and in trigonometry.

In this topic, you will be looking at functions and seeing if they can be inverses of themselves. The notation used for this procedure is $f^{-1}(x)$ is the inverse of $f(x)$. In practice, this works as follows:

$$f^{-1}[f(x)] = x$$

This is a very useful tool used many times over in math. If there are two functions $f(x)$ and $g(x)$ that are inverses of each other (if their composites "undo" each other's function), their composite functions look like:

$$g(f(x)) = x \text{ and } f(g(x)) = x$$

Example 11.3.1

Are the functions $f(x) = 2x + 20$ and $g(x) = \dfrac{x}{2} - 10$ inverses of each other?

Test if either $g(f(x)) = x$ or $f(g(x)) = x$:

$$\begin{aligned} g(f(x)) &= \frac{2x+20}{2} - 10 \\ &= x + 10 - 10 \\ &= x \end{aligned}$$

These two functions are inverses of each other. If you had tested $f(g(x))$, you would have gotten the same result, x.

Example 11.3.2

Are the functions $f(x) = (3x+4)^{\frac{1}{3}} and g(x) = \dfrac{x^3-4}{3}$ inverses of each other?

Test if either $g(f(x)) = x$ or $f(g(x)) = x$.

For this problem, it would be easier to work with $g(f(x)) = x$, since x^3 will cancel out the radical in the $f(x)$.

$$\begin{aligned} g(f(x)) &= [(3x+4)^{\frac{1}{3}}]^3 - 4 \\ g(f(x)) &= \frac{3x+4-4}{3} \\ g(f(x)) &= \frac{3x}{3} \\ g(f(x)) &= x \end{aligned}$$

These functions are inverses of each other.

One of the strategies that is used to find the inverse of another function involves the substitution of the x and y variables of an equation. This is shown in the next few examples.

Example 11.3.3

Find the inverse function of $y = x^3 - 8$.

The inverse function is found by substituting y for all x values and x for all y values in the original equation and then isolating for y.

From the equation $y = x^3 - 8$, you now get $x = y^3 - 8$.

Isolating for y yields $y^3 = x + 8$, which simplifies to $y = (x+8)^{\frac{1}{3}}$.

These equations can be also written as $f(x) = x^3 - 8$ and $f^{-1}(x) = (x+8)^{\frac{1}{3}}$.

Example 11.3.4

Find the inverse function of $f(x) = (x+4)^3 - 2$.

$$\begin{array}{rcl} x & = & (f^{-1}(x)+4)^3 - 2 \\ x+2 & = & (f^{-1}(x)+4)^3 \\ (x+2)^{\frac{1}{3}} & = & f^{-1}(x)+4 \\ f^{-1}(x) & = & (x+2)^{\frac{1}{3}} - 4 \end{array}$$

Questions

State if the given functions are inverses.

1. $g(x) = -x^5 - 3 \text{ and } f(x) = \sqrt[5]{-x-3}$
2. $g(x) = 4 - x \text{ and } f(x) = \dfrac{4}{x}$
3. $g(x) = -10x + 5 \text{ and } f(x) = \dfrac{x-5}{10}$
4. $f(x) = \dfrac{x-5}{10} \text{ and } h(x) = 10x + 5$
5. $f(x) = \dfrac{-2}{x+3} \text{ and } g(x) = \dfrac{3x+2}{x+2}$
6. $f(x) = \dfrac{-x-1}{x-2} \text{ and } g(x) = \dfrac{-2x+1}{-x-1}$

For questions 7 to 22, find the inverse of each function.

7. $f(x) = (x-2)^5 + 3$
8. $g(x) = \sqrt[3]{x+1} + 2$
9. $g(x) = \dfrac{4}{x+2}$
10. $f(x) = \dfrac{-3}{x-3}$
11. $f(x) = \dfrac{-2x-2}{x+2}$
12. $g(x) = \dfrac{9+x}{3}$
13. $f(x) = \dfrac{10-x}{5}$
14. $f(x) = \dfrac{5x-15}{2}$
15. $g(x) = -(x-1)^3$

16. $f(x) = \dfrac{12 - 3x}{4}$
17. $f(x) = (x - 3)^3$
18. $g(x) = \sqrt[5]{-x} + 2$
19. $g(x) = \dfrac{x}{x - 1}$
20. $f(x) = \dfrac{-3 - 2x}{x + 3}$
21. $f(x) = \dfrac{x - 1}{x + 1}$
22. $h(x) = \dfrac{x}{x + 2}$

Answer Key 11.3

102. 11.4 Exponential Functions

As our study of algebra gets more advanced, we begin to study more involved functions. One pair of inverse functions you will look at are exponential and logarithmic functions.

Exponential functions are functions in which the variable is in the exponent, such as $f(x)=a^x$.

Solving exponential equations cannot be done using the techniques used prior. For example, if $3^x=9$, one cannot take the x^{th} root of 9 because we do not know what the index is. However, if you notice that 9 is 3^2, you can then conclude that, if $3^x=3^2$, then $x=2$. This is the process is used to solve exponential functions.

If the problem is rewritten so the bases are the same, then the exponents must also equal each other.

Example 11.4.1

Solve for the exponent x in the equation $5^5=5^{x+2}$.

Since the bases for these exponents are the same, then the exponents must equal each other. Thus:

$$\begin{array}{rcl} 5 &=& x+2 \\ x &=& 3 \end{array}$$

Generally, manipulating the bases on each side of an exponential function to make them equal. These types of problems are as follows:

Example 11.4.2

Solve for the exponent x in the equation $5^{2x+1}=125$.

Since the bases for these exponents are not equal, then the first challenge is to find the lowest common base. For this problem, 125 is the same as 5^3.

Therefore, the equation is rewritten as $5^{2x+1}=5^3$. Thus:

$$\begin{array}{rrrcr} 2x & + & 1 & = & 3 \\ & - & 1 & & -1 \\ & & 2x & = & 2 \\ & & x & = & 1 \end{array}$$

Example 11.4.3

Solve for the exponent x in the equation $8^{3x} = 32$.

Finding the common base is a bit more complicated for this problem, but this issue is easily resolved if terms are reduced to their prime factorization of $8 = 2^3$ and $32 = 2^5$. Use this to rewrite the original equation as $(2^3)^{3x} = 2^5$.

With identical bases, now solve for the exponents:

$$\begin{aligned} 3(3x) &= 5 \\ 9x &= 5 \\ x &= \frac{5}{9} \end{aligned}$$

Example 11.4.4

Solve for the exponent x in the equation $5^{4x} \cdot 5^{2x-1} = 5^{3x+11}$.

Since the bases already equal each other, simplify both sides before beginning to solve this problem. $5^{4x} \cdot 5^{2x-1}$ reduces to 5^{6x-1}, and 5^{3x+11} is already reduced.

With the bases simplified, now solve:

$$5^{6x-1} = 5^{3x+11}$$

$$\begin{array}{rcrcrcr} 6x & - & 1 & = & 3x & + & 11 \\ -3x & + & 1 & & -3x & + & 1 \\ & & 3x & = & 12 & & \\ & & x & = & 4 & & \end{array}$$

Example 11.4.5

Solve for the exponent x in the equation $\left(\frac{1}{9}\right)^{2x} = 3^{7x-1}$.

First, since $\frac{1}{9} = 3^{-2}$, the common base is 3. Rewriting the equation in the base of 3 yields:

$$\begin{array}{rcl} (3^{-2})^{2x} & = & 3^{7x-1} \\ \\ (-2)2x & = & 7x-1 \\ \\ -4x & = & 7x-1 \\ -7x & & -7x \\ -11x & = & -1 \\ \\ x & = & \dfrac{1}{11} \end{array}$$

Questions

Solve each equation.

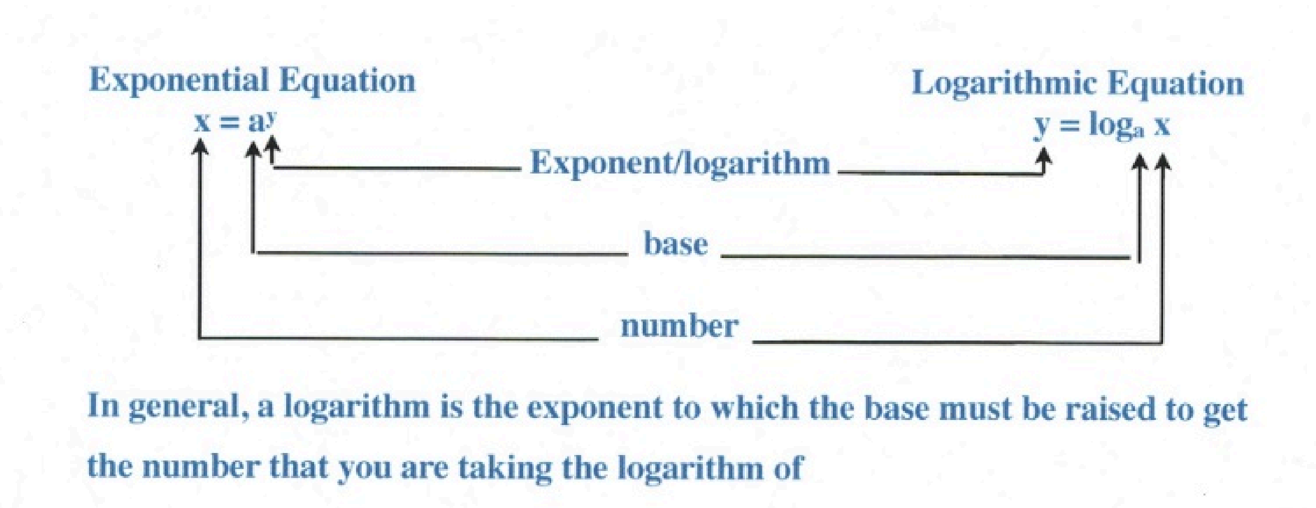
1. $3^{1-2n} = 3^{1-3n}$
2. $4^{2x} = \frac{1}{16}$
3. $4^{2a} = 1$
4. $16^{-3p} = 64^{-3p}$
5. $\left(\frac{1}{25}\right)^{-k} = 125^{-2k-2}$
6. $625^{-n-2} = \frac{1}{125}$
7. $6^{2m+1} = \frac{1}{36}$

8. $6^{2r-3} = 6^{r-3}$
9. $6^{-3x} = 36$
10. $5^{2n} = 5^{-n}$
11. $64^{b} = 2^{5}$
12. $216^{-3v} = 36^{3v}$
13. $\left(\frac{1}{4}\right)^{x} = 16$
14. $27^{-2n-1} = 9$
15. $4^{3a} = 4^{3}$
16. $4^{-3v} = 64$
17. $36^{3x} = 216^{2x+1}$
18. $64^{x+2} = 16$
19. $9^{2n+3} = 243$
20. $16^{2k} = \frac{1}{64}$
21. $3^{3x-2} = 3^{3x+1}$
22. $243^{p} = 27^{-3p}$
23. $3^{-2x} = 3^{3}$
24. $4^{2n} = 4^{2-3n}$
25. $5^{m+2} = 5^{-m}$
26. $625^{2x} = 25$
27. $\left(\frac{1}{36}\right)^{b-1} = 216$
28. $216^{2n} = 36$
29. $6^{2-2x} = 6^{2}$
30. $\left(\frac{1}{4}\right)^{3v-2} = 64^{1-v}$

Answer Key 11.4

103. 11.5 Logarithmic Functions

Logarithms come from a rich history, extending from the Babylonians around 1500–2000 BC, through the Indian mathematician Virasena around 700–800 AD, and later rapidly growing and expanding in European science from the mid-1500s and on. Logarithms were developed to reduce multiplication and division to correspond to adding and subtracting numbers on a number line. Quite simply, logarithms reduced the complexity of these functions and retained significance until the advent of the computer. Even so, logarithms are still in use today in many functions. This topic is taught here, since the logarithmic function is the inverse to the exponential function (shown in 11.4). We will use this feature to solve both exponential and logarithmic functions.



In general, a logarithm is the exponent to which the base must be raised to get the number that you are taking the logarithm of. Using logarithms and exponents together, we can start to identify useful relations.

Consider 2^3. $2^3 = 2 \times 2 \times 2 = 8$, when written using logarithmic functions, will look like $\log_2 8 = 3$. You read this as the log base 2 of 8 equals 3. This means that, if you are using the base 2 and are looking to find the exponent that yields 8, the power needed on the base is 3. You can quantify this relation in either the one of the two equations: $x = a^y$ or $y = \log_a x$. Writing this relationship in either form is illustrated in the following examples.

Example 11.5.1

Write the logarithmic equation for each given exponential relation.

a. $x^3 = 12$ In a logarithmic equation looks like $\log_x 12 = 3$
b. $4^3 = 64$ In a logarithmic equation looks like $\log_4 64 = 3$
c. $7^2 = 49$ In a logarithmic equation looks like $\log_7 49 = 2$
d. $y^6 = 102$ In a logarithmic equation looks like $\log_y 102 = 6$

Example 11.5.2

Write the exponential relation for each given logarithmic equation.

a.	$\log_x 42 = 5$	In exponential form looks like	$x^5 = 42$
b.	$\log_4 624 = 5$	In exponential form looks like	$4^5 = 624$
c.	$\log_3 18 = 2$	In exponential form looks like	$3^2 = 18$
d.	$\log_y 12 = 4$	In exponential form looks like	$y^4 = 12$

A further illustration of this relationship is shown below for the exponents and logarithms for the common base values of 2 and 10.

Examples of Exponents and Logarithms for Base 2 and 10

$2^0 = 1$	$\log_2 1 = 0$	$10^0 = 1$	$\log_{10} 1 = 0$
$2^1 = 2$	$\log_2 2 = 1$	$10^1 = 10$	$\log_{10} 10 = 1$
$2^2 = 4$	$\log_2 4 = 2$	$10^2 = 100$	$\log_{10} 100 = 2$
$2^3 = 8$	$\log_2 8 = 3$	$10^3 = 1000$	$\log_{10} 1000 = 3$
$2^4 = 16$	$\log_2 16 = 4$	$10^4 = 10000$	$\log_{10} 10000 = 4$
$2^5 = 32$	$\log_2 32 = 5$	$10^5 = 100000$	$\log_{10} 100000 = 5$
$2^6 = 64$	$\log_2 64 = 6$	$10^6 = 1000000$	$\log_{10} 1000000 = 6$
$2^7 = 128$	$\log_2 128 = 7$	$10^7 = 10000000$	$\log_{10} 10000000 = 7$
$2^8 = 256$	$\log_2 256 = 8$	$10^8 = 100000000$	$\log_{10} 100000000 = 8$

In the following examples, we evaluate logarithmic functions by converting the logarithms to exponential form.

Example 11.5.3

Evaluate the logarithmic equation $\log_2 64 = x$.

The exponential form of this logarithm is $2^x = 64$.

Since 64 equals 2^6, rewrite this as $2^x = 2^6$, which means that $x = 6$.

Often, you are asked to evaluate a logarithm that is not in the form of an equation; rather, it is given as a simple logarithm. For this type of question, set the logarithm to equal x and then solve as we did above.

Example 11.5.4

Evaluate the logarithmic equation $\log_{125} 5$.

First, we set this logarithm to equal x, so $\log_{125} 5 = x$.

The exponential form of this logarithm is $5^x = 125$.

Since 125 equals 5^3, we rewrite this as $5^x = 5^3$, which means that $x = 3$.

Logarithmic equations that appear more complicated are solved using a somewhat similar strategy as above, except that you often employ algebraic methods. For instance:

Example 11.5.5

Evaluate the logarithmic equation $\log_2(3x + 5) = 4$.

The exponential form of this logarithm is $2^4 = 3x + 4$.

This now becomes an algebraic equation to solve:

$$\begin{array}{rcrcr} 16 & = & 3x & + & 4 \\ -4 & & & - & 4 \\ \dfrac{12}{3} & = & \dfrac{3x}{3} & & \\ & & & & \\ x & = & 4 & & \end{array}$$

The most common form of logarithm uses base 10. This can be compared to the most common radical of the square root. When encountering base 10 logarithms, they are often written without the base 10 shown. To solve these, rewrite in exponential form using the base 10.

Example 11.5.6

Evaluate the logarithmic equation $\log x = -2$.

The exponential form of this logarithm is $10^{-2} = x$.

Since $10^{-2} = \frac{1}{100}$, this means that $x = \frac{1}{100}$.

Questions

Rewrite each equation in exponential form.

1. $\log_9 81 = 2$
2. $\log_b a = -16$
3. $\log_7 \frac{1}{49} = -2$
4. $\log_{16} 256 = 2$
5. $\log_{13} 169 = 2$
6. $\log_{11} 1 = 0$

Rewrite each equation in logarithmic form.

7. $8^0 = 1$
8. $17^{-2} = \frac{1}{289}$
9. $15^2 = 225$
10. $144^{\frac{1}{2}} = 12$
11. $64^{\frac{1}{6}} = 2$
12. $19^2 = 361$

Evaluate each expression.

13. $\log_{125} 5$
14. $\log_5 125$
15. $\log_{343} \frac{1}{7}$
16. $\log_7 1$
17. $\log_4 16$
18. $\log_4 \frac{1}{64}$
19. $\log_6 36$
20. $\log_{36} 6$
21. $\log_2 64$
22. $\log_3 243$

Solve each equation.

23. $\log_5 x = 1$
24. $\log_8 k = 3$
25. $\log_2 x = -2$
26. $\log n = 3$
27. $\log_{11} k = 2$
28. $\log_4 p = 4$
29. $\log_9(n + 9) = 4$
30. $\log_{11}(x - 4) = -1$
31. $\log_5(-3m) = 3$
32. $\log_2 -8r = 1$
33. $\log_{11}(x + 5) = -1$
34. $\log_7 -3n = 4$
35. $\log_4(6b + 4) = 0$
36. $\log_{11}(10v + 1) = -1$
37. $\log_5(-10x + 4) = 4$
38. $\log_9(7 - 6x) = -2$
39. $\log_2(10 - 5a) = 3$
40. $\log_8(3k - 1) = 1$

Answer Key 11.5

104. 11.6 Compound Interest

An application of exponential functions is compound interest. When money is invested in an account (or given out on loan), a certain amount is added to the balance. The money added to the balance is called interest. Once that interest is added, the balance will earn more interest during the next compounding period. This idea of earning interest on interest is called compound interest.

For example, if you invest $\$100$ at 10% interest compounded annually, after one year, you will earn $\$10$ in interest, giving you a new balance of $\$110$. The next year, you will earn another 10% or $\$11$, giving you a new balance of $\$121$. The third year, you will earn another 10% or $\$12.10$, giving you a new balance of $\$133.10$. This pattern will continue each year until you close the account.

There are several ways interest can be paid. The first way, as described above, is compounded annually. In this model, the interest is paid once per year. But interest can be compounded more often. Some common compounds include compounded semiannually (twice per year), quarterly (four times per year), monthly (12 times per year), weekly (52 times per year), or even daily (365 times per year).

When interest is compounded in any of these ways, we can calculate the balance after any amount of time using the following formula:

Compound Interest Formula: $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$

$$\begin{array}{rrcl} \text{where} & A & = & \text{final amount} \\ & P & = & \text{principal (starting balance)} \\ & r & = & \text{interest rate (as a decimal)} \\ & n & = & \text{number of compounds per year} \\ & t & = & \text{time (in years)} \end{array}$$

The following examples will illustrate the use of the compound interest formula.

Example 11.6.1

If you take a car loan for $\$25,000$ with an interest rate of 6.5% compounded quarterly, no payments required for the first five years, what will your balance be at the end of those five years?

$$A = \text{final amount} \qquad P = \$25,000 \qquad r = 0.065 \qquad n = 4 \qquad t = 5 \qquad nt = 20$$

$$A = \$25,000\left(1+\frac{0.065}{4}\right)^{20}$$

$$A = \$25,000(1.38041977\ldots)$$

$$A = \$34,510.49$$

Example 11.6.2

What principal will amount to $\$3000$ if invested at 6.5% compounded weekly for 4 years?

$$\begin{aligned} A &= \$3000 \qquad P = \text{principal} \qquad r = 0.065 \qquad n = 52 \qquad t = 4 \qquad nt = 208 \\ \$3000 &= P\left(1+\frac{0.065}{52}\right)^{208} \\ \$3000 &= P(1.00125)^{208} \\ \$3000 &= P(1.296719528\ldots) \\ P &= \frac{\$3000}{1.296719528\ldots} \\ P &= \$2313.53 \end{aligned}$$

Questions

1. Find each of the following:
   a. $\$500$ invested at 4% compounded annually for 10 years

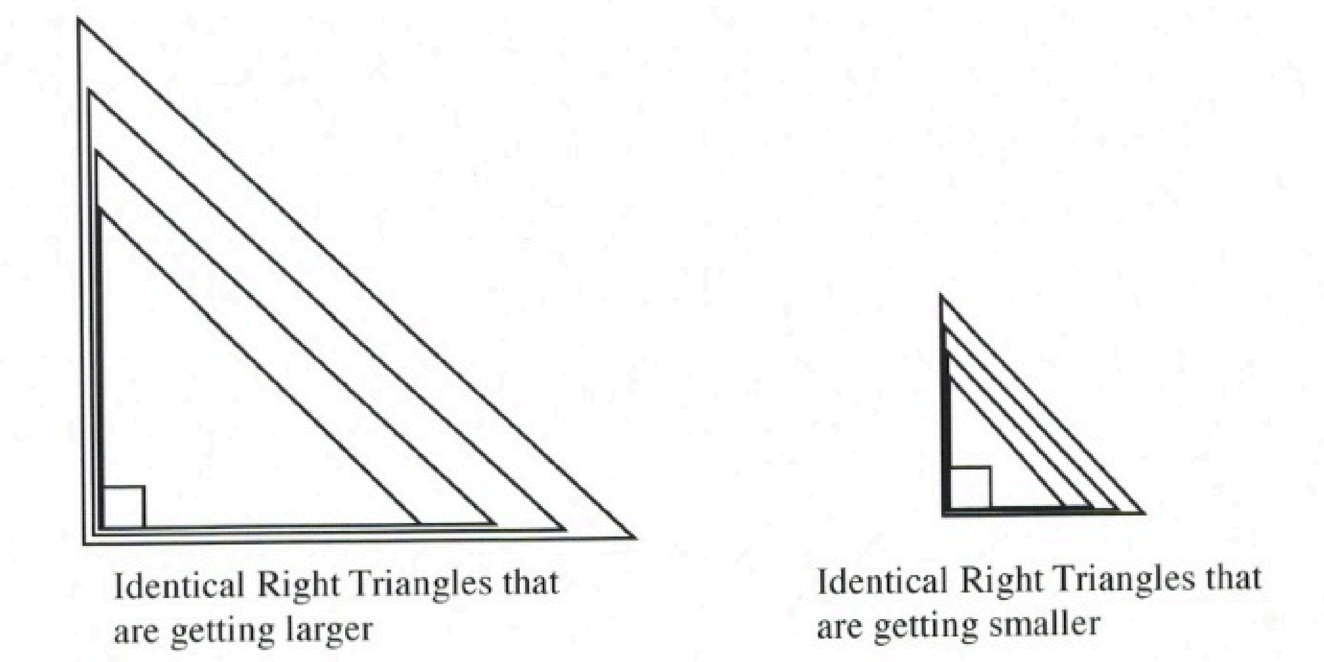
b. $\$600$ invested at 6% compounded annually for 6 years
c. $\$750$ invested at 3% compounded annually for 8 years
d. $\$1500$ invested at 4% compounded semiannually for 7 years
e. $\$900$ invested at 6% compounded semiannually for 5 years
f. $\$950$ invested at 4% compounded semiannually for 12 years
g. $\$2000$ invested at 5% compounded quarterly for 6 years
h. $\$2250$ invested at 4% compounded quarterly for 9 years
i. $\$3500$ invested at 6% compounded quarterly for 12 years

2. If $\$10,000$ is left in a bank savings account drawing 4% interest, compounded quarterly for 10 years, what is the balance at the end of that time?
3. If $\$27,500$ is invested in an account earning 6% interest compounded monthly, what is the balance at the end of 9 years?
4. Suppose that you lend $\$55,000$ at 10% compounded monthly. If the debt is repaid in 18 months, what is the total owed at the time of repayment?
5. What principal will amount to $\$20,000$ if it is invested at 6% interest compounded semiannually for 5 years?
6. What principal will amount to $\$4200$ if it is invested at 4% interest compounded quarterly for 5 years?

Answer Key 11.6

105. 11.7 Trigonometric Functions

Introductory trigonometry is based on identical the similarities between identical right angled (one angle is 90°) of different sizes. If the angles of a triangle are identical then all of this triangle is simply larger or smaller copies of each other.



In all of the cases shown above if you take any two sides of any triangle shown and divide them by each other, that number will be exactly the same for the same two sides chosen from any of the triangles

These triangle ratios have defined names:

$$\text{sine} = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \text{cosine} = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \text{tangent} = \frac{\text{opposite}}{\text{adjacent}}$$

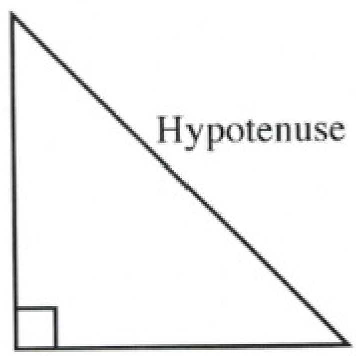
You often see these equations shortened to:

$$\sin = \frac{\text{opp}}{\text{hyp}} \qquad \cos = \frac{\text{adj}}{\text{hyp}} \qquad \tan = \frac{\text{opp}}{\text{adj}}$$

And memorized as:

$$\text{SOH} \qquad \text{CAH} \qquad \text{TOA}$$

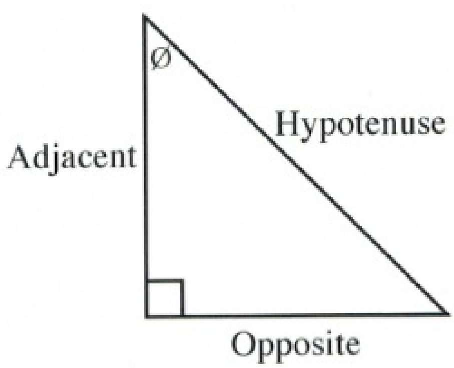
Defining the sides of a triangle follows a set pattern:

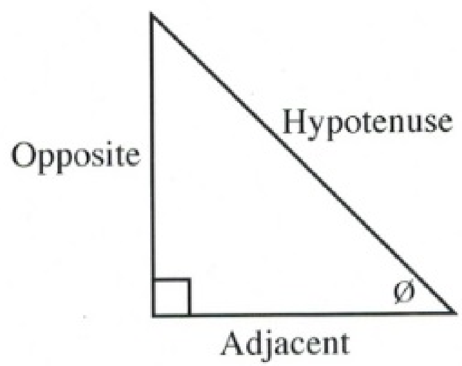


1st: The side of a triangle that is opposite to the right angle is called the hypotenuse.

2nd: The opposite and adjacent sides are then defined by the angle you are going to work with. One of the sides will be opposite this angle and the other side will be beside (adjacent to) this side.

For example: The following sides are defined by the right angle and the angle you are going to work with Ø. You will have to define the adjacent and opposite sides for every right triangle you work with.





The other right-angled trigonometric rations are the reciprocals of sine, cosine and tangent:

$$\text{cosecant} = \frac{1}{\text{sine}} \qquad \text{secant} = \frac{1}{\text{cosine}} \qquad \text{cotangent} = \frac{1}{\text{tangent}}$$

Or formally defined as:

$$\text{cosecant} = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \text{secant} = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \text{cotangent} = \frac{\text{adjacent}}{\text{opposite}}$$

You often see these equations shortened to:

$$\text{csc} = \frac{\text{hyp}}{\text{opp}} \qquad \text{sec} = \frac{\text{hyp}}{\text{adj}} \qquad \text{cot} = \frac{\text{adj}}{\text{opp}}$$

These reciprocal trigonometric functions are commonly used in calculus, specifically in integration and when working with polar coordinates. Anyone taking higher levels of mathematics will encounter these reciprocal trigonometric functions.

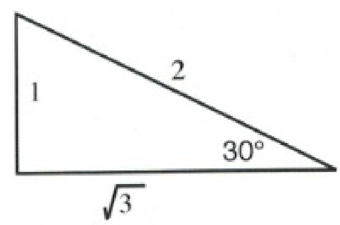
Using the Pythagorean theorem for 30°, 45° and 60° right angle triangles, you can get the exact values of the trigonometric relationship (and the reciprocal values). It is standard to see exams where students are required to draw these 30°, 45° and 60° right angle triangles and use the side lengths to generate exact values.

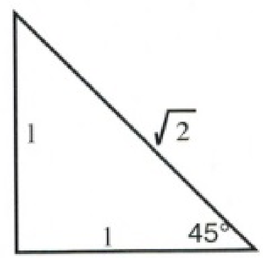
Standard Reference Angles

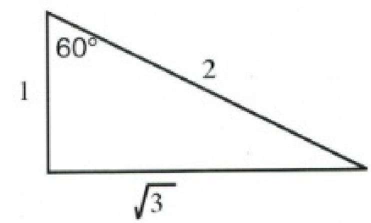
$\sin 30^\circ = \frac{1}{2}$ $\sin 45^\circ = \frac{1}{\sqrt{2}}$ $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$ $\cos 45^\circ = \frac{1}{\sqrt{2}}$ $\cos 60^\circ = \frac{1}{2}$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$ $\tan 45^\circ = 1$ $\tan 60^\circ = \sqrt{3}$







Another common sight is to see trigonometric tables being used for approximations of the trig ratios of standard angles from 1° to 90°. For these tables, choose the value that lines up the trigonometric function you wish to use with the angle that you are using. Basic scientific calculators have essentially made these tables obsolete.

Example 11.7.1

Find the values that correspond to the following trigonometric functions and angles:

$\sin 19^\circ = X$ $\cos 67^\circ = Y$ $\tan 38^\circ = Z$

Solution:

$X = 0.326$ $Y = 0.391$ $Z = 0.781$

It is also possible to work in reverse—that is, given the trigonometric ration of two sides, you can find the angle that you are working with.

Example 11.7.2

Find the angles that correspond to the following trigonometric values:

$\sin\ \emptyset = 0.829 \qquad \cos\ \emptyset = 0.940 \qquad \tan\ \emptyset = 3.732$

Solution:

$\emptyset = 56^\circ \qquad \emptyset = 20^\circ \qquad \emptyset = 75^\circ$

Sometimes, you do not have a value that matches up. For these cases, you choose the value that is closest to what you have.

Example 11.7.3

Find the angles that are closest to the following trigonometric values:

$\sin\ \emptyset = 0.297 \qquad \cos\ \emptyset = 0.380 \qquad \tan\ \emptyset = 0.635$

Solution:

$\emptyset = 17^\circ \qquad \emptyset = 68^\circ \qquad \emptyset = 32^\circ$

Trigonometric Tables

Angle	Sin	Cos	Tan	Csc
1	0.017	1.000	0.017	57.299
2	0.035	0.999	0.035	28.654
3	0.052	0.999	0.052	19.107
4	0.070	0.998	0.070	14.336
5	0.087	0.996	0.087	11.474
6	0.105	0.995	0.105	9.567
7	0.122	0.993	0.123	8.206
8	0.139	0.990	0.141	7.185
9	0.156	0.988	0.158	6.392
10	0.174	0.985	0.176	5.759
11	0.191	0.982	0.194	5.241
12	0.208	0.978	0.213	4.810
13	0.225	0.974	0.231	4.445
14	0.242	0.970	0.249	4.134
15	0.259	0.966	0.268	3.864
16	0.276	0.961	0.287	3.628
17	0.292	0.956	0.306	3.420
18	0.309	0.951	0.325	3.236
19	0.326	0.946	0.344	3.072
20	0.342	0.940	0.364	2.924
21	0.358	0.934	0.384	2.790
22	0.375	0.927	0.404	2.669
23	0.391	0.921	0.424	2.559

Angle	Sin	Cos	Tan	Csc
46	0.719	0.695	1.036	1.390
47	0.731	0.682	1.072	1.36
48	0.743	0.669	1.111	1.346
49	0.755	0.656	1.150	1.325
50	0.766	0.643	1.192	1.305
51	0.777	0.629	1.235	1.287
52	0.788	0.616	1.280	1.269
53	0.799	0.602	1.327	1.252
54	0.809	0.588	1.376	1.236
55	0.819	0.574	1.428	1.221
56	0.829	0.559	1.483	1.206
57	0.839	0.545	1.540	1.192
58	0.848	0.530	1.600	1.179
59	0.857	0.515	1.664	1.167
60	0.866	0.500	1.732	1.155
61	0.875	0.485	1.804	1.143
62	0.883	0.469	1.881	1.133
63	0.891	0.454	1.963	1.122
64	0.899	0.438	2.050	1.113
65	0.906	0.423	2.145	1.103
66	0.914	0.407	2.246	1.095
67	0.921	0.391	2.356	1.086
68	0.927	0.375	2.475	1.079

Angle	Sin	Cos	Tan	Csc
24	0.407	0.914	0.445	2.459
25	0.423	0.906	0.466	2.366
26	0.438	0.899	0.488	2.281
27	0.454	0.891	0.510	2.203
28	0.469	0.883	0.532	2.130
29	0.485	0.875	0.554	2.063
30	0.500	0.866	0.577	2.000
31	0.515	0.857	0.601	1.942
32	0.530	0.848	0.625	1.887
33	0.545	0.839	0.649	1.836
34	0.559	0.829	0.675	1.788
35	0.574	0.819	0.700	1.743
36	0.588	0.809	0.727	1.701
37	0.602	0.799	0.754	1.662
38	0.616	0.788	0.781	1.624
39	0.629	0.777	0.810	1.589
40	0.643	0.766	0.839	1.556
41	0.656	0.755	0.869	1.524
42	0.669	0.743	0.900	1.494
43	0.682	0.731	0.933	1.466
44	0.695	0.719	0.966	1.440
45	0.707	0.707	1.000	1.414

Angle	Sin	Cos	Tan	Csc
69	0.934	0.358	2.605	1.071
70	0.940	0.342	2.747	1.064
71	0.946	0.326	2.904	1.058
72	0.951	0.309	3.078	1.051
73	0.956	0.292	3.271	1.046
74	0.961	0.276	3.487	1.040
75	0.966	0.259	3.732	1.035
76	0.970	0.242	4.011	1.031
77	0.974	0.225	4.331	1.026
78	0.978	0.208	4.705	1.022
79	0.982	0.191	5.145	1.019
80	0.985	0.174	5.671	1.015
81	0.988	0.156	6.314	1.012
82	0.990	0.139	7.115	1.010
83	0.993	0.122	8.144	1.008
84	0.995	0.105	9.514	1.006
85	0.996	0.087	11.430	1.004
86	0.998	0.070	14.301	1.002
87	0.999	0.052	19.081	1.001
88	0.999	0.035	28.636	1.001
89	1.000	0.017	57.290	1.000
90	1.000	0.000		1.000

Questions

Find the value of each of the following trigonometric functions to 6 digits using your scientific calculator.

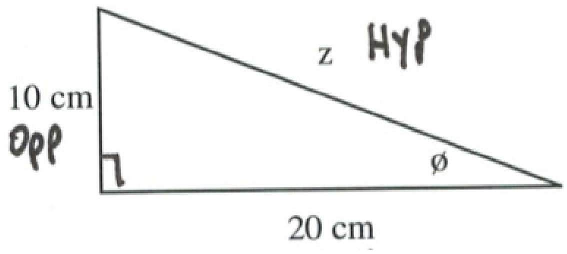
1. sin 48°
2. sin 29°
3. cos 25°
4. cos 61°
5. tan 11°
6. tan 57°
7. sin 11°
8. cos 57°

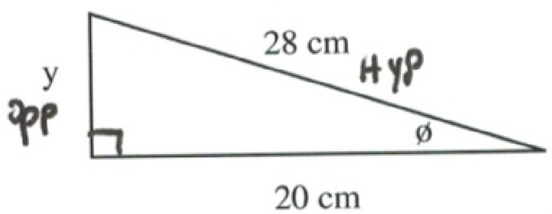
Use your scientific calculator to find each angle to the nearest hundredth of a degree.

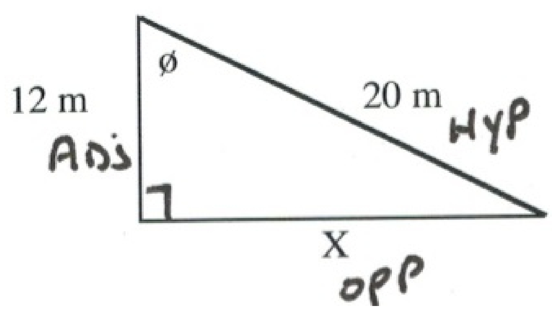
9. sin Ø = 0.4848
10. sin Ø = 0.6293
11. cos Ø = 0.6561

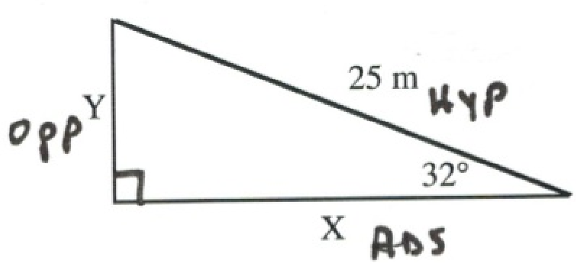
12. cos Ø = 0.6157
13. tan Ø = 0.6561
14. tan Ø = 0.1562
15. sin Ø = 0.6561
16. cos Ø = 0.1562

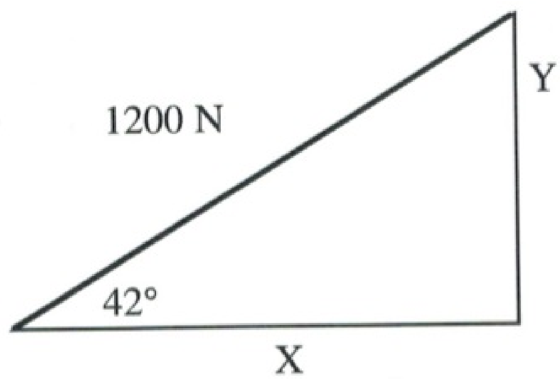
Solve for all unknowns in the following right triangles.

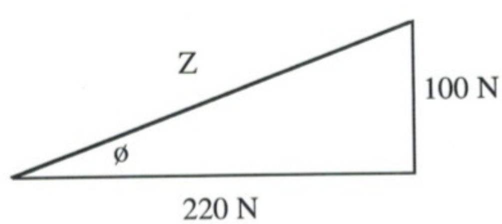
1. 

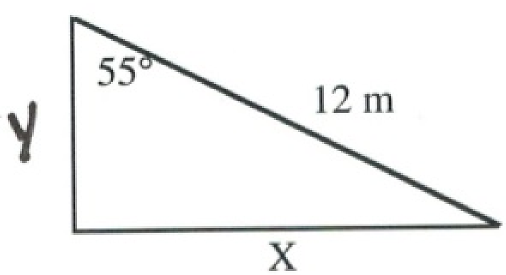
2. 

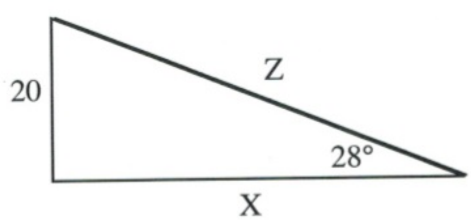
3. 

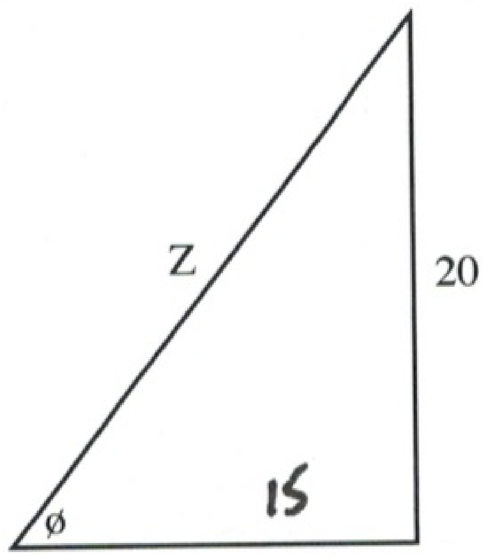
4. 

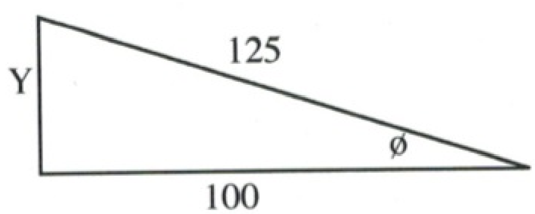
5. 

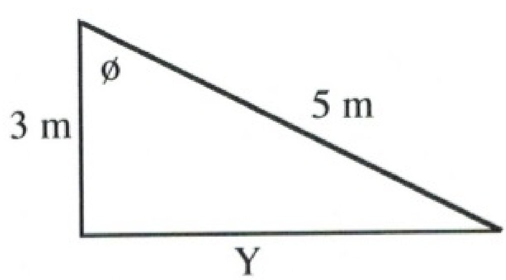
6. 

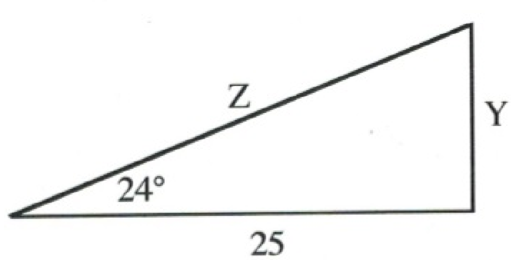
7. 

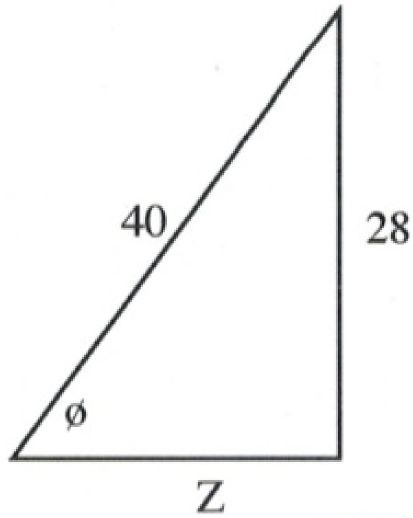
8. 

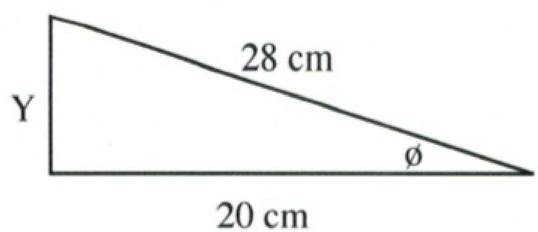
9. 

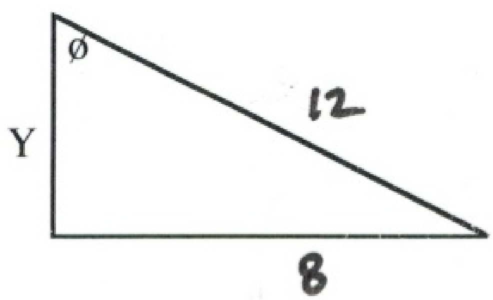
10. 

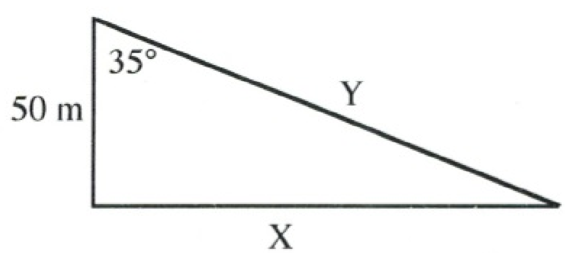
11. 

12. 

13. 

14. 

15. 

16. 

Answer Key 11.7

106. 11.8 Sine and Cosine Laws

Right angle trigonometry is generally limited to triangles that contain a right angle. It is possible to use trigonometry with non-right triangles using two laws: the sine law and the cosine law.

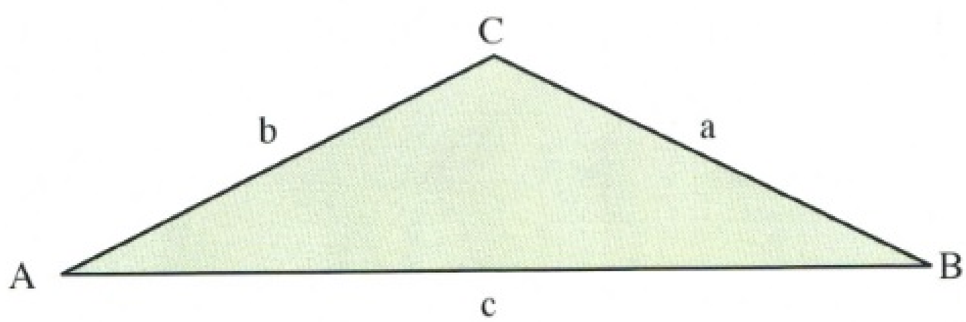
The Law of Sines

The sine law is a ratio of sines and opposite sides. The law takes the following form:

$$\frac{a}{\sin\ A} = \frac{b}{\sin\ B} = \frac{c}{\sin\ C}$$

Sometimes, it is written and used as the reciprocal of the above:

$$\frac{\sin\ A}{a} = \frac{\sin\ B}{b} = \frac{\sin\ C}{c}$$



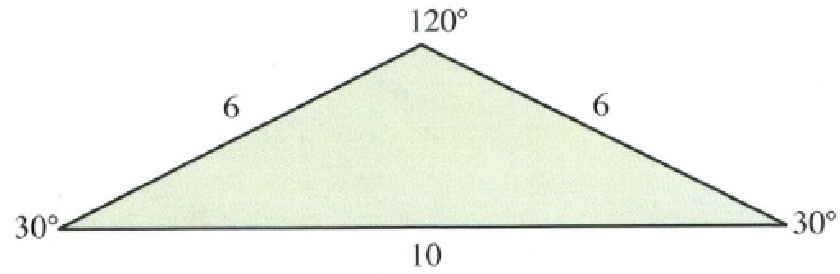
The law of sine is used when either two sides and one opposite angle of one of the sides are known, or when there are two angles and one side of one of the angles. If there are two given angles of a triangle, then all three angles are known, since $A^\circ + B^\circ + C^\circ = 180^\circ$.

The sine law is a very useful law with one caveat in that it is possible to sometimes have two triangles (one larger and one smaller) that generate the same result. This is termed the ambiguous case and is described later in this section.

There are also textbook errors where the data given for the triangle is impossible to create. For instance:

Example 11.8.1

Can the following triangle exist?



If this triangle can exist, then the ratio of sines for the angles to the opposite sides should equate.

$$\frac{6}{\sin\ 30^\circ} = \frac{6}{\sin\ 30^\circ} = \frac{10}{\sin\ 120^\circ}$$

Reducing this yields:

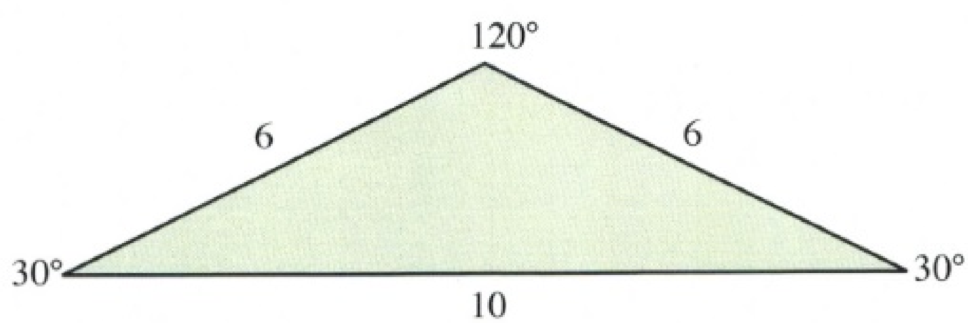
$$\frac{6}{0.5} = \frac{6}{0.5} = \frac{10}{0.866}$$

In checking this out, we find that 12 = 12 ≠ 11.55.

This means that this triangle cannot exist.

Example 11.8.2

Find the correct length of the side opposite 120° in the triangle shown below.



For this triangle, the ratio to solve is:

$$\frac{6}{\sin\ 30°} = \frac{6}{\sin\ 30°} = \frac{x}{\sin\ 120°}$$

We only need to use one portion of this, so:

$$\frac{6}{\sin\ 30°} = \frac{x}{\sin\ 120°}$$

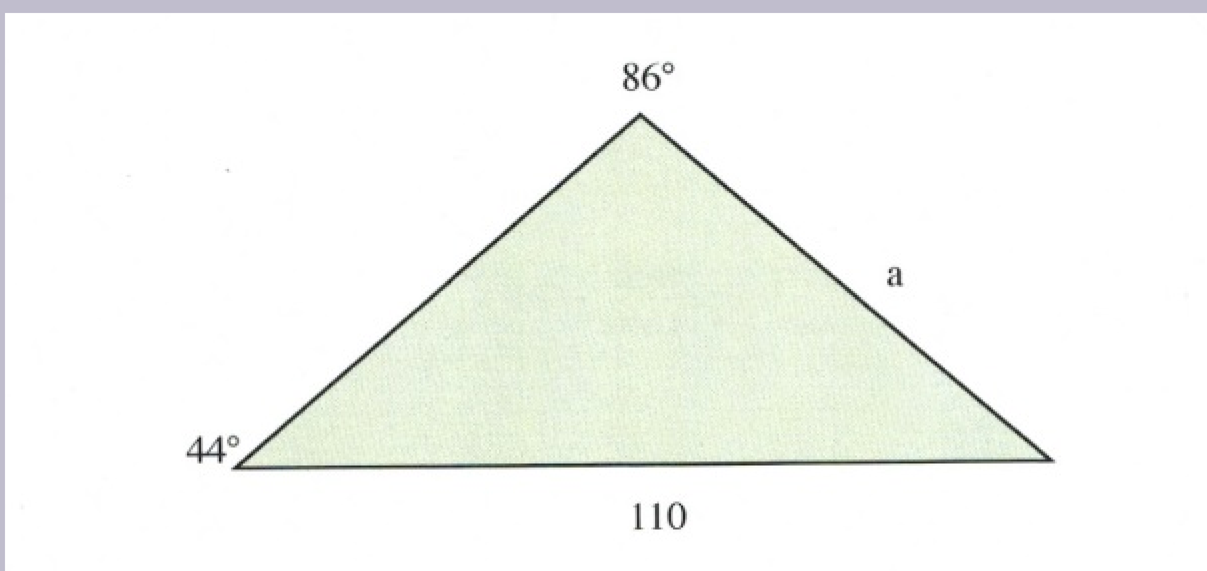
Multiplying both sides of this by sin 120°, we are left with:

$$x = \frac{6\ \sin\ 120°}{\sin\ 30°}$$

This leaves us with $x = 10.29$.

Example 11.8.3

Find the correct length of the side opposite 120° in the triangle shown below.



For this triangle, the ratio to solve is:

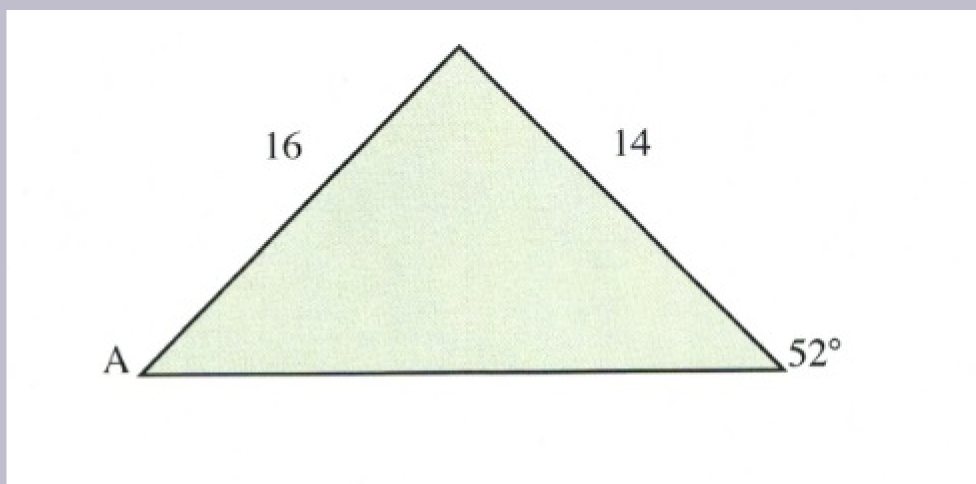
$$\frac{a}{\sin\ 44°} = \frac{110}{\sin\ 86°}$$

Multiplying both sides by sin 44° leaves us with:

$$a = \frac{110 \text{ sin } 44°}{\text{sin } 86°}$$

Example 11.8.4

Find the unknown angle shown in the triangle shown below.



For this triangle, the ratio to solve is:

$$\frac{14}{\text{sin } A} \quad = \quad \frac{16}{\text{sin } 52°}$$

Isolating sin A yields:

$$\text{sin } A = \frac{14 \text{ sin } 52°}{16}$$

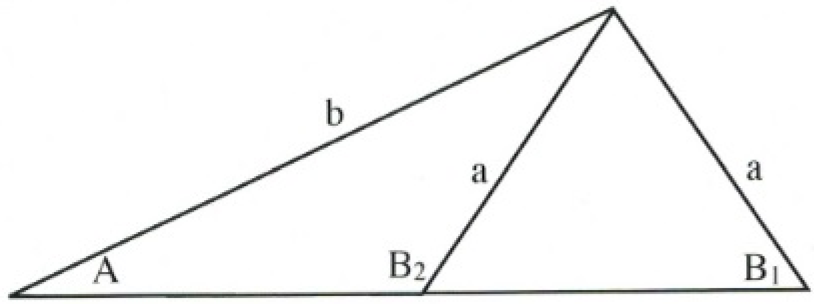
We now need to take the inverse sin of both sides to solve for A:

$$A = \sin^{-1}\left(\frac{14 \text{ sin } 52°}{16}\right)$$

$$A = 43.6°$$

The Ambiguous Case

It is possible, when given the right data, to create two different triangles.



You can see from the triangle shown above that it is possible to have two angles, B_1 and B_2, for side b. Using the sine law, you will always end up solving for B_1, the angle for the largest triangle. If you are trying to solve for the smaller triangle, then you only need to subtract B_1 from 180°.

For example, if $B_1 = 50°$, then $B_2 = 180° - B_1$. This means $B_2 = 180° - 50°$ or 130°.

If the angle you solve for when using the sine law is smaller than it should be, then correct for it as we just did above.

The Law of Cosines

The Law of Cosines is the generalized law of the Pythagorean Theorem $(a^2 + b^2 = c^2)$.

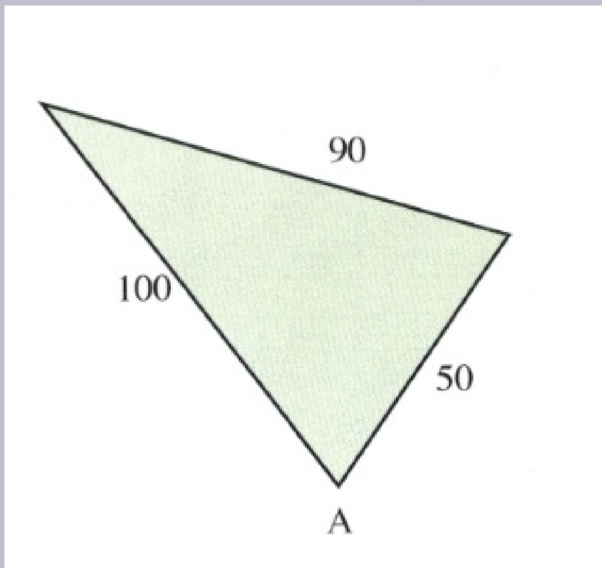
The Law of Cosines is generally written in three different forms, which are as follows:

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

All forms revert back to one of the three regular forms of the Pythagorean theorem $(a^2 = b^2 + c^2, b^2 = a^2 + c^2, c^2 = a^2 + b^2)$ if A, B or C is 90°, since $\cos 90° = 0$. The following examples illustrate the usage of the cosine law in trigonometry

Example 11.8.5

Find the unknown angle shown in the triangle shown below.



For this triangle:

$$A = \text{find}$$

$$a = 90$$

$$b = 50$$

$$c = 100$$

Note that b and c could be switched around. Now, to solve:

$$\begin{array}{rclclcl}
a^2 & = & b^2 & + & c^2 & - & 2bc \cos A \\
\\
90^2 & = & 50^2 & + & 100^2 & - & 2(50)(100) \cos A \\
8100 & = & 2500 & + & 10000 & - & 10000 \cos A \\
-2500 & & -2500 & - & 10000 & & \\
-4400 & = & -10000 & \cos & A & &
\end{array}$$

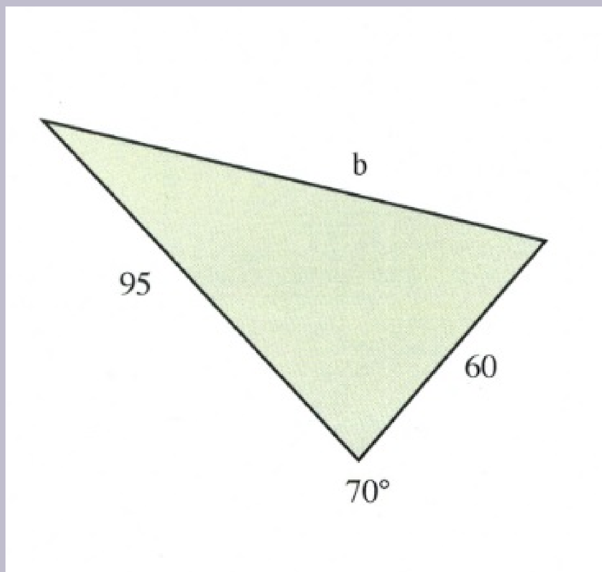
$$\cos A = \frac{-4400}{-10000}$$

$$B = \cos^{-1} 0.44$$

$$B = 63.9°$$

Example 11.8.6

Find the unknown side shown in the triangle shown below.



For this triangle:

$$B = 70°$$

$$a = 95$$

$$b = \text{find}$$

$$c = 60$$

Note that b and c could be switched around. Now, to solve:

$$b^2 = a^2 + c^2 - 2ac\ \cos\ B$$

$$b^2 = 95^2 + 60^2 - 2(95)(60)\ \cos\ 70°$$
$$b^2 = 9025 + 3600 - 11400(0.34202)$$
$$b^2 = 12625 - 3899$$
$$b^2 = 8726$$

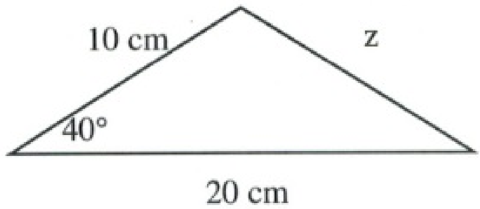
$$b = \sqrt{8726}$$
$$b = 93.4$$

Unlike with the law of sines, there should be no ambiguous cases with the law of cosines.

Questions

Solve all unknowns in the following non-right triangles using either the law of sines or cosines.

1.



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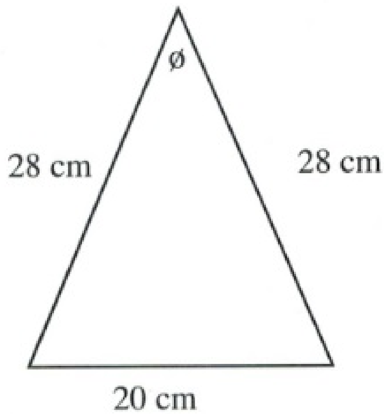

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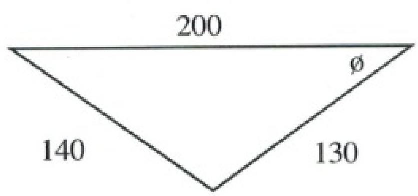

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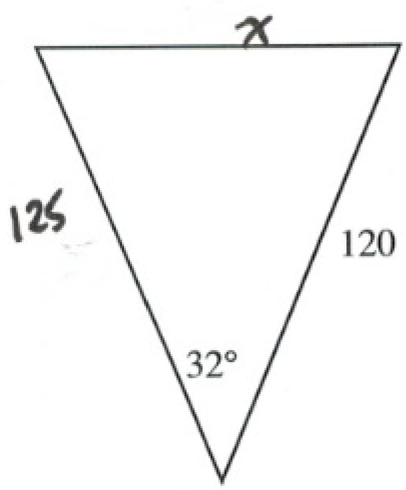

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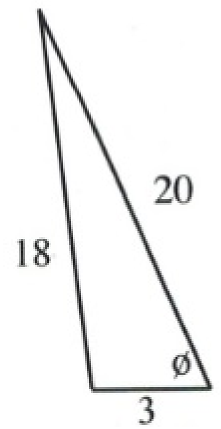

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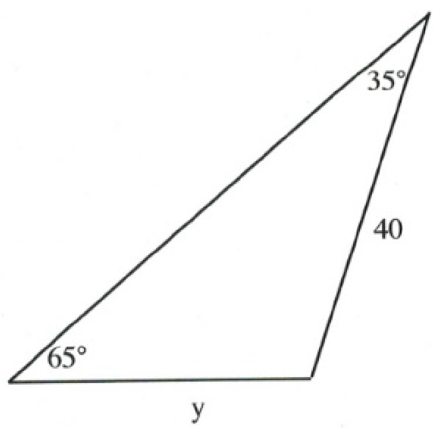

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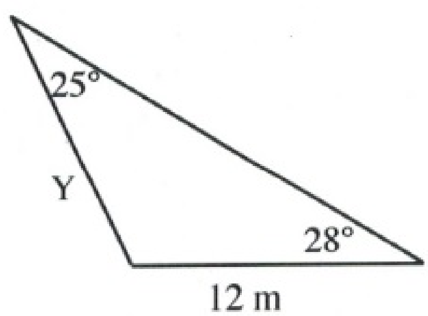

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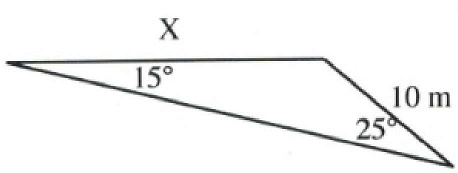

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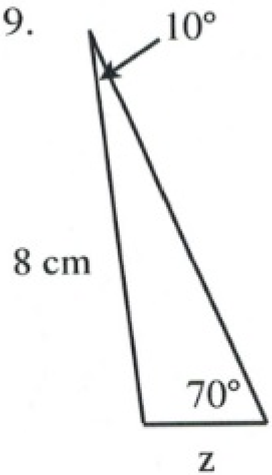

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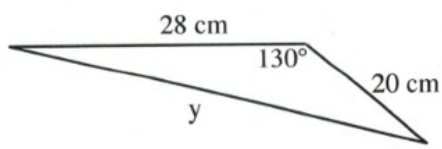

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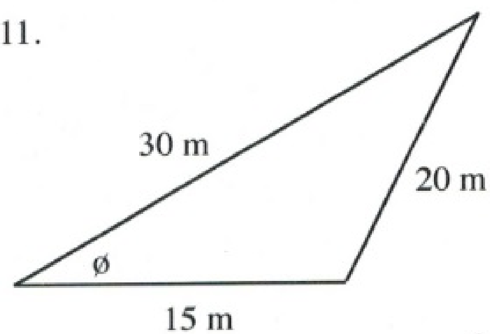

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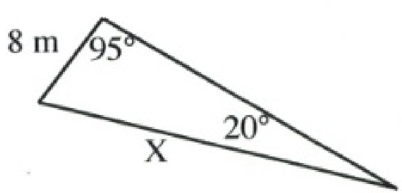

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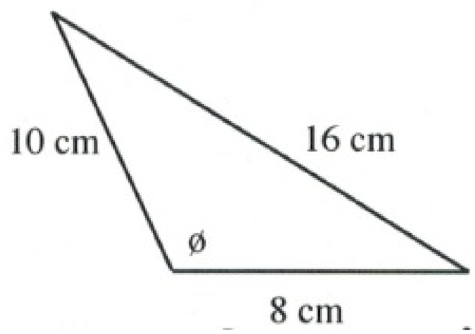

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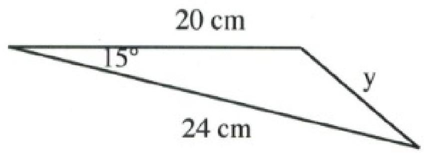

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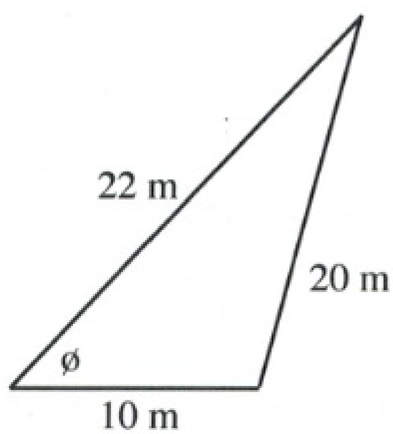

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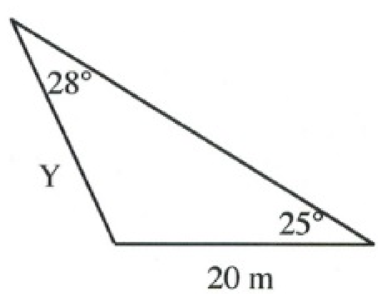

Answer Key 11.8

107. 11.9 Your Next Year's Valentine?

Fill in the data tables and draw the graphs for each. Use the suggested scaling for each axis.

1. $y = \frac{1}{x}$Scaling:
   x-axis: 1 square = 1
   y-axis: 1 square = 1

x	y
10	
8	
5	
4	
2	
1	
0.5	
0.25	
0.2	
0.125	
0.1	

2. $x^2 + y^2 = 9$Scaling:
   x-axis: 2 squares = 1
   y-axis: 2 squares = 1

x	y
3	
2.5	
2	
1.5	
1	
0.5	
0	
−0.5	
−1	
−1.5	
−2	
−2.5	
−3	

3. $y = |-2x|$Scaling:

x-axis: 1 square = 1
y-axis: 1 square = 1

x	y
5	
4	
3	
2	
1	
0	
−1	
−2	
−3	
−4	
−5	

4. $x = -3|\sin\, y|$Scaling:
x-axis: 2 squares = 1
y-axis: 1 square = 30°

x	y
	180°
	150°
	120°
	90°
	60°
	30°
	0°
	−30°
	−60°
	−90°
	−120°
	−150°
	−180°

Answer Key 11.9

PART XV

FINAL EXAM PREPARATION

This chapter contains several sections that will help with preparation for the final exam:

Final Exam Composition
Final Exam Sample A
Final Exam Sample B

Final Exam: Composition

The final exam will be composed of thirty questions covering chapters 0 to 9. Twenty-four questions are algebra questions, and six questions are word problems.

Part One: Topics Covered in the First Midterm

- **Questions 1–8 will be drawn from:**
  - Chapter 1: Algebra Review
    - 1.1 Integers
    - 1.2 Fractions
    - 1.3 Order of Operations
    - 1.4 Properties of Algebra
    - 1.5 Terms and Definitions
  - Chapter 2: Linear Equations
    - 2.1 Elementary Linear Equations
    - 2.2 Solving Linear Equations
    - 2.3 Intermediate Linear Equations
    - 2.4 Fractional Linear Equations
    - 2.5 Absolute Value Equations
    - 2.6 Working with Formulae
  - Chapter 3: Graphing
    - 3.1 Points and Coordinates
    - 3.2 Midpoint and Distance Between Points
    - 3.3 Slopes and Their Graphs
    - 3.4 Graphing Linear Equations
    - 3.5 Constructing Linear Equations

        - 3.6 Perpendicular and Parallel Lines
    - Chapter 4: Inequalities
        - 4.1 Solve and Graph Linear Inequalities
        - 4.2 Compound Inequalities
        - 4.3 Linear Absolute Value Inequalities
        - 4.4 2D Inequality and Absolute Value Graphs
- **Questions 9 and 10 will be drawn from:**
    - 1.6 Unit Conversion Word Problems
    - 2.7 Variation Word Problems
    - 3.7 Numeric Word Problems
    - 4.5 Geometric Word Problems

Part Two: Topics Covered in the Second Midterm

- **Questions 11–18 will be drawn from:**
    - Chapter 5: Systems of Equations
        - 5.1 Graphed Solutions
        - 5.2 Substitution Solutions
        - 5.3 Addition and Subtraction Solutions
        - 5.4 Solving for Three Variables
    - Chapter 6: Polynomials
        - 6.1 Working with Exponents
        - 6.2 Negative Exponents
        - 6.3 Scientific Notation
        - 6.4 Basic Operations Using Polynomials
        - 6.5 Multiplication of Polynomials
        - 6.6 Special Products
        - 6.7 Dividing Polynomials
    - Chapter 7: Factoring
        - 7.1 Greatest Common Factor
        - 7.2 Factoring by Grouping
        - 7.3 Factoring Trinomials where $a = 1$
        - 7.4 Factoring Trinomials where $a \neq 1$
        - 7.5 Factoring Special Products
        - 7.6 Factoring Quadratics of Increasing Difficulty
        - 7.7 Choosing the Correct Factoring Strategy
        - 7.8 Solving Quadratic Equations by Factoring
- **Questions 19 and 20 will be drawn from:**
    - 5.5 Monetary Word Problems
    - 6.8 Mixture and Solution Word Problems
    - 7.9 Age Word Problems

Part Three: Topics Covered in the Third Midterm

- **Questions 21–28 will be drawn from:**
    - Chapter 8: Rational Expressions
        - 8.1 Reducing Rational Expressions
        - 8.2 Multiplication and Division of Rational Expressions
        - 8.3 Least Common Denominators
        - 8.4 Addition and Subtraction of Rational Expressions
        - 8.5 Reducing Complex Fractions
        - 8.6 Solving Complex Fractions
        - 8.7 Solving Rational Equations
    - Chapter 9: Radicals
        - 9.1 Reducing Square Roots
        - 9.2 Reducing Higher Power Roots
        - 9.3 Adding and Subtracting Radicals
        - 9.4 Multiplication and Division of Radicals
        - 9.5 Rationalizing Denominators
        - 9.6 Radicals and Rational Exponents
        - 9.8 Radicals of Mixed Index
        - 9.9 Complex Numbers
    - Chapter 10: Quadratics
        - 10.1 Solving Radical Equations
        - 10.2 Solving Exponential Equations
        - 10.3 Completing the Square
        - 10.4 The Quadratic Formula
        - 10.5 Solving Quadratic Equations Using Substitution
        - 10.6 Graphing Quadratic Equations—Vertex and Intercept Method
        - 10.8 Construct a Quadratic Equation from its Roots
- **Questions 29 and 30 will be drawn from:**
    - 8.8 Rate Word Problems: Speed, Distance and Time
    - 9.10 Rate Word Problems: Work and Time
    - 10.7 Quadratic Word Problems: Age and Numbers

Students will be allowed to use MATQ 1099 Data Booklets & Glossaries for both Midterms and Final Exam.

108. Final Exam: Version A

Questions from Chapters 1 to 3

1. Evaluate $-b - \sqrt{b^2 - 4ac}$ if $a = 4, b = 6$ and $c = 2$.

For problems 2 and 3, solve for x.

2. $6(x+4) = 5(7-x) - 4(2-3x)$
3. $\dfrac{x+4}{2} - \dfrac{1}{2} = \dfrac{x+2}{4}$
4. Write an equation of the vertical line that passes through the point (−2, −3).
5. Find the distance between the points (−4, −2) and (2, 6).
6. Graph the relation $2x - 3y = 6$.

For problems 7 and 8, find the solution set and graph it.

7. $x - 2(x-5) \leq 3(6+x)$
8. $\left|\dfrac{3x-2}{7}\right| < 1$

In problems 9 and 10, set up each problem algebraically and solve. Be sure to state what your variables represent.

9. The time (t) required to empty a tank varies inversely to the rate of pumping (r). If a pump can empty a tank in 45 minutes at the rate of 600 kL/min, how much time will it take the pump to empty the same tank at the rate of 1000 kL/min?
10. Find two consecutive odd integers such that their sum is 12 less than four times the first integer.

Questions from Chapters 4 to 6

For problems 1–3, find the solution set of each system by any convenient method.

1. $\begin{cases} 2x + 5y = -18 \\ y - 6 = 2x \end{cases}$
2. $\begin{cases} 8x + 7y = 51 \\ 5x + 2y = 20 \end{cases}$
3. $\begin{cases} x + y + 6z = 5 \\ 2x - 3z = 4 \\ 3y + 4z = 9 \end{cases}$

For problems 4–6, perform the indicated operations and simplify.

4. $24 + \{-3x - [6x - 3(5-2x)]^0\} + 3x$

5. $2ab^3(a-4)(a+4)$
6. $\left(\dfrac{xy^{-3}}{x^{-2}y^4}\right)^{-1}$

For problems 7 and 8, factor each expression completely.

7. $3x^2+11x+8$
8. $64x^3-y^3$
9. A 50 kg mixture of two different grades of coffee costs $\$191.25.$ If grade A is worth $\$3.95$ per kg and grade B is worth $\$3.70$ per kg, how many kg of each type were used?
10. Kyra gave her brother Mark a logic question to solve: If she has 16 coins in her pocket worth $\$2.35,$ and if the coins are only dimes and quarters, how many of each kind of coin does she have?

Questions from Chapters 7 to 9

In problems 1–3, perform the indicated operations and simplify.

1. $\dfrac{15s^3}{3t^2} \div \dfrac{5t}{17s^3} \div \dfrac{34s^4}{3t^3}$
2. $\dfrac{2x}{x-2} - \dfrac{4x}{x-2} + \dfrac{20}{x^2-4}$
3. $\dfrac{\dfrac{x^2}{y^2}-9}{\dfrac{x+3y}{y^3}}$

For questions 4–6, simplify each expression.

4. $3\sqrt{25x}-2\sqrt{72x}-\sqrt{16x^3}$
5. $\dfrac{\sqrt{m^6n}}{\sqrt{3n}}$
6. $\left(\dfrac{a^0b^4}{c^8d^{-12}}\right)^{\frac{1}{4}}$

For questions 7 and 8, solve x by any convenient method.

7. $x^2-4x-5=0$
8. $\dfrac{x-3}{x}=\dfrac{x}{x-3}$

In problems 9 and 10, find the solution set of each system by any convenient method.

9. The base of a right triangle is 6 cm longer than its height. If the area of this triangle is 20 cm^2, find the length of both the base and the height.

10. Find three consecutive even integers such that the product of the first two is 8 more than six times the third number.

Final Exam: Version A Answer Key

109. Final Exam: Version B

Questions from Chapters 1 to 3

1. Evaluate $-2b - \sqrt{b^2 - 4ac}$ if $a = 4$, $b = -3$ and $c = -1$.

For problems 2 and 3, solve for x.

2. $6(3x-5) = 3\left[4(1-x)-7\right]$
3. $\dfrac{x+4}{2} - \dfrac{1}{3} = \dfrac{x+2}{6}$
4. Find the equation that has a slope of $\dfrac{2}{3}$ and passes through the point (1, 4).
5. Find the distance between the points (−4, −2) and (4, 4).
6. Graph the relation $3x - 2y = 6$.

For problems 7 and 8, find the solution set and graph it.

7. $3 \leq 6x + 3 < 9$
8. $\left|\dfrac{3x+1}{4}\right| = 2$

In problems 9 and 10, set up each problem algebraically and solve. Be sure to state what your variables represent.

9. The weight (w_m) of an object on Mars varies directly with its weight (w_e) on Earth. A person who weighs 95 lb on Earth weighs 38 lb on Mars. How much would a 240 lb person weigh on Mars?
10. Find two consecutive even integers such that their sum is 20 less than the second integer.

Questions from Chapters 4 to 6

For problems 1–3, find the solution set of each system by any convenient method.

1. $\begin{cases} 4x - 3y = 13 \\ 6x + 5y = -9 \end{cases}$
2. $\begin{cases} 3x - 4y = -5 \\ x + y = -1 \end{cases}$
3. $\begin{cases} x + 2y = 0 \\ y - 2z = 0 \\ x - 4z = 0 \end{cases}$

For problems 4–6, perform the indicated operations and simplify.

4. $28 - \{5x^0 - [6x - 3(5-2x)]^0\} + 5x^0$

5. $(x^2 - 3x + 8)(x - 4)$
6. $\left(\dfrac{x^{3n}x^{-6}}{x^{3n}}\right)^{-1}$

For problems 7 and 8, factor each expression completely.

7. $25y^3 - 15y^2 + 5y$
8. $x^3 + 8y^3$
9. How many litres of club soda (carbonated water) must be added to 2 litres of 35% fruit juice to turn it into a carbonated drink diluted to 8% fruit juice?
10. Kyra has 14 coins with a total value of $\$1.85$. If all the coins are dimes and quarters, how many of each kind of coin does she have?

Questions from Chapters 7 to 9

In problems 1–3, perform the indicated operations and simplify.

1. $\dfrac{9s^2}{7y^3} \cdot \dfrac{15t}{13s^2} \cdot \dfrac{26s}{9t}$
2. $\dfrac{2a}{a^2 - 36} - \dfrac{5}{a^2 - 7a + 6}$
3. $\dfrac{1 - \dfrac{8}{x}}{\dfrac{3}{x} - \dfrac{24}{x^2}}$

For questions 4–6, simplify each expression.

4. $\sqrt{x^5y^7} + 2xy\sqrt{16xy^3} - \sqrt{xy^3}$
5. $\dfrac{2 + x}{1 - \sqrt{7}}$
6. $\left(\dfrac{a^6b^3}{c^0d^{-9}}\right)^{\frac{2}{3}}$

For questions 7 and 8, solve x by any convenient method.

7. $x^2 - 2x - 15 = 0$
8. $\dfrac{2x - 1}{3x} = \dfrac{x - 3}{x}$

In problems 9 and 10, find the solution set of each system by any convenient method.

9. The length of a rectangle is 5 cm longer than twice the width. If the area of the rectangle is 75 cm^2, find its length and width.
10. Find three consecutive odd integers such that the product of the first and the second is 25 less than 8 times the

third.

Final Exam: Version B

This is where you can add appendices or other back matter.

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Common Powers
(and not so common)

Squares Cubes 4th Power 5th Power 6th Power 7th Power
22 = 4 23 = 8 24 = 16 25 = 32 26 = 64 27 = 128
32 = 9 33 = 27 34 = 81 35 = 243 36 = 729 37 = 2187
42 = 16 43 = 64 44 = 256 45 = 1024 46 = 4096 47 = 16384
52 = 25 53 = 125 54 = 625 55 = 3125 56 = 15625 57 = 78125
62 = 36 63 = 216 64 = 1296 65 = 7776 66 = 46656 67 = 279936
72 = 49 73 = 343 74 = 2401 75 = 16807 76 = 117649 77 = 823543
82 = 64 83 = 512 84 = 4096 85 = 32768 86 = 262144 87 = 2097152
92 = 81 93 = 729 94 = 6561 95 = 59049 96 = 531441 97 = 4782969
102 = 100 103 = 1000 104 = 10000 105 = 100000 106 = 1000000 107 = 10000000

112 = 121 122 = 144 132 = 169 142 = 196 152 = 225 202 = 400

Greek Alphabet

A α Alpha N ν Nu
B β Beta Ξ ξ Xi
Γ γ Gamma O ο Omicron
Δ δ Delta ∏ π Pi
E ε Epsilon P ρ Rho
Z ζ Zeta ∑ σ Sigma
H η Eta T τ Tau
Θ θ Theta Y ν Upsilon
I i Iota Φ φ Phi

K k Kappa X χ Chi
Λ λ Lambda Ψ ψ Psi
M μ Mu Ω ω Omega

SI Unit Prefixes

Factor Name Symbol Factor Name Symbol
10-18 atto a 10-1 deci d
10-15 femto f 10 deca da
10-12 pico p 102 hecto h
10-9 nano n 103 kilo k
10-6 micro μ 106 mega M
10-3 milli m 109 giga G
10-2 centi c 1012 tera T

Linear Inequalities

Interval Notation Set Builder Notation Graph of the Inequality
(a, + ∞) {x | x > a}
[a, + ∞) {x | x ≥ a}
(- ∞, a) {x | x < a}
(- ∞, a] {x | x ≤ a}
[a, b] {x | a ≤ x ≤ b}
(a, b) {x | a < x < b}
[a, b) {x | a ≤ x < b}
(a, b] {x | a < x ≤ b}
(- ∞, + ∞) {x | x ∈ R}
(- ∞, b) or (a, + ∞) {x | x < a or x > b}
(- ∞, a] or [a, + ∞) {x | x < a or x > b}
(- ∞, a] or [a, + ∞) {x | x < a or x > b}
(- ∞, a] or [a, + ∞) {x | x < a or x > b}

Properties of Absolute Values

If | X | = k, then X = k or X = -k
If | X | < k, then -k < X < k
If | X | > k, then X > k or X < -kMetric to English (US) Conversions

Distance:
12 in = 1 ft
3 ft = 1 yd 10 mm = 1 cm
1760 yds = 1 mi 100 cm = 1 m
5280 ft = 1 mi 1000m = 1 km
(English-Metric conversions: 1 inch = 2.54 cm; 1 mile = 1.61 km)

Area:
144 in2 = 1 ft2 10,000 cm2 = 1 m2
43,560 ft2 = 1 acre 10,000 m2 = 1 hectare
640 acres = 1 mi2 100 hectare = 1 km2
(English-Metric conversions: 1 in2 = 6.45 cm2; 1 mi2 = 2.59 km2)

Volume:
57.75 in3 = 1 qt 1 cm3 = 1 ml
4 qt = 1 gal 1000 ml = 1 liter
42 gal (petroleum) = 1 barrel 1000 liter = 1 m3
(English-Metric conversions: 16.39 cm3 = 1 in3; 3.79 liters = 1 gal)

Mass:

437.5 grains = 1 oz 1000 mg = 1 g

16 oz = 1 lb 1000 g = 1 kg

2000 lb = 1 short ton 1000 kg = 1 metric ton

(English-Metric conversions: 453 g = 1 lb; 2.2 lb = 1 kg)

Temperature:

(Fahrenheit – Celsius Conversions: °C = 5/9 (°F – 32) and °F = 9/5 °C + 32)

Plane Geometry Formula

Circle Square Rectangle

Area = πr^2 Area = s^2 Area = l w

Perimeter = $2 \pi r$ Perimeter = 4 s Perimeter = 2l + 2w

Triangle Rhombus Trapezoid

Area = 1/2 b h Area = b h Area = $1/2 (l_1 + l_2) h$

Perimeter = $s_1 + s_2 + s_3$ Perimeter = 4 b Perimeter = $l_1 + l_2 + h_1 + h_2$

Parallelogram Regular Polygon (n-gon)

Area = b h Area = (1/2 s h) (number of sides)

Perimeter = $2 h_1 + 2 b$ Perimeter = s (number of sides)

Solid Geometry Formula

Cube Right Rectangular Prism Right Cylindrical Prism

Volume = s^3 Volume = l w h Volume = $\pi r^2 h$

S. A. = $6 s^2$ S. A. = 2 l w + 2 h w + 2 l h S. A. = $2 \pi r h + 2 \pi r^2$

Sphere Torus Right Triangular Prism

Volume = $4/3 \pi r^3$ Volume = $2 \pi^2 r^2 R$ Volume = (1/2 b h) l

S. A. = $4 \pi r^2$ S. A. = $4 \pi^2 r R$ S. A. = b h + 2 l s + l b

Right Circular Cone General Cone/Pyramid Square Pyramid

Volume = $1/3 (\pi r^2) h$ Volume = 1/3 (base area) h Volume = $1/3 (s^2) h$

S. A. = $\pi r (r^2 + h^2)^{1/2} + \pi r^2$ S.A. = $s [s + (s^2 + 4h^2)$

Pythagorean Theorem (Variations)

For any right triangle a, b and c:

$a^2 + b^2 = c^2$

For any non-right triangle a, b and c:

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

For any rectangular prism a, b and c, the diagonal (d) length is:

$d^2 = a^2 + b^2 + c^2$

Linear Equations

An Ordered Pair: (x, y)

Distance between Two Ordered Pairs: $d^2 = \Delta x^2 + \Delta y^2$ or $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Midpoint between Two Ordered Pairs: $[(x_1 + x_2) , (y_1 + y_2)]$

2 2

Slope: $m = \Delta y$ or $m = (y_2 - y_1)$ … where $\Delta y = y_2 - y_1$, $\Delta x = x_2 - x_1$ and $\Delta x \neq 0$

Δx $(x_2 - x_1)$

The slope for Two Parallel Lines: $m_1 = m_2$

The slope for Two Perpendicular Lines: $m_1 . m_2 = -1$ or $m_1 = -1/m_2$

To find the Linear Equation Using Two Ordered Pairs: $(x_2 - x_1) m = (y_2 - y_1)$

General Form of a Linear Equation: Ax + By + C = 0 (A, B, C are integers, A is positive)

Slope Intercept Form of a Linear Equation: y = mx + b

Conic Sections

Conic Equations (Standard Form):

Circle: $(x - h)2 + (y - k)2 = r2$ (h, k) is the center point, r is the radius from the center to the circles (x, y) coordinates

Parabolas: $y - k = a(x - h)2$ Parabolas, commonly written as $y = ax2 + bx + c$
$x - h = a(y - k)2$

Ellipse: $(x - h)2 + (y - k)2 = 1$ (h, k) is the center point, rx is the radius length in rx2 ry2 the ± x direction, ry is the radius length in the ± y direction

Hyperbola: $(x - h)2 - (y - k)2 = 1$ (h, k) is the center point, rx is the distance from the rx2 ry2 center to the hyperbola's ± x asymptote. ry is the distance from the center to the hyperbola's ± x asymptote.Polynomials

Quadratic Solutions:

The solution for x from a quadratic equation $ax2 + bx + c = 0$, (where $a \neq 0$), can be found from:

Factoring:

$a2 - b2 = (a + b)(a - b)$ $a2 + b2$ … cannot be factored

$a3 - b3 = (a - b)(a2 + ab + b2)$ $a3 + b3 = (a + b)(a2 - ab + b2)$

Binomial Expansions:

$(a + b)0 = 1$ $(a - b)0 = 1$

$(a + b)1 = a + b$ $(a - b)1 = a - b$

$(a + b)2 = a2 + 2ab + b2$ $(a - b)2 = a2 - 2ab + b2$

$(a + b)3 = a3 + 3a2b + 3ab2 + b3$ $(a - b)3 = a3 - 3a2b + 3ab2 - b3$

$(a + b)4 = a4 + 4a3b + 6a2b2 + 4ab3 + b4$ $(a - b)4 = a4 - 4a3b + 6a2b2 - 4ab3 + b4$

$(a + b)5 = a5 + 5a4b + 10a3b2 + 10a2b3 + 5ab4 + b5$

$(a - b)5 = a5 - 5a4b + 10a3b2 - 10a2b3 + 5ab4 - b5$

$(a + b)6 = a6 + 6a5b + 15a4b2 + 20a3b3 + 15a2b4 + 6ab5 + b5$

$(a - b)6 = a6 - 6a5b + 15a4b2 - 20a3b3 + 15a2b4 - 6ab5 + b5$

Properties of Complex Numbers

$(a + bi) + (c + di) = a + c + (b + d)i$ $(a + bi) - (c + di) = a - c + (b - d)i$

$(a + bi) (c + di) = ac - bd + (ab + bd)i$ $(a + bi) (a - bi) = a2 + b2$

$(-a)1/2 = i(a)1/2, a \geq 0$

Properties of Exponents

Properties of Rational Exponents and Radicals

Basic Trigonometric Functions & Values

Basic Trigonometric Ratios

Sin = Opposite Cos = Adjacent Tan = Opposite
Hypotenuse Hypotenuse Adjacent

Sec = Hypotenuse Csc = Hypotenuse Cot = Adjacent
Opposite Adjacent Opposite

Trigonometric Identities

Reciprocal Identities:

$\sin \theta = 1/\csc \theta$ $\tan \theta = 1/\cot \theta$ $\cos \theta = 1/\sec \theta$

$\csc \theta = 1/\sin \theta$ $\cot \theta = 1/\tan \theta$ $\sec \theta = 1/\cos \theta$

Tangent and Cotangent Identities:

$\tan \theta = \sin \theta / \cos \theta$ $\cot \theta = \cos \theta / \sin \theta$

Pythagorean Identities:

$\sin2 \theta + \cos2 \theta =1$ $\tan2 \theta + 1 = \sec2 \theta$ $1 + \cot2 \theta = \csc2 \theta$

Double Angle Formulas:

$\sin 2\theta = 2 \sin \theta \cos \theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\cos 2\theta = 2\cos^2\theta - 1$ $\cos 2\theta = 1 - 2\sin^2\theta$

$\tan 2\theta = 2\tan\theta / 1 - \tan^2\theta$

Sum and Difference Formulas:

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha + \beta) = \tan\alpha + \tan\beta / 1 - \tan\alpha\tan\beta$ $\tan(\alpha - \beta) = \tan\alpha - \tan\beta / 1 + \tan\alpha\tan\beta$

Graphs of Basic Trigonometric Ratios

Trigonometric Tables

Angle	Sin	Cos	Tan	Csc	Angle	Sin	Cos	Tan	Csc
1	0.017	1.000	0.017	57.299	46	0.719	0.695	1.036	1.390
2	0.035	0.999	0.035	28.654	47	0.731	0.682	1.072	1.36
3	0.052	0.999	0.052	19.107	48	0.743	0.669	1.111	1.346
4	0.070	0.998	0.070	14.336	49	0.755	0.656	1.150	1.325
5	0.087	0.996	0.087	11.474	50	0.766	0.643	1.192	1.305
6	0.105	0.995	0.105	9.567	51	0.777	0.629	1.235	1.287
7	0.122	0.993	0.123	8.206	52	0.788	0.616	1.280	1.269
8	0.139	0.990	0.141	7.185	53	0.799	0.602	1.327	1.252
9	0.156	0.988	0.158	6.392	54	0.809	0.588	1.376	1.236
10	0.174	0.985	0.176	5.759	55	0.819	0.574	1.428	1.221
11	0.191	0.982	0.194	5.241	56	0.829	0.559	1.483	1.206
12	0.208	0.978	0.213	4.810	57	0.839	0.545	1.540	1.192
13	0.225	0.974	0.231	4.445	58	0.848	0.530	1.600	1.179
14	0.242	0.970	0.249	4.134	59	0.857	0.515	1.664	1.167
15	0.259	0.966	0.268	3.864	60	0.866	0.500	1.732	1.155
16	0.276	0.961	0.287	3.628	61	0.875	0.485	1.804	1.143
17	0.292	0.956	0.306	3.420	62	0.883	0.469	1.881	1.133
18	0.309	0.951	0.325	3.236	63	0.891	0.454	1.963	1.122
19	0.326	0.946	0.344	3.072	64	0.899	0.438	2.050	1.113
20	0.342	0.940	0.364	2.924	65	0.906	0.423	2.145	1.103
21	0.358	0.934	0.384	2.790	66	0.914	0.407	2.246	1.095
22	0.375	0.927	0.404	2.669	67	0.921	0.391	2.356	1.086
23	0.391	0.921	0.424	2.559	68	0.927	0.375	2.475	1.079
24	0.407	0.914	0.445	2.459	69	0.934	0.358	2.605	1.071
25	0.423	0.906	0.466	2.366	70	0.940	0.342	2.747	1.064
26	0.438	0.899	0.488	2.281	71	0.946	0.326	2.904	1.058
27	0.454	0.891	0.510	2.203	72	0.951	0.309	3.078	1.051
28	0.469	0.883	0.532	2.130	73	0.956	0.292	3.271	1.046
29	0.485	0.875	0.554	2.063	74	0.961	0.276	3.487	1.040
30	0.500	0.866	0.577	2.000	75	0.966	0.259	3.732	1.035
31	0.515	0.857	0.601	1.942	76	0.970	0.242	4.011	1.031
32	0.530	0.848	0.625	1.887	77	0.974	0.225	4.331	1.026
33	0.545	0.839	0.649	1.836	78	0.978	0.208	4.705	1.022
34	0.559	0.829	0.675	1.788	79	0.982	0.191	5.145	1.019
35	0.574	0.819	0.700	1.743	80	0.985	0.174	5.671	1.015
36	0.588	0.809	0.727	1.701	81	0.988	0.156	6.314	1.012
37	0.602	0.799	0.754	1.662	82	0.990	0.139	7.115	1.010
38	0.616	0.788	0.781	1.624	83	0.993	0.122	8.144	1.008
39	0.629	0.777	0.810	1.589	84	0.995	0.105	9.514	1.006

40	0.643	0.766	0.839	1.556	85	0.996	0.087	11.430	1.004
41	0.656	0.755	0.869	1.524	86	0.998	0.070	14.301	1.002
42	0.669	0.743	0.900	1.494	87	0.999	0.052	19.081	1.001
43	0.682	0.731	0.933	1.466	88	0.999	0.035	28.636	1.001
44	0.695	0.719	0.966	1.440	89	1.000	0.017	57.290	1.000
45	0.707	0.707	1.000	1.414	90	1.000	0.000		1.000

Properties of Logarithmic Functions

$x = a^y$ is equivalent to $y = \log_a x$ $e^x = y$ is equivalent to $\ln y = x$

$\log_a (xy) = \log_a x + \log_a y$ $\log_a (x/y) = \log_a x - \log_a y$ $\log_a (1/x) = -\log_a x$

$\ln (xy) = \ln x - \ln y$ $\ln (x/y) = \ln x - \ln y$ $\ln (1/x) = -\ln x$

$\log_a x = \log x / \log a$ $\log_a a = 1$ $\log_a 1 = 0$ $\log_a x^y = y \log_a x$

$\log_a x = \ln x / \ln a$ $\ln e = 1$ $\ln 1 = 0$ $\ln x^y = y \ln x$

Common Logarithm Table

N	0	1	2	3	4	5	6	7	8	9
1.0	0.0000	0.0043	0.0086	0.0128	0.0170	0.0212	0.0253	0.0294	0.0334	0.0374
1.1	0.0414	0.0453	0.0492	0.0531	0.0569	0.0607	0.0645	0.0682	0.0719	0.0755
1.2	0.0792	0.0828	0.0864	0.0899	0.0934	0.0969	0.1004	0.1038	0.1072	0.1106
1.3	0.1139	0.1173	0.1206	0.1239	0.1271	0.1303	0.1335	0.1367	0.1399	0.1430
1.4	0.1461	0.1492	0.1523	0.1553	0.1584	0.1614	0.1644	0.1673	0.1703	0.1732
1.5	0.1761	0.1790	0.1818	0.1847	0.1875	0.1903	0.1931	0.1959	0.1987	0.2014
1.6	0.2041	0.2068	0.2095	0.2122	0.2148	0.2175	0.2201	0.2227	0.2253	0.2279
1.7	0.2304	0.2330	0.2355	0.2380	0.2405	0.2430	0.2455	0.2480	0.2504	0.2529
1.8	0.2553	0.2577	0.2601	0.2625	0.2648	0.2672	0.2695	0.2718	0.2742	0.2765
1.9	0.2788	0.2810	0.2833	0.2856	0.2878	0.2900	0.2923	0.2945	0.2967	0.2989
2.0	0.3010	0.3032	0.3054	0.3075	0.3096	0.3118	0.3139	0.3160	0.3181	0.3201
2.1	0.3222	0.3243	0.3263	0.3284	0.3304	0.3324	0.3345	0.3365	0.3385	0.3404
2.2	0.3424	0.3444	0.3464	0.3483	0.3502	0.3522	0.3541	0.3560	0.3579	0.3598
2.3	0.3617	0.3636	0.3655	0.3674	0.3692	0.3711	0.3729	0.3747	0.3766	0.3784
2.4	0.3802	0.3820	0.3838	0.3856	0.3874	0.3892	0.3909	0.3927	0.3945	0.3962
2.5	0.3979	0.3997	0.4014	0.4031	0.4048	0.4065	0.4082	0.4099	0.4116	0.4133
2.6	0.4150	0.4166	0.4183	0.4200	0.4216	0.4232	0.4249	0.4265	0.4281	0.4298
2.7	0.4314	0.4330	0.4346	0.4362	0.4378	0.4393	0.4409	0.4425	0.4440	0.4456
2.8	0.4472	0.4487	0.4502	0.4518	0.4533	0.4548	0.4564	0.4579	0.4594	0.4609
2.9	0.4624	0.4639	0.4654	0.4669	0.4683	0.4698	0.4713	0.4728	0.4742	0.4757
3.0	0.4771	0.4786	0.4800	0.4814	0.4829	0.4843	0.4857	0.4871	0.4886	0.4900

N	0	1	2	3	4	5	6	7	8	9
3.1	0.4914	0.4928	0.4942	0.4955	0.4969	0.4983	0.4997	0.5011	0.5024	0.5038
3.2	0.5051	0.5065	0.5079	0.5092	0.5105	0.5119	0.5132	0.5145	0.5159	0.5172
3.3	0.5185	0.5198	0.5211	0.5224	0.5237	0.5250	0.5263	0.5276	0.5289	0.5302
3.4	0.5315	0.5328	0.5340	0.5353	0.5366	0.5378	0.5391	0.5403	0.5416	0.5428
3.5	0.5441	0.5453	0.5465	0.5478	0.5490	0.5502	0.5514	0.5527	0.5539	0.5551
3.6	0.5563	0.5575	0.5587	0.5599	0.5611	0.5623	0.5635	0.5647	0.5658	0.5670
3.7	0.5682	0.5694	0.5705	0.5717	0.5729	0.5740	0.5752	0.5763	0.5775	0.5786
3.8	0.5798	0.5809	0.5821	0.5832	0.5843	0.5855	0.5866	0.5877	0.5888	0.5899
3.9	0.5911	0.5922	0.5933	0.5944	0.5955	0.5966	0.5977	0.5988	0.5999	0.6010
4.0	0.6021	0.6031	0.6042	0.6053	0.6064	0.6075	0.6085	0.6096	0.6107	0.6117
4.1	0.6128	0.6138	0.6149	0.6160	0.6170	0.6180	0.6191	0.6201	0.6212	0.6222
4.2	0.6232	0.6243	0.6253	0.6263	0.6274	0.6284	0.6294	0.6304	0.6314	0.6325
4.3	0.6335	0.6345	0.6355	0.6365	0.6375	0.6385	0.6395	0.6405	0.6415	0.6425

4.4 0.6435 0.6444 0.6454 0.6464 0.6474 0.6484 0.6493 0.6503 0.6513 0.6522
4.5 0.6532 0.6542 0.6551 0.6561 0.6571 0.6580 0.6590 0.6599 0.6609 0.6618
4.6 0.6628 0.6637 0.6646 0.6656 0.6665 0.6675 0.6684 0.6693 0.6702 0.6712
4.7 0.6721 0.6730 0.6739 0.6749 0.6758 0.6767 0.6776 0.6785 0.6794 0.6803
4.8 0.6812 0.6821 0.6830 0.6839 0.6848 0.6857 0.6866 0.6875 0.6884 0.6893
4.9 0.6902 0.6911 0.6920 0.6928 0.6937 0.6946 0.6955 0.6964 0.6972 0.6981
5.0 0.6990 0.6998 0.7007 0.7016 0.7024 0.7033 0.7042 0.7050 0.7059 0.7067
5.1 0.7076 0.7084 0.7093 0.7101 0.7110 0.7118 0.7126 0.7135 0.7143 0.7152
5.2 0.7160 0.7168 0.7177 0.7185 0.7193 0.7202 0.7210 0.7218 0.7226 0.7235
5.3 0.7243 0.7251 0.7259 0.7267 0.7275 0.7284 0.7292 0.7300 0.7308 0.7316
N 0 1 2 3 4 5 6 7 8 9
5.4 0.7324 0.7332 0.7340 0.7348 0.7356 0.7364 0.7372 0.7380 0.7388 0.7396
5.5 0.7404 0.7412 0.7419 0.7427 0.7435 0.7443 0.7451 0.7459 0.7466 0.7474
5.6 0.7482 0.7490 0.7497 0.7505 0.7513 0.7520 0.7528 0.7536 0.7543 0.7551
5.7 0.7559 0.7566 0.7574 0.7582 0.7589 0.7597 0.7604 0.7612 0.7619 0.7627
5.8 0.7634 0.7642 0.7649 0.7657 0.7664 0.7672 0.7679 0.7686 0.7694 0.7701
5.9 0.7709 0.7716 0.7723 0.7731 0.7738 0.7745 0.7752 0.7760 0.7767 0.7774
6.0 0.7782 0.7789 0.7796 0.7803 0.7810 0.7818 0.7825 0.7832 0.7839 0.7846
6.1 0.7853 0.7860 0.7868 0.7875 0.7882 0.7889 0.7896 0.7903 0.7910 0.7917
6.2 0.7924 0.7931 0.7938 0.7945 0.7952 0.7959 0.7966 0.7973 0.7980 0.7987
6.3 0.7993 0.8000 0.8007 0.8014 0.8021 0.8028 0.8035 0.8041 0.8048 0.8055
6.4 0.8062 0.8069 0.8075 0.8082 0.8089 0.8096 0.8102 0.8109 0.8116 0.8122
6.5 0.8129 0.8136 0.8142 0.8149 0.8156 0.8162 0.8169 0.8176 0.8182 0.8189
6.6 0.8195 0.8202 0.8209 0.8215 0.8222 0.8228 0.8235 0.8241 0.8248 0.8254
6.7 0.8261 0.8267 0.8274 0.8280 0.8287 0.8293 0.8299 0.8306 0.8312 0.8319
6.8 0.8325 0.8331 0.8338 0.8344 0.8351 0.8357 0.8363 0.8370 0.8376 0.8382
6.9 0.8388 0.8395 0.8401 0.8407 0.8414 0.8420 0.8426 0.8432 0.8439 0.8445
7.0 0.8451 0.8457 0.8463 0.8470 0.8476 0.8482 0.8488 0.8494 0.8500 0.8506
7.1 0.8513 0.8519 0.8525 0.8531 0.8537 0.8543 0.8549 0.8555 0.8561 0.8567
7.2 0.8573 0.8579 0.8585 0.8591 0.8597 0.8603 0.8609 0.8615 0.8621 0.8627
7.3 0.8633 0.8639 0.8645 0.8651 0.8657 0.8663 0.8669 0.8675 0.8681 0.8686
7.4 0.8692 0.8698 0.8704 0.8710 0.8716 0.8722 0.8727 0.8733 0.8739 0.8745
7.5 0.8751 0.8756 0.8762 0.8768 0.8774 0.8779 0.8785 0.8791 0.8797 0.8802
7.6 0.8808 0.8814 0.8820 0.8825 0.8831 0.8837 0.8842 0.8848 0.8854 0.8859
N 0 1 2 3 4 5 6 7 8 9
7.7 0.8865 0.8871 0.8876 0.8882 0.8887 0.8893 0.8899 0.8904 0.8910 0.8915
7.8 0.8921 0.8927 0.8932 0.8938 0.8943 0.8949 0.8954 0.8960 0.8965 0.8971
7.9 0.8976 0.8982 0.8987 0.8993 0.8998 0.9004 0.9009 0.9015 0.9020 0.9025
8.0 0.9031 0.9036 0.9042 0.9047 0.9053 0.9058 0.9063 0.9069 0.9074 0.9079
8.1 0.9085 0.9090 0.9096 0.9101 0.9106 0.9112 0.9117 0.9122 0.9128 0.9133
8.2 0.9138 0.9143 0.9149 0.9154 0.9159 0.9165 0.9170 0.9175 0.9180 0.9186
8.3 0.9191 0.9196 0.9201 0.9206 0.9212 0.9217 0.9222 0.9227 0.9232 0.9238
8.4 0.9243 0.9248 0.9253 0.9258 0.9263 0.9269 0.9274 0.9279 0.9284 0.9289
8.5 0.9294 0.9299 0.9304 0.9309 0.9315 0.9320 0.9325 0.9330 0.9335 0.9340
8.6 0.9345 0.9350 0.9355 0.9360 0.9365 0.9370 0.9375 0.9380 0.9385 0.9390
8.7 0.9395 0.9400 0.9405 0.9410 0.9415 0.9420 0.9425 0.9430 0.9435 0.9440
8.8 0.9445 0.9450 0.9455 0.9460 0.9465 0.9469 0.9474 0.9479 0.9484 0.9489
8.9 0.9494 0.9499 0.9504 0.9509 0.9513 0.9518 0.9523 0.9528 0.9533 0.9538

9.0	0.9542	0.9547	0.9552	0.9557	0.9562	0.9566	0.9571	0.9576	0.9581	0.9586
9.1	0.9590	0.9595	0.9600	0.9605	0.9609	0.9614	0.9619	0.9624	0.9628	0.9633
9.2	0.9638	0.9643	0.9647	0.9652	0.9657	0.9661	0.9666	0.9671	0.9675	0.9680
9.3	0.9685	0.9689	0.9694	0.9699	0.9703	0.9708	0.9713	0.9717	0.9722	0.9727
9.4	0.9731	0.9736	0.9741	0.9745	0.9750	0.9754	0.9759	0.9763	0.9768	0.9773
9.5	0.9777	0.9782	0.9786	0.9791	0.9795	0.9800	0.9805	0.9809	0.9814	0.9818
9.6	0.9823	0.9827	0.9832	0.9836	0.9841	0.9845	0.9850	0.9854	0.9859	0.9863
9.7	0.9868	0.9872	0.9877	0.9881	0.9886	0.9890	0.9894	0.9899	0.9903	0.9908
9.8	0.9912	0.9917	0.9921	0.9926	0.9930	0.9934	0.9939	0.9943	0.9948	0.9952
9.9	0.9956	0.9961	0.9965	0.9969	0.9974	0.9978	0.9983	0.9987	0.9991	0.9996

List of Links by Chapter for Print User

Preface

- Wallace: Elementary and Introductory Algebra : http://www.wallace.ccfaculty.org/book/Beginning_and_Intermediate_Algebra.pdf

Chapter 3

- Null Island: https://en.wikipedia.org/wiki/Null_Island
- Derivation of a Slope: https://services.math.duke.edu//education/webfeats/Slope/Slopederiv.html
- Why UPS Drivers Don't Turn Left and You Probably Shouldn't Either: http://theconversation.com/why-ups-drivers-dont-turn-left-and-you-probably-shouldnt-either-71432

Answer Key 1.1

1. −2
2. 5
3. 2
4. 2
5. −6
6. −5
7. 8
8. 0
9. −2
10. −5
11. 4
12. −7
13. 3
14. −9
15. −2
16. −9
17. −1
18. −2
19. −3
20. 2
21. −7
22. 0
23. 11
24. 9
25. −3
26. −4
27. −3
28. 4
29. 0
30. −8
31. −4
32. −35
33. −80
34. 14
35. 8
36. 6
37. −56
38. −6
39. −36
40. 63
41. −10
42. 4
43. −20
44. 27

45. −3
46. 7
47. 3
48. 2
49. 5
50. 2
51. 9
52. 7
53. −10
54. −4
55. 10
56. −8
57. 6
58. −6

Answer Key 1.2

1. $\frac{42}{12} \div \frac{6}{6} = \frac{7}{2}$
2. $\frac{25}{20} \div \frac{5}{5} = \frac{5}{4}$
3. $\frac{35}{25} \div \frac{5}{5} = \frac{7}{5}$
4. $\frac{24}{8} \div \frac{8}{8} = 3$
5. $\frac{54}{36} \div \frac{9}{9} = \frac{6}{4} \div \frac{2}{2} = \frac{3}{2}$
6. $\frac{30}{24} \div \frac{6}{6} = \frac{5}{4}$
7. $\frac{45}{36} \div \frac{9}{9} = \frac{5}{4}$
8. $\frac{36}{27} \div \frac{9}{9} = \frac{4}{3}$
9. $\frac{27}{18} \div \frac{9}{9} = \frac{3}{2}$
10. $\frac{48}{18} \div \frac{6}{6} = \frac{8}{3}$
11. $\frac{40}{16} \div \frac{8}{8} = \frac{5}{2}$
12. $\frac{48}{42} \div \frac{6}{6} = \frac{8}{7}$
13. $\frac{63}{18} \div \frac{9}{9} = \frac{7}{2}$
14. $\frac{16}{12} \div \frac{4}{4} = \frac{4}{3}$
15. $\frac{80}{60} \div \frac{20}{20} = \frac{4}{3}$
16. $\frac{72}{48} \div \frac{12}{12} = \frac{6}{4} \div \frac{2}{2} = \frac{3}{2}$
17. $\frac{72}{60} \div \frac{12}{12} = \frac{6}{5}$
18. $\frac{126}{108} \div \frac{9}{9} = \frac{14}{12} \div \frac{2}{2} = \frac{7}{6}$
19. $9 \cdot \frac{8}{9} = 8$
20. $-2 - 1 \cdot -\frac{5}{63} = \frac{5}{3}$
21. $2 \cdot -\frac{2}{9} = -\frac{4}{9}$
22. $-2 \cdot \frac{1}{3} = -\frac{2}{3}$
23. $-2 - 1 \cdot \frac{13}{84} = -\frac{13}{4}$

24. $\frac{3}{2} \cdot \frac{1}{2} = \frac{3}{4}$

25. $-\frac{63}{5} \cdot -\frac{11}{84} = \frac{33}{20}$

26. $-\frac{3}{7} \cdot -\frac{11}{8} = \frac{33}{56}$

27. $84 \cdot \frac{1}{21} = 4$

28. $-2 \cdot -\frac{9}{7} = \frac{18}{7}$

29. $\frac{21}{31} \cdot \frac{31}{42} = \frac{1}{2}$

30. $-\frac{17}{93} \cdot -\frac{31}{5} = \frac{17}{15}$

31. $21 \cdot \frac{3}{21} = 3$

32. $\frac{17}{93} \cdot -\frac{31}{5} = -\frac{17}{15}$

33. $\frac{1}{2} \cdot -\frac{7}{5} = -\frac{7}{10}$

34. $\frac{1}{2} \cdot \frac{5}{7} = \frac{5}{14}$

35. $\frac{51}{2} \cdot -\frac{0}{51} = -\frac{0}{2}$ or 0

36. $\underbrace{\frac{6}{0}}_{\text{undefined}} \cdot \frac{6}{7} = \text{no solution}$

37. $-2 \cdot \frac{4}{7} = -\frac{8}{7}$

38. $-\frac{124}{7} \cdot -\frac{5}{93} = \frac{20}{21}$

39. $-\frac{1}{9} \cdot -\frac{2}{1} = \frac{2}{9}$

40. $-2 \cdot -\frac{2}{3} = \frac{4}{3}$

41. $-\frac{3}{2} \cdot \frac{7}{13} = -\frac{21}{26}$

42. $\frac{5}{3} \cdot \frac{5}{7} = \frac{25}{21}$

43. $-1 \cdot \frac{3}{2} = -\frac{3}{2}$

44. $\frac{105}{9} \cdot -\frac{1}{63} = -\frac{5}{27}$

45. $\frac{8}{9} \cdot \frac{5}{1} = \frac{40}{9}$

46. $\frac{1}{62} \cdot -\frac{31}{5} = -\frac{1}{10}$

47. $-\frac{9}{7} \cdot \frac{5}{1} = -\frac{45}{7}$

48. $-\frac{13}{81} \cdot -\frac{81}{15} = \frac{13}{15}$

49. $-\frac{2}{9} \cdot -\frac{2}{3} = \frac{4}{27}$

50. $-\frac{4}{5} \cdot -\frac{8}{13} = \frac{32}{65}$

51. $\frac{1}{105} \cdot \frac{21}{3} = \frac{1}{15}$

52. $\frac{51}{31} \cdot \frac{31}{51} = 1$

53. $\frac{1}{3} - \frac{4}{3} = -\frac{3}{3}$ or -1

54. $\frac{1}{7} - \frac{11}{7} = -\frac{10}{7}$

55. $\frac{3}{7} - \frac{1}{7} = \frac{2}{7}$

56. $\frac{1}{3} + \frac{5}{3} = \frac{6}{3}$ or 2

57. $\frac{11}{6} + \frac{7}{6} = \frac{18}{6}$ or 3

58. $-2 - \frac{15}{8} \Rightarrow -\frac{16}{8} - \frac{15}{8} = -\frac{31}{8}$

59. $\frac{3}{5} + \frac{5}{4} \Rightarrow \frac{12}{20} + \frac{25}{20} = \frac{37}{20}$

60. $-1 - \frac{2}{3} \Rightarrow -\frac{3}{3} - \frac{2}{3} = -\frac{5}{3}$

61. $\frac{2}{5} + \frac{5}{4} \Rightarrow \frac{8}{20} + \frac{25}{20} = \frac{33}{20}$

62. $\frac{12}{7} - \frac{9}{7} = \frac{3}{7}$

63. $\frac{9}{8} - \frac{2}{7} \Rightarrow \frac{63}{56} - \frac{16}{56} = \frac{47}{56}$

64. $-2 + \frac{5}{6} \Rightarrow -\frac{12}{6} + \frac{5}{6} = -\frac{7}{6}$

65. $1 - \frac{1}{3} \Rightarrow \frac{3}{3} - \frac{1}{3} = \frac{2}{3}$

66. $\frac{1}{2} - \frac{11}{6} \Rightarrow \frac{3}{6} - \frac{11}{6} = -\frac{8}{6}$ or $-\frac{4}{3}$

67. $-\frac{1}{2} + \frac{3}{2} = \frac{2}{2}$ or 1

68. $\frac{11}{8} - \frac{1}{22} \Rightarrow \frac{121}{88} - \frac{4}{88} = \frac{117}{88}$

69. $\frac{1}{5} + \frac{3}{4} \Rightarrow \frac{4}{20} + \frac{15}{20} = \frac{19}{20}$

70. $\frac{6}{5} - \frac{8}{5} = -\frac{2}{5}$

Answer Key 1.3

1. $-6 \cdot -\frac{4}{1}$

   $-6 \cdot -4$

   24
2. $(-1)^3 = -1$
3. $3 + 8 \div 4$

   $3 + 2$

   5
4. $5(1) \cdot 36$

   $5 \cdot 36$

   180
5. $8 \div 4 \cdot 2$

   $2 \cdot 2$

   4
6. $2 + 6 = 8$
7. $[-9 - (2 - 5)] \div -6$

   $[-9 - (-3)] \div -6$

   $[-6] \div -6$

   1
8. $(-2 \cdot 8 \cdot 2) \div (-4)$

   $-32 \div -4$

   8
9. $-6 + (-6)^2 \div 3$

   $-6 + 36 \div 3$

   $-6 + 12$

   6
10. $-12 \div [-2 - 2 + 6]$

    $-12 \div 2$

    -6
11. $4 - 2|9 - 16|$

    $4 - 2(7)$

    $4 - 14$

    -10
12. $-16 \div 4 - 5$

    $-4 - 5$

    -9
13. $(-1 + 5)(5)$

    $4(5)$

    20

14. $-3-\{3-[-3(6)+2]\}$
$-3-\{3-[-18+2]\}$
$-3-\{3-[-16]\}$
$-3-\{19\}$
-22

15. $[2+4|7+4|]\div[8+15]$
$[2+4(11)]\div 23$
$2+44\div 23$
$46\div 23$
2

16. $-4-[2-24-4-22-10]$
$-4-[-58]$
54

17. $[12+2+6]\,(-5+|-3|)$
$(20)(-5+3)$
$(20)(-2)$
-40

18. $-6+3-6\,[-2-(-4)]$
$-3-6\,[-2+4]$
$-3-6\,[2]$
$-3-12$
-15

19. $$\frac{-15}{2--1-6-[-1+3]}$$
$-15\div(-3-[2])$
$-15\div(-5)$
3

20. $$\frac{25+25}{|16-32|-6}$$
$$\frac{50}{|-16|-6}$$
$$\frac{50}{16-6}$$
$$\frac{50}{10}$$
5

21. $\dfrac{-48-4-4-[-4+3]}{(16+9)\div 5}$

$\dfrac{-56-[-1]}{25\div 5}$

$\dfrac{-55}{5}$

-11

22. $\dfrac{-18-(-3)}{1-(-1)+3}$

$-\dfrac{15}{5}$

-3

23. $\dfrac{8+4}{-24-4-[-25]}$

$-\dfrac{12}{3}$

-4

24. $\dfrac{13+9-12+1-[-10+6]}{\{9\div[16-9(1)-8]\}+12}$

$\dfrac{11-[-4]}{\{9\div[-1]\}+12}$

$\dfrac{15}{-9+12}$

$\dfrac{15}{3}$

5

Answer Key 1.4

1. $r+1$
2. $-4x-2$
3. $2n$
4. $11b+7$
5. $15v$
6. $7x$
7. $-9x$
8. $-7a-1$
9. $k+5$
10. $-3p$
11. $-5x-9$
12. $-10n-9$
13. $-m$
14. $-r-5$
15. $-8x+32$
16. $24r+27$
17. $8n^2+72n$
18. $-9a+5$
19. $-7k^2+42k$
20. $20x^2+10x$
21. $-36x-6$
22. $-2n-2$
23. $-8m^2+40m$
24. $-18p^2+2p$
25. $9x^2-36x$
26. $32n-8$
27. $-9b^2+90b$
28. $-28r-4$
29. $9b+90+5b$
    $14b+90$
30. $4v-7+56v$
    $60v-7$
31. $-3x+12x^2-4x^2$
    $8x^2-3x$
32. $-8x-81x+81$
    $-89x+81$
33. $-4k^2-64k^2-8k$
    $-68k^2-8k$
34. $-9-10-90a$
    $-90a-19$
35. $1-35-49p$
    $-49p-34$
36. $-10x+20-3$
    $-10x+17$

37. $-10 - 4n + 20$
$-4n + 10$
38. $-30 + 6m + 3m$
$9m - 30$
39. $4x + 28 + 8x + 32$
$12x + 60$
40. $-2r - 8r^2 - 8r^2 + 32r$
$-16r^2 + 30r$
41. $-8n - 48 - 8n^2 - 64n$
$-8n^2 - 72n - 48$
42. $54b + 45 - 4b^2 - 12b$
$-4b^2 + 42b + 45$
43. $49 + 21v + 30 - 100v$
$-79v + 79$
44. $-28x + 42 + 20x - 20$
$-8x + 22$
45. $-20n^2 + 10n - 42 + 70n$
$-20n^2 + 80n - 42$
46. $-12 - 3a + 54a^2 + 60a$
$54a^2 + 57a - 12$
47. $5 - 30k + 10k - 80$
$-20k - 75$
48. $-28x - 21 - 100x - 100$
$-128x - 121$
49. $8n^2 - 3n - 5 - 4n^2$
$4n^2 - 3n - 5$
50. $7x^2 - 3 - 5x^2 - 6x$
$2x^2 - 6x - 3$
51. $5p - 6 + 1 - p$
$4p - 5$
52. $3x^2 - x - 7 + 8x$
$3x^2 + 7x - 7$
53. $2 - 4v^2 + 3v^2 + 2v$
$-v^2 + 2v + 2$
54. $2b - 8 + b - 7b^2$
$-7b^2 + 3b - 8$
55. $4 - 2k^2 + 8 - 2k^2$
$-4k^2 + 12$
56. $7a^2 + 7a - 6a^2 - 4a$
$a^2 + 3a$
57. $x^2 - 8 + 2x^2 - 7$
$3x^2 - 15$
58. $3 - 7n^2 + 6n^2 + 3$
$-n^2 + 6$

Answer Key 1.6

1. $$\begin{aligned} 7 \text{ mi} &= 7 \text{ mi} \times \frac{1760 \text{ yd}}{\text{mi}} \\ &= 12{,}320 \text{ yd} \end{aligned}$$

2. $$\begin{aligned} 234 \text{ oz} &= 234 \text{ oz} \times \frac{1 \text{ lb}}{16 \text{ oz}} \times \frac{1 \text{ ton}}{2000 \text{ lb}} \\ &= 0.0073 \text{ tons} \end{aligned}$$

3. $$\begin{aligned} 11.2 \text{ mg} &= 11.2 \text{ mg} \times \frac{1 \text{ g}}{1000 \text{ mg}} \\ &= 0.0112 \text{ g} \end{aligned}$$

4. $$\begin{aligned} 1.35 \text{ km} &= 1.35 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{100 \text{ cm}}{1 \text{ m}} \\ &= 135{,}000 \text{ cm} \end{aligned}$$

5. $$\begin{aligned} 9{,}800{,}000 \text{ mm} &= 9{,}800{,}000 \text{ mm} \times \frac{1 \text{ m}}{1000 \text{ mm}} \times \frac{1 \text{ km}}{1000 \text{ m}} \times \frac{1 \text{ mi}}{1.61 \text{ km}} \\ &= 6.09 \text{ mi (rounded)} \end{aligned}$$

6. $$\begin{aligned} 4.5 \text{ ft}^2 &= 4.5 \text{ ft}^2 \times \frac{(1 \text{ yd})^2}{(3 \text{ ft})^2} \\ &= 0.5 \text{ yd}^2 \end{aligned}$$

7. $$\begin{aligned} 435{,}000 \text{ m}^2 &= 435{,}000 \text{ m}^2 \times \frac{(1 \text{ km}^2)}{(1000 \text{ m})^2} \\ &= 0.435 \text{ km}^2 \end{aligned}$$

8. $$\begin{aligned} 8 \text{ km}^2 &= 8 \text{ km}^2 \times \frac{1 \text{ mi}^2}{(1.61 \text{ km})^2} \times \frac{(5280 \text{ ft})^2}{(1 \text{ mi})^2} \\ &= 86{,}000{,}000 \text{ ft}^2 \text{ (rounded)} \end{aligned}$$

9. $$\begin{aligned} 0.0065 \text{ km}^3 &= 0.0065 \text{ km}^3 \times \frac{(1000 \text{ m})^3}{(1 \text{ km})^3} \\ &= 6{,}500{,}000 \text{ m}^3 \end{aligned}$$

10. $$\begin{aligned} 14.62 \text{ in}^2 &= 14.62 \text{ in}^2 \times \frac{(2.54 \text{ cm})^2}{(1 \text{ in})^2} \\ &= 94.3 \text{ cm}^2 \end{aligned}$$

11. $$5500 \text{ cm}^3 = 5500 \text{ cm}^3 \times \frac{(1 \text{ in})^3}{(2.54 \text{ cm})^3} \times \frac{(1 \text{ yd})^3}{(36 \text{ in})^3} = 0.0072 \text{ yd}^3$$

12. $$3.5 \text{ mi/h} = \frac{3.5 \text{ mi}}{\text{h}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 5.13 \text{ ft/s}$$

13. $$185 \text{ yd/min.} = \frac{185 \text{ yd}}{\text{min.}} \times \frac{1 \text{ mi}}{1760 \text{ yd}} \times \frac{60 \text{ min.}}{1 \text{ h}} = 6.31 \text{ mi/h}$$

14. $$153 \text{ ft/s} = \frac{153 \text{ ft}}{\text{s}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \times \frac{3600 \text{ s}}{1 \text{ h}} = 104.3 \text{ mi/h}$$

15. $$248 \text{ mi/h} = \frac{248 \text{ mi}}{\text{h}} \times \frac{1.61 \text{ km}}{1 \text{ mi}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 111 \text{ m/s}$$

16. $$186,000 \text{ mi/h} = \frac{186,000 \text{ mi}}{\text{h}} \times \frac{1.61 \text{ km}}{1 \text{ mi}} \times \frac{24 \text{ h}}{1 \text{ day}} \times \frac{365 \text{ days}}{1 \text{ yr}} = 2,620,000,000 \text{ km/yr (rounded)}$$

17. $$7.50 \text{ t/yd}^2 = \frac{7.50 \text{ t}}{\text{yd}^2} \times \frac{2000 \text{ lb}}{1 \text{ t}} \times \frac{(1 \text{ yd})^2}{(36 \text{ in})^2} = 11.57 \text{ lb/in}^2$$

18. $$16 \text{ ft/s}^2 = \frac{16 \text{ ft}}{\text{s}^2} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \times \frac{1.61 \text{ km}}{1 \text{ mi}} \times \frac{(3600 \text{ s})^2}{(1 \text{ h})^2} = 63,200 \text{ km/h}^2 \text{ (rounded)}$$

19.

mpg: $\frac{260 \text{ mi}}{8 \text{ gal}} \Rightarrow 32.5$ mpg

$$32.5 \text{ mi/gal} = \frac{32.5 \text{ mi}}{\text{gal}} \times \frac{1.602 \text{ km}}{1 \text{ mi}} \times \frac{1 \text{ gal}}{3.785 \text{ litres}} = 13.8 \text{ km/litre}$$

$$32.5 \text{ mpg} = 13.8 \text{ km/l}$$

20. $$12 \text{ pg/min} = \frac{12 \text{ pg}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \Rightarrow 720 \text{ pg/hr}$$

$$720 \text{ pg/hr} = \frac{720 \text{ pg}}{\text{hr}} \times \frac{24 \text{ hr}}{1 \text{ d}} \Rightarrow 17,280 \text{ pg/d}$$

21. $$60 \text{ beats/min} = \frac{60 \text{ beats}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{365.24 \text{ days}}{\text{yr}} \times \frac{86 \text{ yrs}}{\text{life}}$$

$$60 \text{ beats/min} = 2,713,879,296 \text{ beats/life}$$

$$= 2.71 \text{ million beats}$$

22. $$128 \text{ mg/dL} = \frac{128 \text{ mg}}{\text{dL}} \times \frac{1 \text{ g}}{1000 \text{ mg}} \times \frac{10 \text{ dL}}{\text{L}}$$

$$128 \text{ mg/dL} = 1.28 \text{ g/L}$$

23. $$38 \text{ ft} = 38 \text{ ft} \times \frac{1 \text{ yd}}{3 \text{ ft}} \Rightarrow 12\frac{2}{3} \text{ yd}$$

$$40 \text{ ft} = 40 \text{ ft} \times \frac{1 \text{ yd}}{3 \text{ ft}} \Rightarrow 13\frac{1}{3} \text{ yd}$$

$$\text{Area} = 12\frac{2}{3} \text{ yd} \times 13\frac{1}{3} \text{ yd} \Rightarrow 168\frac{8}{9} \text{ yd}^2$$

$$\text{Cost} = \$18/\text{yd}^2 \times 168\frac{8}{9} \text{ yd}^2 \Rightarrow \$3040$$

24. $$\text{Volume} = 50 \text{ ft} \times 10 \text{ ft} \times 8 \text{ ft} \Rightarrow 4000 \text{ ft}^3$$

$$4000 \text{ ft}^3 = 4000 \text{ ft}^3 \times \frac{1 \text{ yd}^3}{(3 \text{ ft})^3} \Rightarrow 148 \text{ yd}^3$$

$$148 \text{ yd}^3 = 148 \text{ yd}^3 \times \frac{(0.9144 \text{ m})^3}{1 \text{ yd}^3} \Rightarrow 113 \text{ m}^3$$

25. $$\text{Area of lot} \Rightarrow \frac{1}{3}\left(43,560 \text{ ft}^2\right) \Rightarrow 14,520 \text{ ft}^2$$

$$\text{Now } \frac{1}{4} \text{ of this is} \Rightarrow \frac{1}{4}\left(14,520 \text{ ft}^2\right) \Rightarrow 3630 \text{ ft}^2$$

$$\text{Convert to square metres} \Rightarrow 3630 \text{ ft}^2 \times \frac{(1 \text{ yd})^2}{(3 \text{ ft})^2} \Rightarrow 403\frac{1}{3} \text{ yd}^2$$

26.
$$\text{Car speed} \Rightarrow \frac{23 \text{ km}}{15 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ h}} \Rightarrow \frac{92 \text{ km}}{\text{h}}$$

$$\text{Convert to m/s} \Rightarrow \frac{92 \text{ km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} \Rightarrow 25.6 \text{ m/s}$$

27.
$$3106 \text{ carats} = 3106 \text{ carats} \times \frac{0.20 \text{ g}}{1 \text{ carat}} \Rightarrow 621.2 \text{ g}$$

$$621.2 \text{ g} = 621.2 \text{ g} \times \frac{1000 \text{ mg}}{1 \text{ g}} \Rightarrow 621,200 \text{ mg}$$

$$621.2 \text{ g} = 621.2 \text{ g} \times \frac{0.0022005 \text{ lbs}}{1 \text{ g}} \Rightarrow 1.37 \text{ lbs}$$

Answer Key 1.7

1. Solutions:
    1. $6 - 4 \neq 4$
    2. $5 + 4 = 9$
    3. $0 + 4 = 4$
    4. $8 - 4 = 4$
2. Are the following statements true?

    - Letters A, B, C & D do not appear anywhere in the spellings of 1 to 99.
      True — but also J, K, L, P, Q & Z.
    - Letter D appears for the first time in “hundred.”
      True.
    - Letters A, B & C do not appear anywhere in the spellings of 1 to 999.
      True.
    - Letter A appears for the first time in “thousand.”
      True.
    - Letters B & C do not appear anywhere in the spellings of 1 to 999 999 999.
      True.
    - Letter B appears for the first time in “billion.”
      True.
    - Letter C does not appear anywhere in any word used to count in English.
      False.

Answer Key 2.1

1. $$\begin{aligned} v+9 &= 16 \\ -9 \quad & \quad -9 \\ v &= 7 \end{aligned}$$

2. $$\begin{array}{rcrcr} 14 & = & b & + & 3 \\ -3 & & & - & 3 \\ 11 & = & b & & \end{array}$$

3. $$\begin{array}{rcrcr} x & - & 11 & = & -16 \\ & + & 11 & & +11 \\ & & x & = & -5 \end{array}$$

4. $$\begin{array}{rcrcr} -14 & = & x & - & 18 \\ +18 & & & + & 18 \\ x & = & 4 & & \end{array}$$

5. $$\begin{array}{rcrcr} 30 & = & a & + & 20 \\ -20 & & & - & 20 \\ a & = & 10 & & \end{array}$$

6. $$\begin{array}{rcrcr} -1 & + & k & = & 5 \\ +1 & & & & +1 \\ & & k & = & 6 \end{array}$$

7. $$\begin{array}{rcrcr} x & - & 7 & = & -26 \\ & + & 7 & & +7 \\ & & x & = & -19 \end{array}$$

8. $$\begin{array}{rcrcr} -13 & + & p & = & -19 \\ +13 & & & & +13 \\ & & p & = & -6 \end{array}$$

9. $$\begin{array}{rcrcr} 13 & = & n & - & 5 \\ +5 & & & + & 5 \\ n & = & 18 & & \end{array}$$

10. $$\begin{array}{rcrcr} 22 & = & 16 & + & m \\ -16 & & -16 & & \\ m & = & 6 & & \end{array}$$

11. $$\begin{aligned} \frac{340}{-17} &= \frac{-17x}{-17} \\ x &= -20 \end{aligned}$$

12. $$\begin{aligned} \frac{4r}{4} &= \frac{-28}{4} \\ r &= -7 \end{aligned}$$

13. $$\left(-9 = \frac{n}{12}\right)(12)$$

$$n = -108$$

14. $$\begin{aligned} \frac{27}{9} &= \frac{9b}{9} \\ b &= 3 \end{aligned}$$

15. $$\begin{aligned} \frac{20v}{20} &= \frac{-160}{20} \\ v &= -8 \end{aligned}$$

16. $$\begin{aligned} \frac{-20x}{-20} &= \frac{-80}{-20} \\ x &= 4 \end{aligned}$$

17. $$\begin{aligned} \frac{340}{20} &= \frac{20n}{20} \\ n &= 17 \end{aligned}$$

18. $$\begin{aligned} \frac{12}{8} &= \frac{8a}{8} \\ a &= \frac{3}{2} \end{aligned}$$

19. $$\begin{aligned} \frac{16x}{16} &= \frac{320}{16} \\ x &= 20 \end{aligned}$$

20. $$\begin{aligned} \frac{8k}{8} &= \frac{-16}{8} \\ k &= -2 \end{aligned}$$

21. $$\begin{array}{rcrcr} -16 & + & n & = & -13 \\ +16 & & & & +16 \\ & & n & = & 3 \end{array}$$

22. $$\begin{array}{rcrcr} -21 & = & x & - & 5 \\ +5 & & & + & 5 \\ x & = & -16 & & \end{array}$$

23. $$\begin{array}{rcrcr} p & - & 8 & = & -21 \\ & + & 8 & & +8 \\ & & p & = & -13 \end{array}$$

24. $$\begin{array}{rcrcr} m & - & 4 & = & -13 \\ & + & 4 & & +4 \\ & & m & = & -9 \end{array}$$

25. $$\left(\frac{r}{14} = \frac{5}{14}\right)(14)$$

$$r = 5$$

26. $$\left(\frac{n}{8} = 40\right)(8)$$

$$n = 320$$

27. $$\begin{aligned} \frac{20b}{20} &= \frac{-200}{20} \\ b &= -10 \end{aligned}$$

28. $$\left(-\frac{1}{3} = \frac{x}{12}\right)(12)$$

$$-\frac{1}{3} \cdot 12 = x$$

$$x = -4$$

Answer Key 2.2

1.
$$\begin{array}{rrrcr} 5 & + & \frac{n}{4} & = & 4 \\ -5 & & & & -5 \\ & & 4\left(\frac{n}{4}\right) & = & (-1)4 \\ & & n & = & -4 \end{array}$$

2.
$$\begin{array}{rcrcr} -2 & = & -2m & + & 12 \\ -12 & & & - & 12 \\ \frac{-14}{-2} & = & \frac{-2m}{-2} & & \\ m & = & 7 & & \end{array}$$

3.
$$\begin{array}{rcrcr} 102 & = & -7r & + & 4 \\ -4 & & & - & 4 \\ \frac{98}{-7} & = & \frac{-7r}{-7} & & \\ r & = & -14 & & \end{array}$$

4.
$$\begin{array}{rcrcr} 27 & = & 21 & - & 3x \\ -21 & & -21 & & \\ \frac{6}{-3} & = & \frac{-3x}{-3} & & \\ x & = & -2 & & \end{array}$$

5.
$$\begin{array}{rrrcr} -8n & + & 3 & = & -77 \\ & - & 3 & & -3 \\ & & \frac{-8n}{-8} & = & \frac{-80}{-8} \\ & & n & = & 10 \end{array}$$

6.
$$\begin{array}{rrrcr} -4 & - & b & = & 8 \\ +4 & & & & +4 \\ & & (-b & = & 12)(-1) \\ & & b & = & -12 \end{array}$$

7.
$$\begin{array}{rcr} \frac{0}{-6} & = & \frac{-6v}{-6} \\ v & = & 0 \end{array}$$

8.
$$\begin{array}{rcrcl} -2 & + & \dfrac{x}{2} & = & 4 \\ +2 & & & & +2 \\ & & 2\left(\dfrac{x}{2}\right) & = & (6)2 \\ & & x & = & 12 \end{array}$$

9.
$$\begin{array}{rcrcl} -8 & = & \dfrac{x}{5} & - & 6 \\ +6 & & & + & 6 \\ 5(-2) & = & \left(\dfrac{x}{5}\right)5 & & \\ x & = & -10 & & \end{array}$$

10.
$$\begin{array}{rcrcl} -5 & = & \dfrac{a}{4} & - & 1 \\ +1 & & & + & 1 \\ 4(-4) & = & \left(\dfrac{a}{4}\right)4 & & \\ a & = & -16 & & \end{array}$$

11.
$$\begin{array}{rcrcl} 0 & = & -7 & + & \dfrac{k}{2} \\ +7 & & +7 & & \\ 2(7) & = & \left(\dfrac{k}{2}\right)2 & & \\ k & = & 14 & & \end{array}$$

12.
$$\begin{array}{rcrcl} -6 & = & 15 & + & 3p \\ -15 & & -15 & & \\ \dfrac{-21}{3} & = & \dfrac{3p}{3} & & \\ p & = & -7 & & \end{array}$$

13.
$$\begin{array}{rcrcl} -12 & + & 3x & = & 0 \\ +12 & & & & +12 \\ & & \dfrac{3x}{3} & = & \dfrac{12}{3} \\ & & x & = & 4 \end{array}$$

14.
$$\begin{array}{rcrcl} -5m & + & 2 & = & 27 \\ & - & 2 & & -2 \\ & & \dfrac{-5m}{-5} & = & \dfrac{25}{-5} \\ & & m & = & -5 \end{array}$$

Answer Key 2.3

1.
$$\begin{array}{rrrrcr} 2 & -(-3a & - & 8) & = & 1 \\ -2 & & & & & -2 \\ & -(-3a & - & 8) & = & -1 \\ & 3a & + & 8 & = & -1 \\ & & - & 8 & & -8 \\ & & & 3a & = & -9 \\ & & & a & = & -3 \end{array}$$

2.
$$\begin{array}{rrrcr} 2(-3n & + & 8) & = & -20 \\ \dfrac{2}{2}(-3n & + & 8) & = & \dfrac{-20}{2} \\ -3n & + & 8 & = & -10 \\ & - & 8 & & -8 \\ & & \dfrac{-3n}{-3} & = & \dfrac{-18}{-3} \\ & & n & = & 6 \end{array}$$

3.
$$\begin{array}{rrrcr} -5(-4 & + & 2v) & = & -50 \\ \dfrac{-5}{-5}(-4 & + & 2v) & = & \dfrac{-50}{-5} \\ -4 & + & 2v & = & 10 \\ +4 & & & & +4 \\ & & \dfrac{2v}{2} & = & \dfrac{14}{2} \\ & & v & = & 7 \end{array}$$

4.
$$\begin{array}{rrrrrcl} 2 & - & 8(-4 & + & 3x) & = & 34 \\ -2 & & & & & & -2 \\ & & \dfrac{-8}{-8}(-4 & + & 3x) & = & \dfrac{32}{-8} \\ & & -4 & + & 3x & = & -4 \\ & & +4 & & & & +4 \\ & & & & 3x & = & 0 \\ & & & & x & = & 0 \end{array}$$

5. $$\begin{array}{rcrcl} \frac{66}{6} & = & \frac{6}{6}(6 & + & 5x) \\ 11 & = & 6 & + & 5x \\ -6 & & -6 & & \\ \frac{5}{5} & = & \frac{5x}{5} & & \\ x & = & 1 & & \end{array}$$

6. $$\begin{array}{rcrcrcl} 32 & = & 2 & - & 5(-4n & + & 6) \\ -2 & & -2 & & & & \\ \frac{30}{-5} & = & \frac{-5}{-5}(-4n & + & 6) & & \\ -6 & = & -4n & + & 6 & & \\ -6 & & & - & 6 & & \\ \frac{-12}{-4} & = & \frac{-4n}{-4} & & & & \\ n & = & 3 & & & & \end{array}$$

7. $$\begin{array}{rcrcrcr} -2 & + & 2(8x & - & 9) & = & -16 \\ +2 & & & & & & +2 \\ & & \frac{2}{2}(8x & - & 9) & = & \frac{-14}{2} \\ & & 8x & - & 9 & = & -7 \\ & & & + & 9 & & +9 \\ & & & & 8x & = & 2 \\ & & & & x & = & \frac{1}{4} \end{array}$$

8. $$\begin{array}{rcrcr} -3 & + & 5n & = & 12 \\ +3 & & & & +3 \\ & & \frac{5n}{5} & = & \frac{15}{5} \\ & & n & = & 3 \end{array}$$

9. $$\begin{array}{rcrclcr} -1 & - & 7m & = & -8m & + & 7 \\ -7 & + & 7m & & +7m & - & 7 \\ & & (-8 & = & -m)(-1) & & \\ & & m & = & 8 & & \end{array}$$

10.
$$\begin{array}{rrrrrrr} 56p & - & 48 & = & 6p & + & 2 \\ -6p & + & 48 & & -6p & + & 48 \\ & & \dfrac{50p}{50} & = & \dfrac{50}{50} & & \\ & & p & = & 1 & & \end{array}$$

11.
$$\begin{array}{rrrrrrr} 1 & - & 12r & = & 29 & - & 8r \\ -1 & + & 8r & & -1 & + & 8r \\ & & \dfrac{-4r}{-4} & = & \dfrac{28}{-4} & & \\ & & r & = & -7 & & \end{array}$$

12.
$$\begin{array}{rrrrrrr} 4 & + & 3x & = & -12x & + & 4 \\ -4 & + & 12x & & +12x & - & 4 \\ & & 15x & = & 0 & & \\ & & x & = & 0 & & \end{array}$$

13.
$$\begin{array}{rrrrrrr} 20 & - & 7b & = & -12b & + & 30 \\ -20 & + & 12b & & +12b & - & 20 \\ & & \dfrac{5b}{5} & = & \dfrac{10}{5} & & \\ & & b & = & 2 & & \end{array}$$

14.
$$\begin{array}{rrrrrrr} -16n & + & 12 & = & 39 & - & 7n \\ +7n & - & 12 & & -12 & + & 7n \\ & & \dfrac{-9n}{-9} & = & \dfrac{27}{-9} & & \\ & & n & = & -3 & & \end{array}$$

15.
$$\begin{array}{rrrrrrrrr} -2 & - & 5(2 & - & 4m) & = & 33 & + & 5m \\ +2 & & & & & & +2 & & \\ & & -10 & + & 20m & = & 35 & + & 5m \\ & & +10 & - & 5m & & +10 & - & 5m \\ & & & & \dfrac{15m}{15} & = & \dfrac{45}{15} & & \\ & & & & m & = & 3 & & \end{array}$$

16.
$$\begin{array}{rrrrrrr} -25 & - & 7x & = & 12x & - & 6 \\ +25 & - & 12x & & -12x & + & 25 \\ & & \dfrac{-19x}{-19} & = & \dfrac{19}{-19} & & \\ & & x & = & -1 & & \end{array}$$

17. $$\begin{array}{rrrrrrrrr} -4n & + & 11 & = & 2 & - & 16n & + & 3n \\ +16n & - & 11 & & -11 & + & 16n & - & 3n \\ -3n & & & & & & & & \\ & & \dfrac{9n}{9} & = & \dfrac{-9}{9} & & & & \\ & & n & = & -1 & & & & \end{array}$$

18. $$\begin{array}{rrrrrrr} -7 & - & 7b & = & -5 & - & 5b \\ +7 & + & 5b & & +7 & + & 5b \\ & & \dfrac{-2b}{-2} & = & \dfrac{2}{-2} & & \\ & & b & = & -1 & & \end{array}$$

19. $$\begin{array}{rrrrrrrrr} -6v & - & 29 & = & -4v & - & 5v & - & 5 \\ +4v & + & 29 & & +4v & + & 5v & + & 29 \\ +5v & & & & & & & & \\ & & \dfrac{3v}{3} & = & \dfrac{24}{3} & & & & \\ & & v & = & 8 & & & & \end{array}$$

20. $$\begin{array}{rrrrrrr} -64r & + & 16 & = & 3r & + & 16 \\ -3r & - & 16 & & -3r & - & 16 \\ & & -67r & = & 0 & & \\ & & r & = & 0 & & \end{array}$$

21. $$\begin{array}{rrrrrrr} 8x & - & 8 & = & -20 & - & 4x \\ +4x & + & 8 & & +8 & + & 4x \\ & & \dfrac{12x}{12} & & \dfrac{-12}{12} & & \\ & & x & = & -1 & & \end{array}$$

22. $$\begin{array}{rrrrrrrrr} -8n & - & 19 & = & -16n & + & 6 & + & 3n \\ +16n & + & 19 & & +16n & + & 19 & - & 3n \\ -3n & & & & & & & & \\ & & \dfrac{5n}{5} & = & \dfrac{25}{5} & & & & \\ & & n & = & 5 & & & & \end{array}$$

23. $$\begin{array}{rrrrrrrrr} -2m & + & 4 & + & 7m & - & 56 & = & -67 \\ & - & 4 & & & + & 56 & & +56 \\ & & & & & & & & -4 \\ & & & & & & \dfrac{5m}{5} & = & \dfrac{-15}{5} \\ & & & & & & m & = & -3 \end{array}$$

24.
$$\begin{array}{rcrrrrrrr} 7 & = & 4n & - & 28 & + & 35n & + & 35 \\ +28 & & & + & 28 & & & - & 35 \\ -35 & & & & & & & & \\ 0 & = & 39n & & & & & & \\ n & = & 0 & & & & & & \end{array}$$

25.
$$\begin{array}{rcrrrrrrr} 50 & = & 56 & + & 56r & - & 4r & - & 6 \\ -56 & & -56 & & & & & + & 6 \\ +6 & & & & & & & & \\ 0 & = & 52r & & & & & & \\ r & = & 0 & & & & & & \end{array}$$

26.
$$\begin{array}{rcrrrrrcr} -48 & - & 48x & - & 12 & + & 24x & = & -12 \\ +48 & & & + & 12 & & & & +12 \\ & & & & & & & & +48 \\ & & & & & & \dfrac{-24x}{-24} & = & \dfrac{48}{-24} \\ & & & & & & x & = & -2 \end{array}$$

Answer Key 2.4

1.
$$\left(\frac{3}{5}\left(1+p\right)=\frac{21}{20}\right)(20)$$
$$\frac{3}{51}\cdot 204(1+p)=\frac{21}{201}\cdot 20$$
$$\begin{array}{rcrcr} 12 & + & 12p & = & 21 \\ -12 & & & & -12 \\ & & \dfrac{12p}{12} & = & \dfrac{9}{12} \\ & & & & \\ & & p & = & \dfrac{3}{4} \end{array}$$

2.
$$\left(-\frac{1}{2}=\frac{3k}{2}+\frac{3}{2}\right)(2)$$
$$\begin{array}{rcrcr} -1 & = & 3k & + & 3 \\ -3 & & & - & 3 \\ \dfrac{-4}{3} & = & \dfrac{3k}{3} & & \\ & & & & \\ k & = & -\dfrac{4}{3} & & \end{array}$$

3.
$$\left(0=-\frac{5}{4}x+\frac{6}{4}\right)(4)$$
$$\begin{array}{rcrcr} 0 & = & -5x & + & 6 \\ +5x & & +5x & & \\ \dfrac{5x}{5} & = & \dfrac{6}{5} & & \\ & & & & \\ x & = & \dfrac{6}{5} & & \end{array}$$

4.
$$\left(\frac{3n}{2}-8=-\frac{29}{12}\right)(12)$$

$$\begin{array}{rrrcr} 18n & - & 96 & = & -29 \\ & + & 96 & & +96 \\ & & \dfrac{18n}{18} & = & \dfrac{67}{18} \\ \\ & & n & = & \dfrac{67}{18} \end{array}$$

5.

$$\left(\frac{3}{4} - \frac{5}{4}m = \frac{108}{24}\right)(24)$$

$$\begin{array}{rrrcr} 18 & - & 30m & = & 108 \\ -18 & & & & -18 \\ & & \dfrac{-30m}{-30} & = & \dfrac{90}{-30} \\ \\ & & m & = & -3 \end{array}$$

6.

$$\left(\frac{11}{4} + \frac{3}{4}r = \frac{160}{32}\right)(32)$$

$$\begin{array}{rcrcr} 11 \cdot 8 & + & 3r \cdot 8 & = & 160 \\ 88 & + & 24r & = & 160 \\ -88 & & & & -88 \\ & & \dfrac{24r}{24} & = & \dfrac{72}{24} \\ \\ & & r & = & 3 \end{array}$$

7.

$$\left(2b + \frac{9}{5} = -\frac{11}{5}\right)(5)$$

$$\begin{array}{rrrcr} 10b & + & 9 & = & -11 \\ & - & 9 & & -9 \\ & & \dfrac{10b}{10} & = & \dfrac{-20}{10} \\ \\ & & b & = & -2 \end{array}$$

8.

$$\left(\frac{3}{2} - \frac{7v}{4} = -\frac{9}{8}\right)(8)$$

$$\begin{array}{rcrcr} 3\cdot 4 & - & 7r\cdot 2 & = & -9 \\ 12 & - & 14r & = & -9 \\ -12 & & & & -12 \\ & & \dfrac{-14r}{-14} & = & \dfrac{-21}{-14} \\ \\ & & r & = & \dfrac{3}{2} \end{array}$$

9.

$$\left(\frac{21n}{6}+\frac{3}{2}=\frac{3}{2}\right)(6)$$

$$\begin{array}{rcrcr} \dfrac{21n}{6} & + & 3\cdot 3 & = & 3\cdot 3 \\ & - & 9 & & -9 \\ & & \dfrac{21n}{6} & = & 0 \\ \\ & & n & = & 0 \end{array}$$

10.

$$\left(\frac{41}{9}=\frac{5}{2}x+\frac{10}{6}-\frac{1}{3}x\right)(18)$$

$$\begin{array}{rcrcrcr} 41\cdot 2 & = & 5x\cdot 9 & + & 10\cdot 3 & - & x\cdot 6 \\ 82 & = & 45x & + & 30 & - & 6x \\ -30 & & & - & 30 & & \\ \dfrac{52}{39} & = & \dfrac{39x}{39} & & & & \\ \\ x & = & \dfrac{4}{3} & & & & \end{array}$$

11.

$$\left(-a+\frac{40a}{12}-\frac{5}{4}=-\frac{19}{4}\right)(12)$$

$$\begin{array}{rcrcrcr} -12a & + & 40a & - & 15 & = & -57 \\ & & & + & 15 & & +15 \\ & & & & \dfrac{28a}{28} & = & \dfrac{-42}{28} \\ \\ & & & & a & = & -\dfrac{3}{2} \end{array}$$

12.

$$\left(-\frac{7k}{12}+\frac{1}{3}-\frac{10k}{3}=-\frac{13}{8}\right)(24)$$

$$\begin{array}{rrrrrrr}
-14k & + & 8 & - & 80k & = & -39 \\
 & - & 8 & & & & -8 \\
 & & & & \dfrac{-94k}{-94} & = & \dfrac{-47}{-94} \\
 & & & & k & = & \dfrac{1}{2}
\end{array}$$

13.

$$\left(\frac{55}{6} = -\frac{15p}{4} + \frac{25}{6}\right)(12)$$

$$\begin{array}{rrrrr}
110 & = & -45p & + & 50 \\
-50 & & & - & 50 \\
\dfrac{60}{-45} & = & \dfrac{-45p}{-45} & & \\
p & = & -\dfrac{4}{3} & &
\end{array}$$

14.

$$\left(-\frac{2x}{6} + \frac{3}{8} - \frac{7x}{2} = -\frac{83}{24}\right)(24)$$

$$\begin{array}{rrrrrrr}
-8x & + & 9 & - & 84x & = & -83 \\
 & - & 9 & & & & -9 \\
 & & & & \dfrac{-92x}{-92} & = & \dfrac{-92}{-92} \\
 & & & & x & = & 1
\end{array}$$

15.

$$\left(-\frac{5}{8} = \frac{5}{4}r - \frac{15}{8}\right)(8)$$

$$\begin{array}{rrrrr}
-5 & = & 10r & - & 15 \\
+15 & & & + & 15 \\
\dfrac{10}{10} & = & \dfrac{10r}{10} & & \\
r & = & 1 & &
\end{array}$$

16.

$$\left(\frac{1}{12} = \frac{4x}{3} + \frac{5x}{3} - \frac{35}{12}\right)(12)$$

$$\begin{array}{rrrrrrr}
1 & = & 16x & + & 20x & - & 35 \\
+35 & & & & & + & 35 \\
\dfrac{36}{36} & = & \dfrac{36x}{36} & & & & \\
x & = & 1 & & & &
\end{array}$$

17.

$$\left(-\frac{11}{3}+\frac{3b}{2}=\frac{5b}{2}-\frac{25}{6}\right)(6)$$

$$\begin{array}{rrrcrrr} -22 & + & 9b & = & 15b & - & 25 \\ +22 & - & 15b & & -15b & + & 22 \\ & & \dfrac{-6b}{-6} & = & \dfrac{-3}{-6} & & \\ \\ & & b & = & \dfrac{1}{2} & & \end{array}$$

18.

$$\left(\frac{7}{6}-\frac{4n}{3}=-\frac{3n}{2}+2n+3\right)(6)$$

$$\begin{array}{rrrcrrrrr} 7 & - & 8n & = & -9n & + & 12n & + & 18 \\ -7 & + & 9n & & +9n & - & 12n & - & 7 \\ & - & 12n & & & & & & \\ & & \dfrac{-11n}{-11} & = & \dfrac{11}{-11} & & & & \\ \\ & & n & = & -1 & & & & \end{array}$$

Answer Key 2.5

1. $x = \pm 8$
2. $n = \pm 7$
3. $b = \pm 1$
4. $x = \pm 2$
5. $$\begin{array}{rcrcr} 5 & + & 8a & = & 53 \\ -5 & & & & -5 \\ & & \dfrac{8a}{8} & = & \dfrac{48}{8} \\ & & a & = & 6 \end{array}$$ $$\begin{array}{rcrcl} 5 & + & 8a & = & -53 \\ -5 & & & & -5 \\ & & \dfrac{8a}{8} & = & \dfrac{-58}{8} \\ & & a & = & -\dfrac{58}{8} \text{ or } -7\dfrac{1}{4} \end{array}$$
6. $$\begin{array}{rcrcl} 9n & + & 8 & = & 46 \\ & - & 8 & & -8 \\ & & \dfrac{9n}{9} & = & \dfrac{38}{9} \\ & & n & = & \dfrac{38}{9} \text{ or } 4\dfrac{2}{9} \end{array}$$ $$\begin{array}{rcrcr} 9n & + & 8 & = & -46 \\ & - & 8 & & -8 \\ & & \dfrac{9n}{9} & = & \dfrac{-54}{9} \\ & & n & = & -6 \end{array}$$
7. $$\begin{array}{rcrcr} 3k & + & 8 & = & 2 \\ & - & 8 & & -8 \\ & & \dfrac{3k}{3} & = & \dfrac{-6}{3} \\ & & k & = & -2 \end{array}$$ $$\begin{array}{rcrcr} 3k & + & 8 & = & -2 \\ & - & 8 & & -8 \\ & & \dfrac{3k}{3} & = & \dfrac{-10}{3} \\ & & k & = & -\dfrac{10}{3} \end{array}$$
8. $$\begin{array}{rcrcl} 3 & - & x & = & 6 \\ -3 & & & & -3 \\ & & (-x & = & 3)(-1) \\ & & x & = & -3 \end{array}$$ $$\begin{array}{rcrcl} 3 & - & x & = & -6 \\ -3 & & & & -3 \\ & & (-x & = & -9)(-1) \\ & & x & = & 9 \end{array}$$
9. $$\begin{array}{rcr} \dfrac{-7}{-7}\,|-3-3r| & = & \dfrac{-21}{-7} \\ |-3-3r| & = & 3 \end{array}$$ $$\begin{array}{rcrcr} -3 & - & 3r & = & 3 \\ +3 & & & & +3 \\ & & \dfrac{-3r}{-3} & = & \dfrac{6}{-3} \\ & & r & = & -2 \end{array}$$ $$\begin{array}{rcrcr} -3 & - & 3r & = & -3 \\ +3 & & & & +3 \\ & & \dfrac{-3r}{-3} & = & \dfrac{0}{-3} \\ & & r & = & 0 \end{array}$$
10. $$\begin{array}{rcrcr} |2+2b| & + & 1 & = & 3 \\ & - & 1 & & -1 \\ |2+2b| & & & = & 2 \end{array}$$

$$\begin{array}{rrrrr} 2 & + & 2b & = & 2 \\ -2 & & & & -2 \\ & & 2b & = & 0 \\ & & b & = & 0 \end{array}$$

$$\begin{array}{rrrrr} 2 & + & 2b & = & -2 \\ -2 & & & & -2 \\ & & \dfrac{2b}{2} & = & \dfrac{-4}{2} \\ & & b & = & -2 \end{array}$$

11. $$\begin{array}{rcl} \dfrac{7}{7}|-7x-3| & = & \dfrac{21}{7} \\ |-7x-3| & = & 3 \end{array}$$

$$\begin{array}{rrrrr} -7x & - & 3 & = & 3 \\ & + & 3 & & +3 \\ & & \dfrac{-7x}{-7} & = & \dfrac{6}{-7} \\ & & x & = & -\dfrac{6}{7} \end{array}$$

$$\begin{array}{rrrrr} -7x & - & 3 & = & -3 \\ & + & 3 & & +3 \\ & & -7x & = & 0 \\ & & x & = & 0 \end{array}$$

12. $$\begin{array}{rrrrr} -4 & - & 3n & = & 2 \\ +4 & & & & +4 \\ & & \dfrac{-3n}{-3} & = & \dfrac{6}{-3} \\ & & n & = & -2 \end{array}$$

$$\begin{array}{rrrrr} -4 & - & 3n & = & -2 \\ +4 & & & & +4 \\ & & \dfrac{-3n}{-3} & = & \dfrac{2}{-3} \\ & & n & = & -\dfrac{2}{3} \end{array}$$

13. $$\begin{array}{rrrrrrr} 8|5p & + & 8| & - & 5 & = & 11 \\ & & & + & 5 & & +5 \\ & & \dfrac{8}{8}|5p & + & 8| & = & \dfrac{16}{8} \\ & & |5p & + & 8| & = & 2 \end{array}$$

$$\begin{array}{rrrrr} 5p & + & 8 & = & 2 \\ & - & 8 & & -8 \\ & & \dfrac{5p}{5} & = & \dfrac{-6}{5} \\ & & p & = & -\dfrac{6}{5} \end{array}$$

$$\begin{array}{rrrrr} 5p & + & 8 & = & -2 \\ & - & 8 & & -8 \\ & & \dfrac{5p}{5} & = & \dfrac{-10}{5} \\ & & p & = & -2 \end{array}$$

14. $$\begin{array}{rrrrrrl} 3 & - & |6n & + & 7| & = & -40 \\ -3 & & & & & & -3 \\ & & (-|6n & + & 7| & = & -43)(-1) \\ & & |6n & + & 7| & = & 43 \end{array}$$

$$\begin{array}{rrrrr} 6n & + & 7 & = & 43 \\ & - & 7 & & -7 \\ & & \dfrac{6n}{6} & = & \dfrac{36}{6} \\ & & n & = & 6 \end{array}$$

$$\begin{array}{rrrrr} 6n & + & 7 & = & -43 \\ & - & 7 & & -7 \\ & & \dfrac{6n}{6} & = & \dfrac{-50}{6} \\ & & n & = & -\dfrac{25}{3} \end{array}$$

15. $$\begin{array}{rrrrrrr} 5|3 & + & 7m| & + & 1 & = & 51 \\ & & & - & 1 & & -1 \\ & & \dfrac{5}{5}|3 & + & 7m| & = & \dfrac{50}{5} \\ & & |3 & + & 7m| & = & 10 \end{array}$$

$$\begin{array}{rrrrr} 3 & + & 7m & = & 10 \\ -3 & & & & -3 \\ & & \dfrac{7m}{7} & = & \dfrac{7}{7} \\ & & m & = & 1 \end{array}$$

$$\begin{array}{rrrrr} 3 & + & 7m & = & -10 \\ -3 & & & & -3 \\ & & \dfrac{7m}{7} & = & \dfrac{-13}{7} \\ & & m & = & -\dfrac{13}{7} \end{array}$$

16. $$\begin{array}{rrrrrrr} 4|r & + & 7| & + & 3 & = & 59 \\ & & & - & 3 & & -3 \\ & & \dfrac{4}{4}|r & + & 7| & = & \dfrac{56}{4} \\ & & |r & + & 7| & = & 14 \end{array}$$

$$\begin{array}{rrrrr} r & + & 7 & = & 14 \\ & - & 7 & & -7 \\ & & r & = & 7 \end{array}$$

$$\begin{array}{rrrrr} r & + & 7 & = & -14 \\ & - & 7 & & -7 \\ & & r & = & -21 \end{array}$$

17. $$\begin{array}{rrrrrrr} -7 & + & 8|-7x & - & 3| & = & 73 \\ +7 & & & & & & +7 \\ & & \dfrac{8}{8}|-7x & - & 3| & = & \dfrac{80}{8} \\ & & |-7x & - & 3| & = & 10 \end{array}$$

$$\begin{array}{rrrrr} -7x & - & 3 & = & 10 \\ & + & 3 & & +3 \\ & & \dfrac{-7x}{-7} & = & \dfrac{13}{-7} \\ & & x & = & -\dfrac{13}{7} \end{array}$$

$$\begin{array}{rrrrr} -7x & - & 3 & = & -10 \\ & + & 3 & & +3 \\ & & \dfrac{-7x}{-7} & = & \dfrac{-7}{-7} \\ & & x & = & 1 \end{array}$$

18.
$$\begin{aligned} 8|3 - 3n| - 5 &= 91 \\ +5 &\quad +5 \\ \frac{8}{8}|3 - 3n| &= \frac{96}{8} \\ |3 - 3n| &= 12 \end{aligned}$$

$$\begin{aligned} 3 - 3n &= 12 \\ -3 &\quad -3 \\ \frac{-3n}{-3} &= \frac{9}{-3} \\ n &= -3 \end{aligned}$$

$$\begin{aligned} 3 - 3n &= -12 \\ -3 &\quad -3 \\ \frac{-3n}{-3} &= \frac{-15}{-3} \\ n &= 5 \end{aligned}$$

19.
$$\begin{aligned} 5x + 3 &= 2x - 1 \\ -2x - 3 &\quad -2x - 3 \\ \frac{3x}{3} &= \frac{-4}{3} \\ x &= -\frac{4}{3} \end{aligned}$$

$$\begin{aligned} 5x + 3 &= -2x + 1 \\ +2x - 3 &\quad +2x - 3 \\ \frac{7x}{7} &= \frac{-2}{7} \\ x &= -\frac{2}{7} \end{aligned}$$

20.
$$\begin{aligned} 2 + 3x &= 4 - 2x \\ -2 + 2x &\quad -2 + 2x \\ \frac{5x}{5} &= \frac{2}{5} \\ x &= \frac{2}{5} \end{aligned}$$

$$\begin{aligned} 2 + 3x &= -4 + 2x \\ -2 - 2x &\quad -2 - 2x \\ x &= -6 \end{aligned}$$

21.
$$\begin{aligned} 3x - 4 &= 2x + 3 \\ -2x + 4 &\quad -2x + 4 \\ x &= 7 \end{aligned}$$

$$\begin{aligned} 3x - 4 &= -2x - 3 \\ +2x + 4 &\quad +2x + 4 \\ \frac{5x}{5} &= \frac{1}{5} \\ x &= \frac{1}{5} \end{aligned}$$

22.
$$\begin{aligned} 2x - 5 &= 3x + 4 \\ -3x + 5 &\quad -3x + 5 \\ -x &= 9 \\ x &= -9 \end{aligned}$$

$$\begin{aligned} 2x - 5 &= -3x - 4 \\ +3x + 5 &\quad +3x + 5 \\ \frac{5x}{5} &= \frac{1}{5} \\ x &= \frac{1}{5} \end{aligned}$$

23.

$$\begin{array}{rrrcrrr} 4x & - & 2 & = & 6x & + & 3 \\ -6x & + & 2 & & -6x & + & 2 \end{array}$$

$$\frac{-2x}{-2} = \frac{5}{-2}$$

$$x = -\frac{5}{2}$$

$$\begin{array}{rrrcrrr} 4x & - & 2 & = & -6x & - & 3 \\ +6x & + & 2 & & +6x & + & 2 \end{array}$$

$$\frac{10x}{10} = \frac{-1}{10}$$

$$x = -\frac{1}{10}$$

24.

$$\begin{array}{rrrcrrr} 3x & + & 2 & = & 2x & - & 3 \\ -2x & - & 2 & & -2x & - & 2 \end{array}$$

$$x = -5$$

$$\begin{array}{rrrcrrr} 3x & + & 2 & = & -2x & + & 3 \\ +2x & - & 2 & & +2x & - & 2 \end{array}$$

$$\frac{5x}{5} = \frac{1}{5}$$

$$x = \frac{1}{5}$$

Answer Key 2.6

1. $(3) + 1 + (4) - (1)$
   $3 + 1 + 4 - 1$
   7
2. $(5)^2 + (5) - (1)$
   $25 + 5 - 1$
   29
3. $(6) - [(6)(5) \div 6]$
   $6 - [30 \div 6]$
   $6 - 5$
   1
4. $[6 + (4) - (1)] \div 3$
   $[6 + 4 - 1] \div 3$
   $9 \div 3$
   3
5. $(5)^2 - ((3) - 1)$
   $25 - (3 - 1)$
   $25 - 2$
   23
6. $(6) + 6(4) - 4(4)$
   $6 + 24 - 16$
   14
7. $5(4) + (2)(5) \div 2$
   $20 + 10 \div 2$
   $20 + 5$
   25
8. $5((6) + (2)) + 1 + (5)$
   $5(6 + 2) + 1 + 5$
   $5(8) + 6$
   $40 + 6$
   46
9. $[4 - ((6) - (4))] \div 2 + (6)$
   $[4 - (6 - 4)] \div 2 + 6$
   $[4 - 2] \div 2 + 6$
   $2 \div 2 + 6$
   $1 + 6$
   7
10. $(4) + (5) - 1$
    $4 + 5 - 1$
    8
11. $$\begin{aligned} \frac{ab}{a} &= \frac{c}{a} \\ b &= \frac{c}{a} \end{aligned}$$

12. $\left(g = \dfrac{h}{i}\right)(i)$

$h = gi$

13. $\left(\left(\dfrac{f}{g}\right)x = b\right)\dfrac{g}{f}$

$x = \dfrac{bg}{f}$

14. $\left(p = \dfrac{3y}{q}\right)\left(\dfrac{q}{3}\right)$

$y = \dfrac{pq}{3}$

15. $\left(3x = \dfrac{a}{b}\right)\left(\dfrac{1}{3}\right)$

$x = \dfrac{a}{3b}$

16. $\left(\dfrac{ym}{b} = \dfrac{c}{d}\right)\left(\dfrac{b}{m}\right)$

$y = \dfrac{bc}{dm}$

17. $\left(V = \dfrac{4}{3}\pi r^3\right)\left(\dfrac{3}{4r^3}\right)$

$\pi = \dfrac{3V}{4r^3}$

18. $\left(E = mv^2\right)\left(\dfrac{1}{v^2}\right)$

$m = \dfrac{E}{v^2}$

19. $\left(c = \dfrac{4y}{m+n}\right)\left(\dfrac{m+n}{4}\right)$

$y = \dfrac{c(m+n)}{4}$

20. $\left(\dfrac{rs}{a-3} = k\right)\left(\dfrac{a-3}{s}\right)$

$r = \dfrac{k(a-3)}{s}$

21. $$\left(V = \frac{\pi D n}{12}\right)\left(\frac{12}{\pi n}\right)$$

$$D = \frac{12V}{\pi n}$$

22. $$\begin{array}{rcl} F & = & kR - kL \\ +kL & & +kL \\ \dfrac{F+kL}{k} & = & \dfrac{kR}{k} \\ \\ R & = & \dfrac{F+kL}{k} \end{array}$$

23. $$\begin{array}{rcl} P & = & np - nc \\ -np & & -np \\ \dfrac{P-np}{-n} & = & \dfrac{-nc}{-n} \\ \\ c & = & \dfrac{P-np}{-n} \end{array}$$

24. $$\begin{array}{rcrcr} S & = & L & + & 2B \\ -2B & & & - & 2B \\ L & = & S & - & 2B \end{array}$$

25.

$$\left(T = \frac{D-d}{L}\right)(L)$$

$$\begin{array}{rcrcr} TL & = & D & - & d \\ +d & & & + & d \\ D & = & TL & + & d \end{array}$$

26.

$$\left(I = \frac{E_a - E_q}{R}\right)(R)$$

$$\begin{array}{rcrcr} IR & = & E_a & - & E_q \\ +E_q & & & + & E_q \\ E_a & = & IR & + & E_q \end{array}$$

27. $$\begin{array}{rcl} \dfrac{L}{1+at} & = & \dfrac{L_o(1+at)}{1+at} \\ \\ L_o & = & \dfrac{L}{1+at} \end{array}$$

28.
$$\begin{aligned} 2m+p &= 4m+q \\ -2m-q & \quad -2m-q \\ \frac{p-q}{2} &= \frac{2m}{2} \\ m &= \frac{p-q}{2} \end{aligned}$$

29.
$$\left(\frac{k-m}{r}=q\right)(r)$$
$$\begin{aligned} k-m &= qr \\ +m & \quad +m \\ k &= qr+m \end{aligned}$$

30.
$$\begin{aligned} R &= aT + b \\ -b & \quad - b \\ \frac{R-b}{a} &= \frac{aT}{a} \\ T &= \frac{R-b}{a} \end{aligned}$$

31.
$$\begin{aligned} Q_1 &= PQ_2 - PQ_1 \\ +PQ_1 & \quad + PQ_1 \\ \frac{Q_1+PQ_1}{P} &= \frac{PQ_2}{P} \\ Q_2 &= \frac{Q_1+PQ_1}{P} \end{aligned}$$

32.
$$\begin{aligned} L &= \pi r_1 + \pi r_2 + 2d \\ -\pi r_2 - 2d & \quad - \pi r_2 - 2d \\ \frac{L-\pi r_2-2d}{\pi} &= \frac{\pi r_1}{\pi} \\ r_1 &= \frac{L-\pi r_2-2d}{\pi} \end{aligned}$$

33.
$$\left(R=\frac{kA(T+T_1)}{d}\right)\left(\frac{d}{kA}\right)$$
$$\begin{aligned} \frac{Rd}{kA} &= T+T_1 \\ -T & \quad -T \\ T_1 &= \frac{Rd}{kA}-T \end{aligned}$$

34.

$$\left(P = \frac{V_1(V_2 - V_1)}{g}\right)\left(\frac{g}{V_1}\right)$$

$$\begin{aligned}
\frac{Pg}{V_1} &= V_2 - V_1 \\
+V_1 &\quad\quad + V_1 \\
V_2 &= \frac{Pg}{V_1} + V_1
\end{aligned}$$

Answer Key 2.7

1. $x = ky$
2. $x = kyz$
3. $x = \frac{k}{y}$
4. $x = ky^2$
5. $x = kzy$
6. $x = \frac{k}{y^3}$
7. $x = ky^2\sqrt{z}$
8. $x = \frac{k}{y^6}$
9. $x = \frac{ky^3}{\sqrt{z}}$
10. $x = \frac{k}{y^2\sqrt{z}}$
11. $x = \frac{kzy}{p^3}$
12. $x = \frac{k}{y^3z^2}$
13. $$\begin{aligned} A &= kB \\ (15) &= k(5) \\ \frac{15}{5} &= \frac{k(5)}{5} \\ k &= 3 \end{aligned}$$
14. $$\begin{aligned} P &= kQR \\ (12) &= k(8)(3) \\ \frac{12}{24} &= \frac{k(8)(3)}{24} \\ k &= \frac{1}{2} \end{aligned}$$

15.
$$\begin{aligned} A &= \frac{k}{B} \\ (7) &= \frac{k}{(4)} \\ (4)7 &= \frac{k}{4}(4) \\ k &= 28 \end{aligned}$$

16.
$$\begin{aligned} A &= kB^2 \\ (6) &= k(3)^2 \\ \frac{6}{9} &= \frac{k(3)^2}{9} \\ k &= \frac{6}{9} \text{ or } \frac{2}{3} \end{aligned}$$

17.
$$\begin{aligned} C &= kAB \\ (24) &= k(3)(2) \\ \frac{24}{6} &= \frac{k(3)(2)}{6} \\ k &= 4 \end{aligned}$$

18.
$$\begin{aligned} y &= \frac{k}{x^3} \\ (54) &= \frac{k}{(3)^3} \\ 54 &= \frac{k}{27} \\ 27 \cdot 54 &= \frac{k}{27} \cdot 27 \\ k &= 1458 \end{aligned}$$

19.
$$\begin{aligned} x &= kY \\ (12) &= k(8) \\ \frac{12}{8} &= \frac{k(8)}{8} \\ k &= \frac{12}{8} \text{ or } \frac{3}{2} \end{aligned}$$

20.
$$\begin{aligned} A &= kB^2\sqrt{C} \\ (25) &= k(5)^2\sqrt{(9)} \\ 25 &= k(75) \\ k &= \frac{25}{75} \\ k &= \frac{1}{3} \end{aligned}$$

21.
$$\begin{aligned} y &= \frac{kmn^2}{d} \\ (10) &= \frac{k(4)(5)^2}{(6)} \\ k &= \frac{105 \cdot 63}{(4)(5)^2} \\ k &= \frac{3}{5} \end{aligned}$$

22.
$$\begin{aligned} P &= \frac{kT}{V} \\ (10) &= \frac{k(250)}{(400)} \\ k &= \frac{10(400)}{250} \\ k &= 16 \end{aligned}$$

23.
$$I = kV$$

1st Data

$$\begin{aligned} I &= 5 \text{ A} \\ V &= 15 \text{ V} \\ k &= \text{find} \end{aligned}$$

$$\begin{aligned} I &= kV \\ 5 \text{ A} &= k(15 \text{ V}) \\ k &= \frac{5 \text{ A}}{15 \text{ V}} \\ k &= \frac{1}{3} \text{ A/V} \end{aligned}$$

2nd Data

$$\begin{aligned} I &= \text{find} \\ k &= \frac{1}{3} \\ V &= 25 \text{ V} \end{aligned}$$

$$\begin{aligned} I &= kV \\ I &= \left(\frac{1}{3}\right)(25) \\ I &= 8\frac{1}{3} \text{ A} \end{aligned}$$

24.

$$I = \frac{k}{R}$$

1st Data

$$\begin{aligned} I &= 12 \text{ A} \\ k &= \text{find} \\ R &= 240\Omega \end{aligned}$$

$$\begin{aligned} I &= \frac{k}{R} \\ 12 \text{ A} &= \frac{k}{240\Omega} \\ k &= (12 \text{ A})(240\Omega) \\ k &= 2880 \text{ A}\Omega \end{aligned}$$

2nd Data

$$\begin{aligned} I &= \text{find} \\ k &= 2880 \\ R &= 540\Omega \end{aligned}$$

$$\begin{aligned} I &= \frac{k}{R} \\ I &= \frac{2880 \text{ A}\Omega}{540\Omega} \\ I &= 5.\bar{3} \text{ A or } 5\frac{1}{3} \end{aligned}$$

25.

$$d_{\text{s}} = km$$

1st Data

$$\begin{aligned} d_{\text{s}} &= 18 \text{ cm} \\ k &= \text{find} \\ m &= 3 \text{ kg} \end{aligned}$$

$$\begin{aligned} 18 \text{ cm} &= k(3 \text{ kg}) \\ k &= \frac{18 \text{ cm}}{3 \text{ kg}} \\ k &= 6 \text{ cm/kg} \end{aligned}$$

2nd Data

$$\begin{aligned} d_{\text{s}} &= \text{find} \\ k &= 6 \text{ cm/kg} \\ m &= 5 \text{ kg} \end{aligned}$$

$$\begin{aligned} d_{\text{s}} &= (6 \text{ cm/kg})(5 \text{ kg}) \\ d_{\text{s}} &= 30 \text{ cm} \end{aligned}$$

26.

$$V = \frac{k}{P}$$

1st Data

$$\begin{aligned} P &= 32 \text{ kg/cm}^2 \\ V &= 200 \text{ cm}^3 \\ k &= \text{find} \end{aligned}$$

$$\begin{aligned} 200 \text{ cm}^3 &= \frac{k}{32 \text{ kg/cm}^2} \\ k &= (200 \text{ cm}^3)(32 \text{ kg/cm}^2) \\ k &= 6400 \text{ kg cm} \end{aligned}$$

2nd Data

$$\begin{aligned} P &= 40 \\ V &= \text{find} \\ k &= 6400 \end{aligned}$$

$$\begin{aligned} V &= \frac{6400}{40} \\ V &= 160 \text{ cm}^3 \end{aligned}$$

27.

$$c = kP$$

1st Data

$$\begin{aligned} c &= 60,000 \\ k &= \text{find} \\ P &= 250 \end{aligned}$$

$$\begin{aligned} 60,000 &= k(250) \\ k &= \frac{60,000}{250} \\ k &= 240 \end{aligned}$$

2nd Data

$$\begin{aligned} c &= \text{find} \\ k &= 240 \\ P &= 1,000,000 \end{aligned}$$

$$\begin{aligned} c &= (240)(1,000,000) \\ c &= 240,000,000 \text{ or } 240 \text{ million} \end{aligned}$$

28.

$$t = \frac{k}{b}$$

1st Data

$$\begin{aligned} t &= 5 \text{ h} \\ k &= \text{find} \\ b &= 7 \end{aligned}$$

$$\begin{aligned} 5 \text{ h} &= \frac{k}{7} \\ k &= (5 \text{ h})(7) \\ k &= 35 \end{aligned}$$

2nd Data

$$\begin{aligned} t &= \text{find} \\ k &= 35 \\ b &= 10 \end{aligned}$$

$$\begin{aligned} t &= \frac{35}{10} \\ t &= 3.5 \text{ h} \end{aligned}$$

29.

$$\lambda = \frac{k}{f}$$

1st Data	2nd Data
$\lambda = 250$ m	$\lambda =$ find
$k =$ find	$k = 300,000$
$f = 1200$ kHz	$f = 60$ kHz

$$250 = \frac{k}{1200} \qquad \lambda = \frac{300,000}{60}$$

$$k = (250)(1200) \qquad \lambda = 5000 \text{ m}$$

$$k = 300,000$$

30.

$$w = km$$

1st Data	2nd Data
$w = 64$ kg	$w =$ find
$k =$ find	$k = \frac{2}{3}$
$m = 96$ kg	$m = 60$ kg

$$64 = k(96)$$

$$k = \frac{64}{96}$$

$$k = \frac{2}{3}$$

$$w = \left(\frac{2}{3}\right)(60 \text{ kg})$$

$$w = 40 \text{ kg}$$

31.

$$t = \frac{d}{v}$$

1st Data	2nd Data
$t = 5$ h	$t = 4.2$ h
$d =$ find	$d = 400$ km
$v = 80$ km/h	$v =$ find

$$5 \text{ h} = \frac{d}{80 \text{ km/h}} \qquad 4.2 = \frac{400}{v}$$

$$d = 5(80) \qquad v = \frac{400}{4.2}$$

$$d = 400 \text{ km}$$

$$v = 95.24 \text{ km/h}$$

32.

$V = khr^2$

$$\begin{array}{rcl} & & \textbf{1st Data} \\ V &=& 33.5 \text{ cm}^3 \\ k &=& \text{find} \\ h &=& 8 \text{ cm} \\ r &=& 2 \text{ cm} \end{array} \qquad \begin{array}{rcl} & & \textbf{2nd Data} \\ V &=& \text{find} \\ k &=& 1.046875 \\ h &=& 6 \text{ cm} \\ r &=& 4 \text{ cm} \end{array}$$

$$\begin{aligned} 33.5 &= k(8)(2)^2 \\ k &= \frac{33.5}{(8)(2)^2} \\ k &= 1.046875 \end{aligned} \qquad \begin{aligned} V &= khr^2 \\ V &= (1.046875)(6)(4)^2 \\ V &= 100.5 \text{ cm}^3 \end{aligned}$$

33.

$F_{\mathrm{e}} = \dfrac{kv^2}{r}$

$$\begin{array}{rcl} & & \textbf{1st Data} \\ F_{\mathrm{e}} &=& 100 \text{ N} \\ k &=& \text{find} \\ v &=& 10 \text{ m/s} \\ r &=& 0.5 \text{ m} \end{array} \qquad \begin{array}{rcl} & & \textbf{2nd Data} \\ F_{\mathrm{e}} &=& \text{find} \\ k &=& 0.5 \\ v &=& 25 \text{ m/s} \\ r &=& 1.0 \text{ m} \end{array}$$

$$\begin{aligned} 100 \text{ N} &= \frac{k(10 \text{ m/s})^2}{0.5 \text{ m}} \\ k &= \frac{(0.5)(100)}{(10)^2} \\ k &= 0.5 \end{aligned} \qquad \begin{aligned} F_{\mathrm{e}} &= \frac{0.5(25)^2}{1.0} \\ F_{\mathrm{e}} &= 312.5 \text{ N} \end{aligned}$$

34.

$L_{\max} = \dfrac{kd^4}{h^2}$

1st Data

$$\begin{aligned} L_{\max} &= 64 \text{ tonnes} \\ k &= \text{find} \\ d &= 2.0 \text{ m} \\ h &= 8.0 \text{ m} \end{aligned}$$

$$\begin{aligned} 64 &= \frac{k(2)^4}{(8)^2} \\ k &= \frac{64(8)^2}{(2)^4} \\ k &= 256 \end{aligned}$$

2nd Data

$$\begin{aligned} L_{\max} &= \text{find} \\ k &= 256 \\ d &= 3.0 \text{ m} \\ h &= 12.0 \text{ m} \end{aligned}$$

$$\begin{aligned} L_{\max} &= \frac{(256)(3.0)^4}{(12.0)^2} \\ L_{\max} &= 144 \text{ tonnes} \end{aligned}$$

35.

$$V = \frac{kT}{P}$$

1st Data

$$\begin{aligned} V &= 225 \text{ cc} \\ k &= \text{find} \\ T &= 300 \text{ K} \\ P &= 100 \text{ N/cm}^2 \end{aligned}$$

$$\begin{aligned} V &= \frac{kT}{P} \\ 225 &= \frac{k(300)}{100} \\ k &= \frac{225(100)}{300} \\ k &= 75 \end{aligned}$$

2nd Data

$$\begin{aligned} V &= \text{find} \\ k &= 75 \\ T &= 270 \\ P &= 150 \end{aligned}$$

$$\begin{aligned} V &= \frac{75(270)}{150} \\ V &= 135 \text{ cc} \end{aligned}$$

36.

$$R = \frac{kl}{d^2}$$

1st Data

$$\begin{aligned} R &= 20\Omega \\ k &= \text{find} \\ l &= 5.0 \text{ m} \\ d &= 0.25 \text{ cm} \end{aligned}$$

$$\begin{aligned} R &= \frac{kl}{d^2} \\ 20\Omega &= \frac{k(5.0 \text{ m})}{(0.25 \text{ cm})^2} \\ k &= \frac{(20\Omega)(0.25 \text{ cm})^2}{5.0 \text{ m}} \\ k &= 0.25 \end{aligned}$$

2nd Data

$$\begin{aligned} R &= \text{find} \\ k &= 0.25 \\ l &= 10.0 \text{ m} \\ d &= 0.50 \text{ cm} \end{aligned}$$

$$\begin{aligned} R &= \frac{(0.25)(10.0 \text{ m})}{(0.50 \text{ cm})^2} \\ R &= 10\Omega \end{aligned}$$

37.

$V = khd^2$

1st Data

$$\begin{aligned} V &= 377 \text{ m}^3 \\ k &= \text{find} \\ h &= 30 \text{ m} \\ d &= 2.0 \text{ m} \end{aligned}$$

$$\begin{aligned} 377 \text{ m}^3 &= k(30)(2.0)^2 \\ k &= \frac{377}{(30)(2.0)^2} \\ k &= 3.1416 \end{aligned}$$

2nd Data

$$\begin{aligned} V &= 225 \text{ m}^3 \\ k &= 3.1416 \\ h &= \text{find} \\ d &= 1.75 \text{ m} \end{aligned}$$

$$\begin{aligned} 225 &= \pi h(1.75)^2 \\ h &= \frac{225}{\pi(1.75)^2} \\ h &= 23.4 \text{ m} \end{aligned}$$

Answer Key 2.8

1st

5

31

3 6

$\Longrightarrow$

$$\begin{aligned} 3 \cdot 5 + 6 \\ 15 + 6 &= 21 \end{aligned}$$

2nd

3

42

11 9

$\Longrightarrow$

$$\begin{aligned} 3 \cdot 11 + 9 \\ 33 + 9 &= 42 \end{aligned}$$

3rd

12

64

5 X

$\Longrightarrow$

$$\begin{array}{rcccr} 5 \cdot 12 & + & x & = & 64 \\ 60 & + & x & = & 64 \\ -60 & & & & -60 \\ & & x & = & 4 \end{array}$$

Answer Key 3.1

1. $A = (4, 4)$ $\quad$ $B = (2, 1)$
   $C = (-3, -1)$ $\quad$ $D = (5, 0)$
   $E = (-5, 3)$ $\quad$ $F = (2, -3)$
   $G = (0, 4)$ $\quad$ $H = (5, -4)$

Answer Key 3.2

1. $$\begin{aligned} d^2 &= \Delta x^2 &&+ \Delta y^2 \\ d^2 &= (6 - -6)^2 &&+ (4 - -1)^2 \\ d^2 &= 12^2 &&+ 5^2 \\ d^2 &= 144 &&+ 25 \\ d^2 &= 169 \\ d^2 &= \sqrt{169} \\ d &= 13 \end{aligned}$$
2. $$\begin{aligned} d^2 &= \Delta x^2 &&+ \Delta y^2 \\ d^2 &= (5 - 1)^2 &&+ (-1 - -4)^2 \\ d^2 &= 4^2 &&+ 3^2 \\ d^2 &= 16 &&+ 9 \\ d^2 &= 25 \\ d^2 &= \sqrt{25} \\ d &= 5 \end{aligned}$$
3. $$\begin{aligned} d^2 &= \Delta x^2 &&+ \Delta y^2 \\ d^2 &= (3 - -5)^2 &&+ (5 - -1)^2 \\ d^2 &= 8^2 &&+ 6^2 \\ d^2 &= 64 &&+ 36 \\ d^2 &= 100 \\ d^2 &= \sqrt{100} \\ d &= 10 \end{aligned}$$
4. $$\begin{aligned} d^2 &= \Delta x^2 &&+ \Delta y^2 \\ d^2 &= (12 - 6)^2 &&+ (4 - -4)^2 \\ d^2 &= 6^2 &&+ 8^2 \\ d^2 &= 36 &&+ 64 \\ d^2 &= 100 \\ d^2 &= \sqrt{100} \\ d &= 10 \end{aligned}$$
5. $$\begin{aligned} d^2 &= \Delta x^2 &&+ \Delta y^2 \\ d^2 &= (4 - -8)^2 &&+ (3 - -2)^2 \\ d^2 &= 12^2 &&+ 5^2 \\ d^2 &= 144 &&+ 25 \\ d^2 &= 169 \\ d^2 &= \sqrt{169} \\ d &= 13 \end{aligned}$$

6. $$\begin{array}{rclcl} d^2 &=& \Delta x^2 &+& \Delta y^2 \\ d^2 &=& (7-3)^2 &+& (1--2)^2 \\ d^2 &=& 4^2 &+& 3^2 \\ d^2 &=& 16 &+& 9 \\ d^2 &=& 25 && \\ d^2 &=& \sqrt{25} && \\ d &=& 5 && \end{array}$$

7. $$\begin{array}{rclcl} d^2 &=& \Delta x^2 &+& \Delta y^2 \\ d^2 &=& (-2--10)^2 &+& (0--6)^2 \\ d^2 &=& 8^2 &+& 6^2 \\ d^2 &=& 64 &+& 36 \\ d^2 &=& 100 && \\ d^2 &=& \sqrt{100} && \\ d &=& 10 && \end{array}$$

8. $$\begin{array}{rclcl} d^2 &=& \Delta x^2 &+& \Delta y^2 \\ d^2 &=& (14-8)^2 &+& (6--2)^2 \\ d^2 &=& 6^2 &+& 8^2 \\ d^2 &=& 36 &+& 64 \\ d^2 &=& 100 && \\ d^2 &=& \sqrt{100} && \\ d &=& 10 && \end{array}$$

9. $$\left(\frac{6+-6}{2}, \frac{5+-1}{2}\right) \Rightarrow \left(\frac{0}{2}, \frac{4}{2}\right) \Rightarrow (0,2)$$

10. $$\left(\frac{5+1}{2}, \frac{-2+-4}{2}\right) \Rightarrow \left(\frac{6}{2}, \frac{-6}{2}\right) \Rightarrow (3,-3)$$

11. $$\left(\frac{3+-5}{2}, \frac{5+-1}{2}\right) \Rightarrow \left(\frac{-2}{2}, \frac{4}{2}\right) \Rightarrow (-1,2)$$

12. $$\left(\frac{12+6}{2}, \frac{4+-4}{2}\right) \Rightarrow \left(\frac{18}{2}, \frac{0}{2}\right) \Rightarrow (9,0)$$

13. $$\left(\frac{-8+6}{2}, \frac{-1+7}{2}\right) \Rightarrow \left(\frac{-2}{2}, \frac{6}{2}\right) \Rightarrow (-1,3)$$

14. $$\left(\frac{1+3}{2}, \frac{-6+-2}{2}\right) \Rightarrow \left(\frac{4}{2}, \frac{-8}{2}\right) \Rightarrow (2,-4)$$

15. $$\left(\frac{-7+3}{2}, \frac{-1+9}{2}\right) \Rightarrow \left(\frac{-4}{2}, \frac{8}{2}\right) \Rightarrow (-2,4)$$

16. $$\left(\frac{2+12}{2}, \frac{-2+4}{2}\right) \Rightarrow \left(\frac{14}{2}, \frac{2}{2}\right) \Rightarrow (7,1)$$

Answer Key 3.3

1. $m = -\frac{3}{5}$
2. $m = \frac{5}{4}$
3. $m = -4$
4. $m = \frac{2}{7}$
5. $m = -\frac{1}{3}$
6. $m = \frac{3}{5}$
7. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{15-10}{-2-2} \Rightarrow \frac{-5}{4}$
8. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{-12-2}{-6-1} \Rightarrow \frac{-14}{-7} \Rightarrow 2$
9. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{0-10}{0--5} \Rightarrow \frac{-10}{5} \Rightarrow -2$
10. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{8--2}{7-2} \Rightarrow \frac{10}{5} \Rightarrow 2$
11. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{-10-6}{-8-4} \Rightarrow \frac{-16}{-12} \Rightarrow \frac{4}{3}$
12. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{-6-6}{9--3} \Rightarrow \frac{-12}{12} \Rightarrow -1$
13. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{-4--4}{10--2} \Rightarrow \frac{0}{12} \Rightarrow 0$
14. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{0-5}{2-3} \Rightarrow \frac{-5}{-1} \Rightarrow 5$
15. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{8-4}{-6--4} \Rightarrow \frac{4}{-2} \Rightarrow -2$
16. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{-7--6}{-7-9} \Rightarrow \frac{-1}{-16} \Rightarrow \frac{1}{16}$
17. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{4--9}{6-2} \Rightarrow \frac{13}{4}$
18. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{0-2}{5--6} \Rightarrow \frac{-2}{11}$
19. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{0-0}{-5--5} \Rightarrow \frac{0}{0} \therefore \text{Undefined}$
20. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{-13-11}{-3-8} \Rightarrow \frac{-24}{-11} \Rightarrow \frac{24}{11}$
21. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{-7-9}{1--7} \Rightarrow \frac{-16}{8} \Rightarrow -2$
22. $m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{7--2}{1-1} \Rightarrow \frac{9}{0} \therefore \text{Undefined}$

23. $m = \dfrac{\Delta y}{\Delta x} \Rightarrow \dfrac{-9 - -4}{-8 - 7} \Rightarrow \dfrac{-5}{-15} \Rightarrow \dfrac{1}{3}$

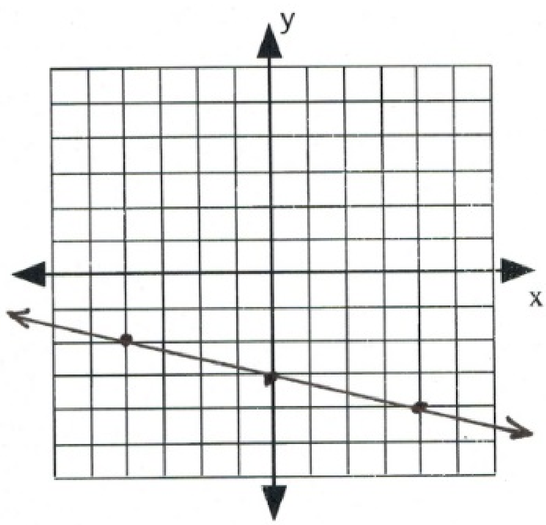
24. $m = \dfrac{\Delta y}{\Delta x} \Rightarrow \dfrac{-3 - -5}{4 - -8} \Rightarrow \dfrac{2}{12} \Rightarrow \dfrac{1}{6}$

25. $m = \dfrac{\Delta y}{\Delta x} \Rightarrow \dfrac{4 - 7}{-8 - -5} \Rightarrow \dfrac{-3}{-3} \Rightarrow 1$

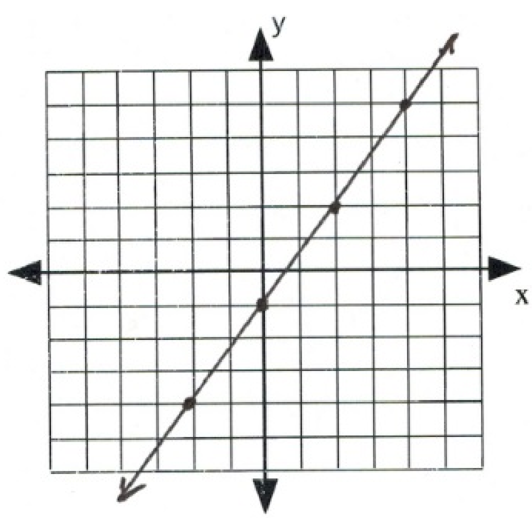
26. $m = \dfrac{\Delta y}{\Delta x} \Rightarrow \dfrac{1 - 5}{5 - 9} \Rightarrow \dfrac{-4}{-4} \Rightarrow 1$

Answer Key 3.4

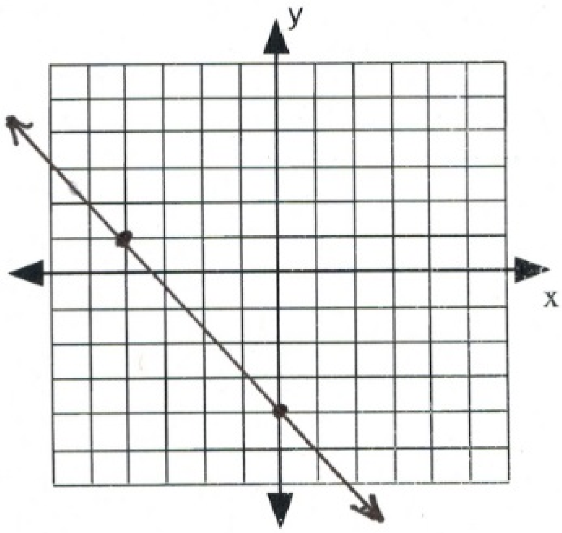
For questions 1 to 10, sketch the linear equation using the slope intercept method.



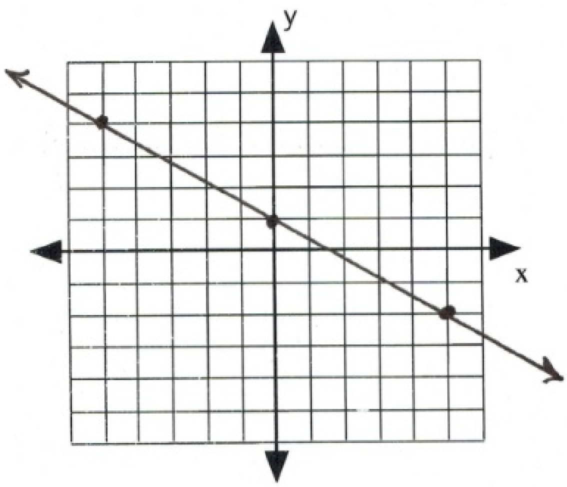
1. $y=-\frac{1}{4}x-3$



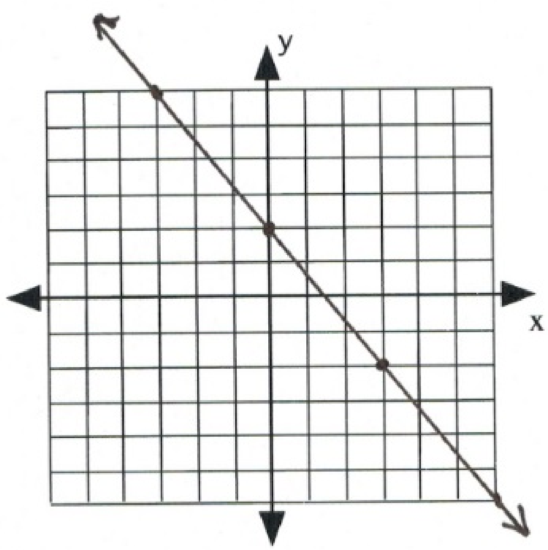
2. $y=\frac{3}{2}x-1$



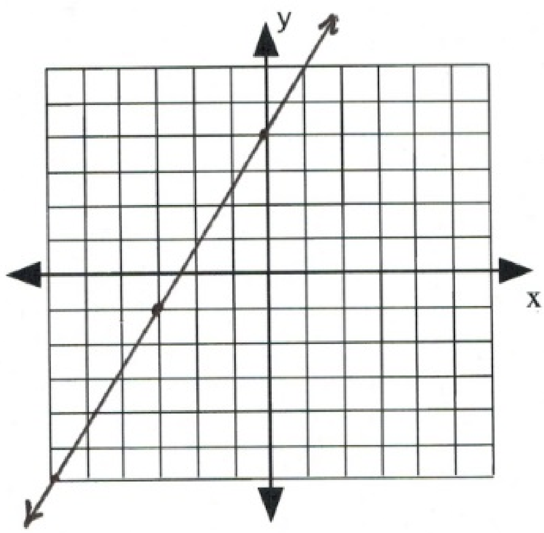
3. $y = -\dfrac{5}{4}x - 4$



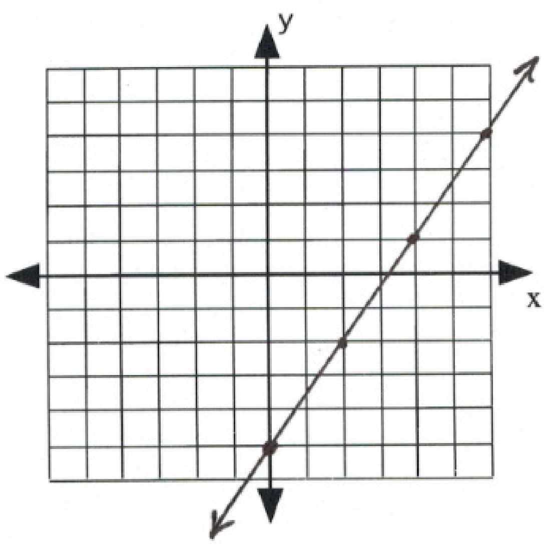
4. $y = -\dfrac{3}{5}x + 1$



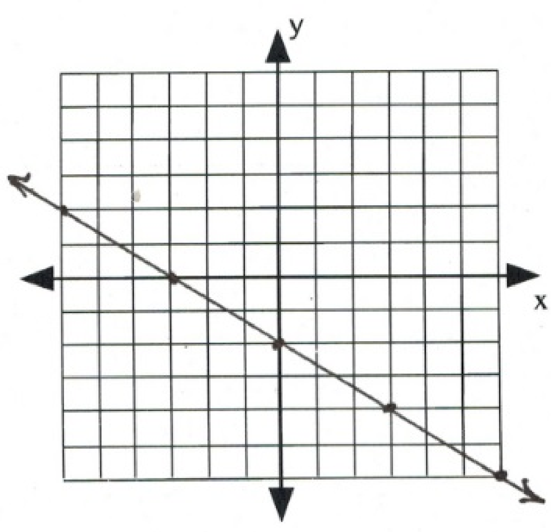
5. $y = -\dfrac{4}{3}x + 2$



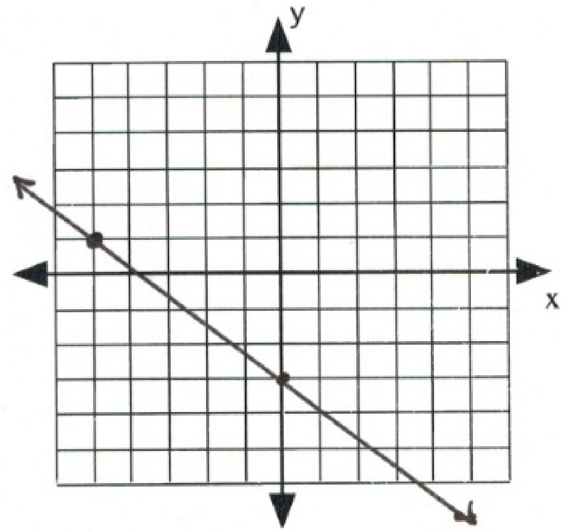
6. $y=\dfrac{5}{3}x+4$



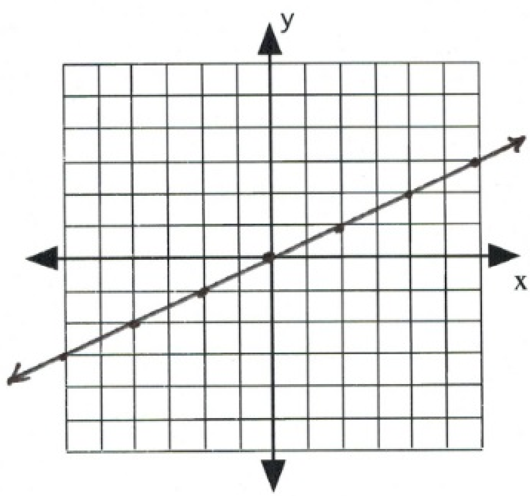
7. $y=\dfrac{3}{2}x-5$



8. $y=-\dfrac{2}{3}x-2$

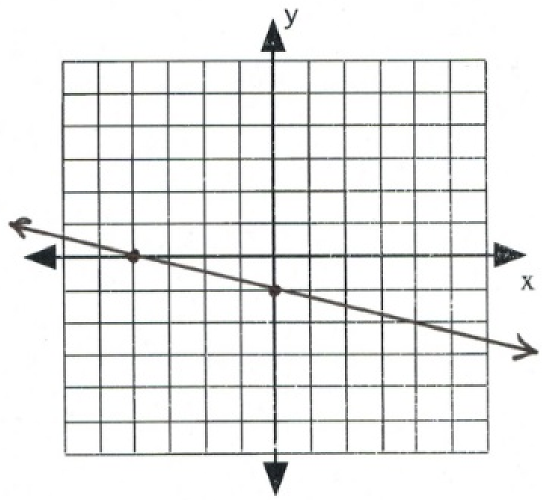


9. $y = -\dfrac{4}{5}x - 3$

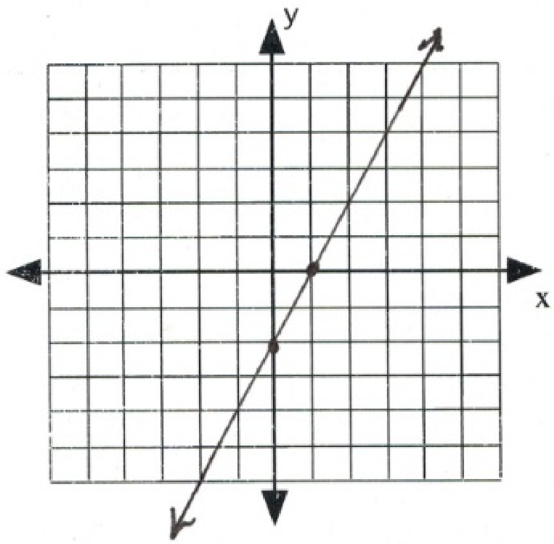


10. $y = \dfrac{1}{2}x$

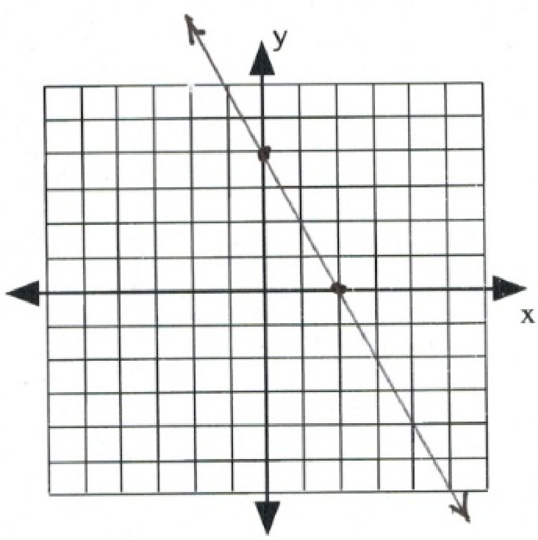
For questions 11 to 20, sketch the linear equation using the x and y intercepts.



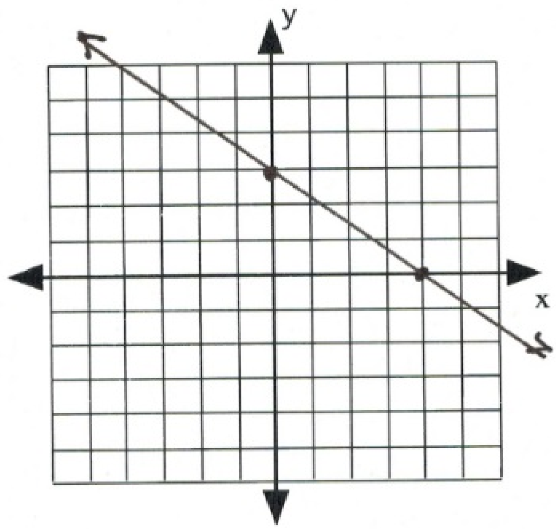
11. $x + 4y = -4$



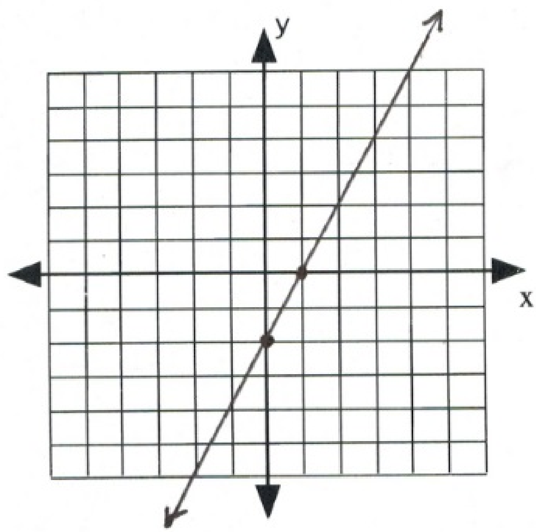
12. $2x - y = 2$



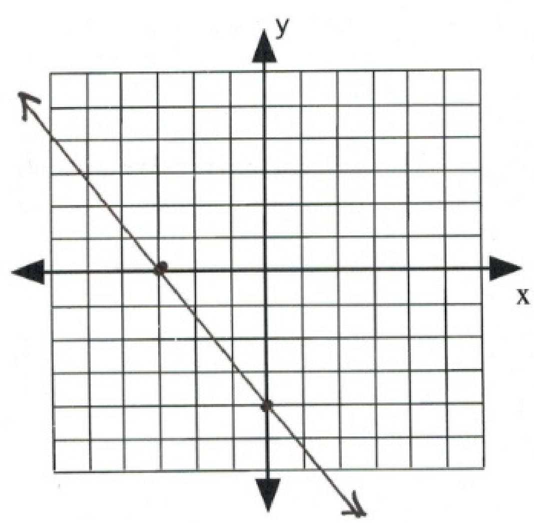
13. $2x + y = 4$



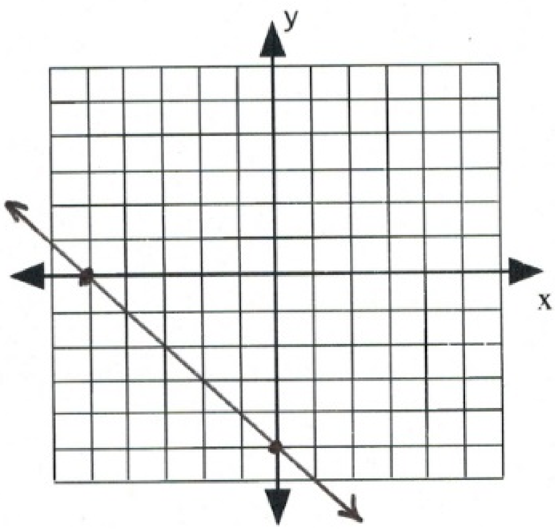
14. $3x + 4y = 12$



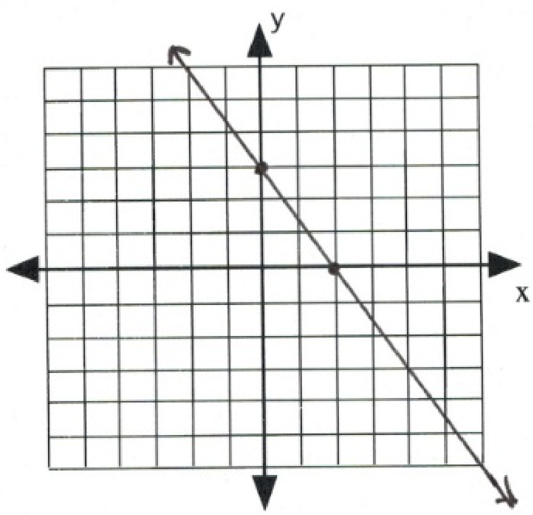
15. $2x - y = 2$



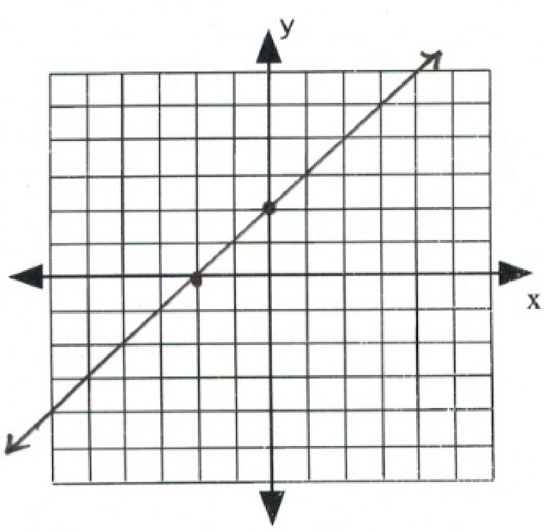
16. $4x + 3y = -12$



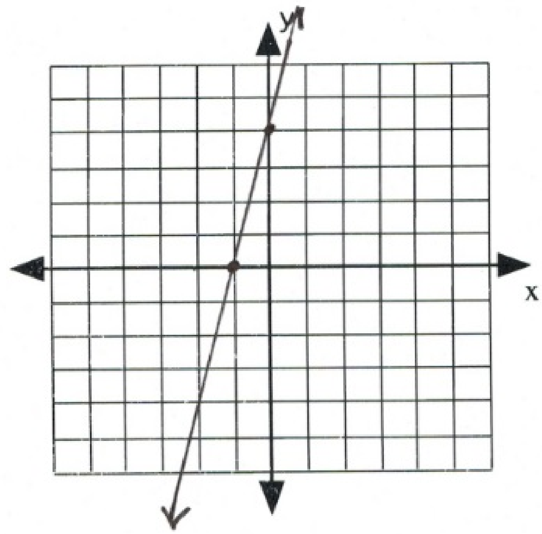
17. $x + y = -5$



18. $3x + 2y = 6$

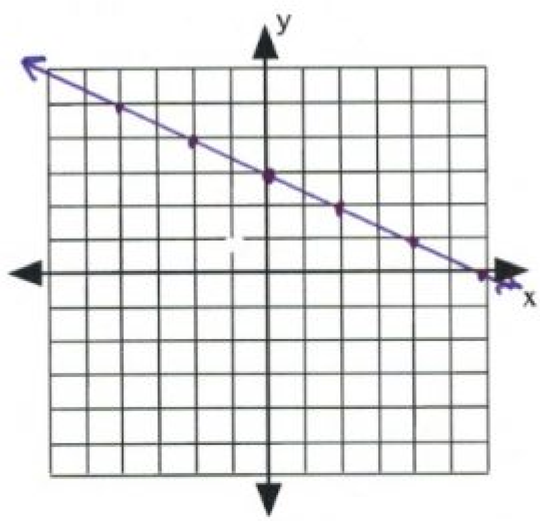


19. $x - y = -2$

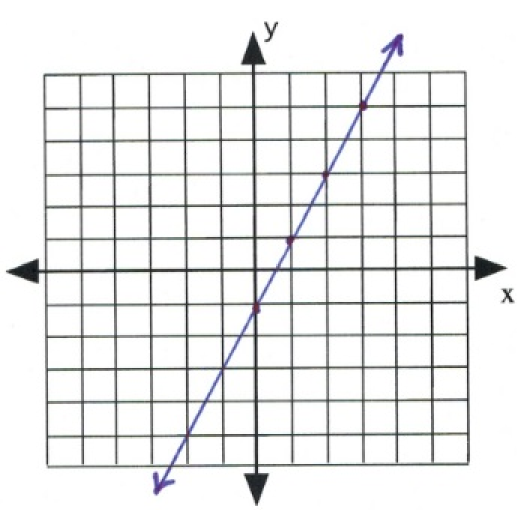


20. $4x - y = -4$

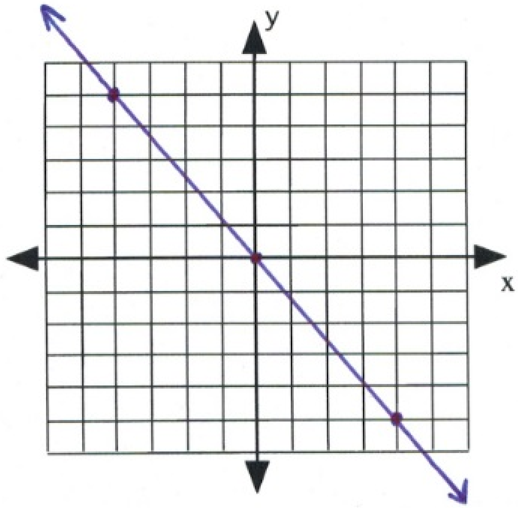
For questions 21 to 28, sketch the linear equation using any method.



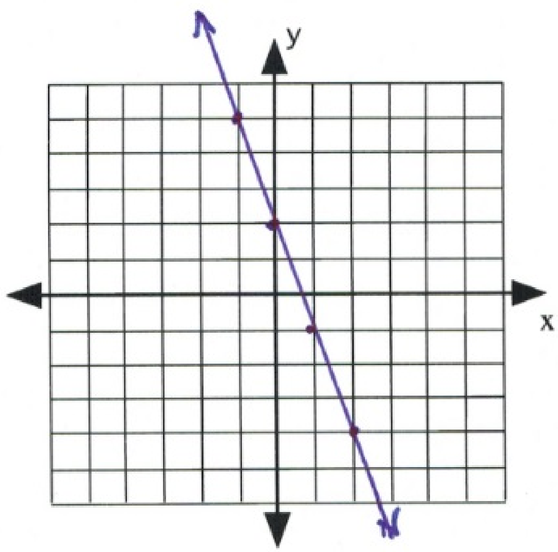
21. $y = -\dfrac{1}{2}x + 3$



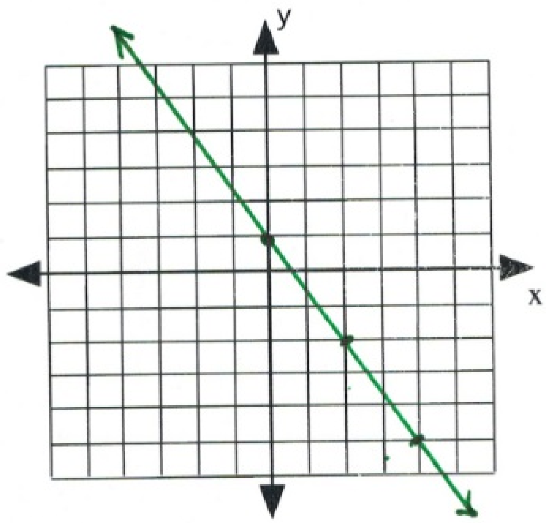
22. $y = 2x - 1$



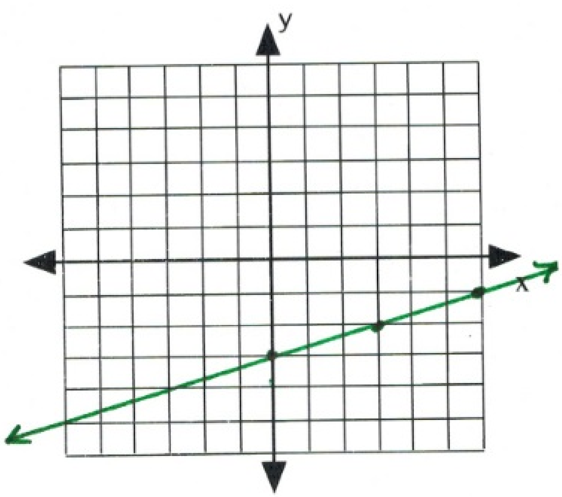
23. $y = -\dfrac{5}{4}x$



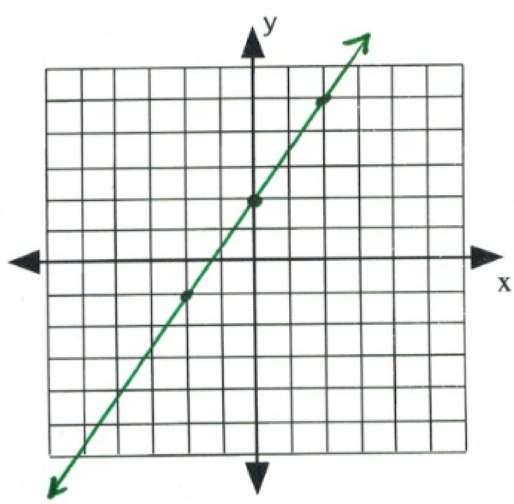
24. $y = -3x + 2$



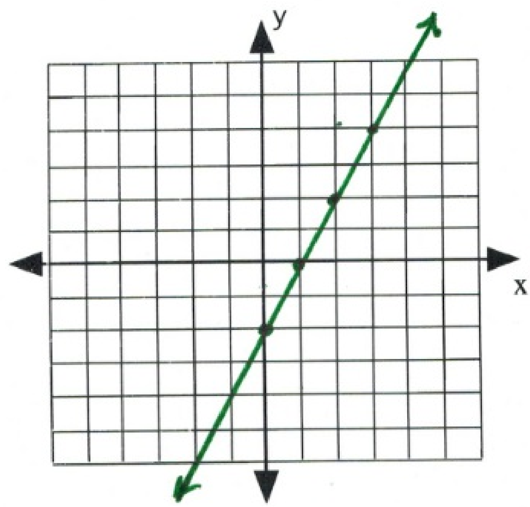
25. $y=-\dfrac{3}{2}x+1$



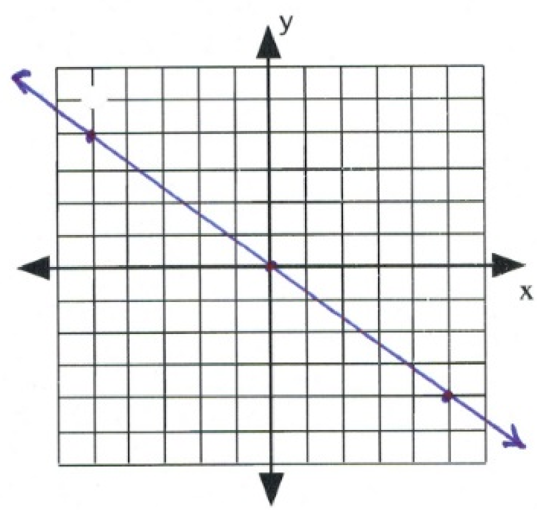
26. $y=\dfrac{1}{3}x-3$



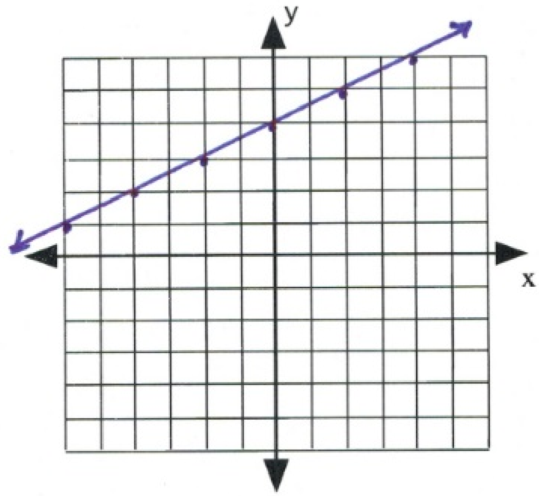
27. $y=\dfrac{3}{2}x+2$



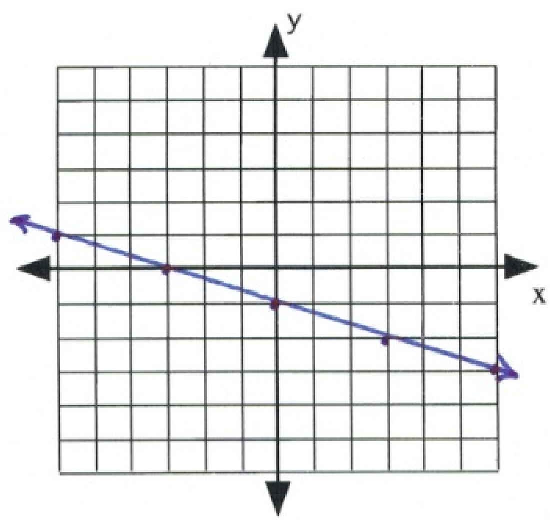
28. $y = 2x - 2$



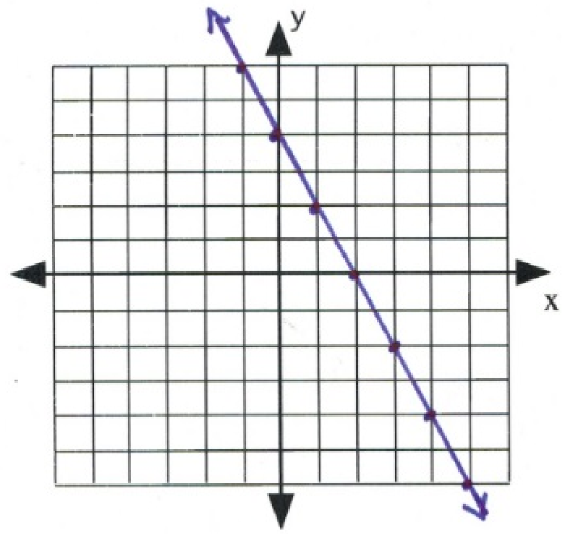
29.
$$\begin{array}{ccccccc} y & + & 3 & = & -\frac{4}{5}x & + & 3 \\ & - & 3 & & & - & 3 \\ & & y & = & -\frac{4}{5}x & & \end{array}$$



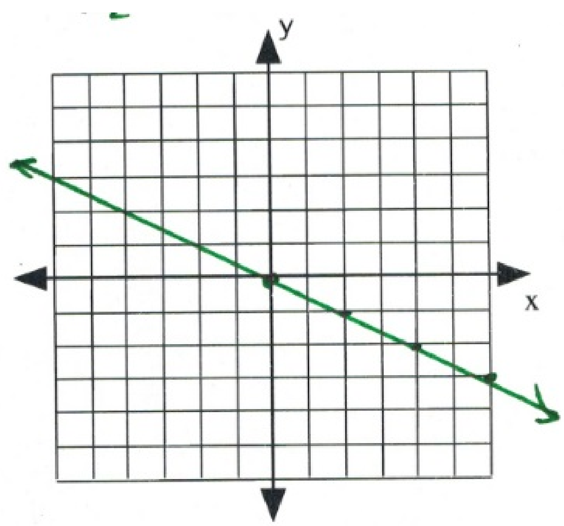
30.
$$\begin{array}{rrrrrrr} y & - & 4 & = & \frac{1}{2}x & & \\ & + & 4 & & & + & 4 \\ & & y & = & \frac{1}{2}x & + & 4 \end{array}$$



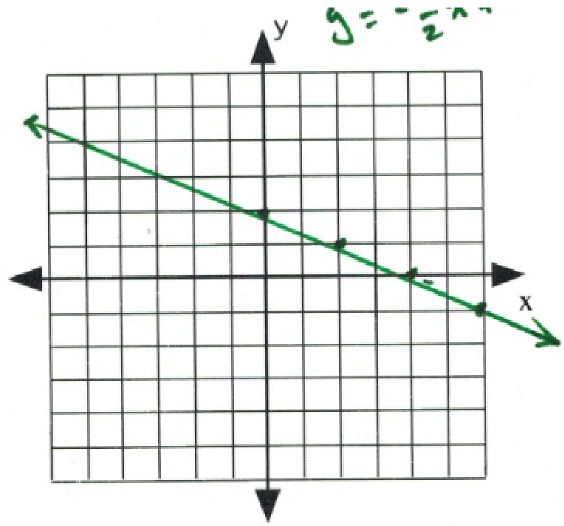
31.
$$\begin{array}{rrrrrrr} x & + & 5y & = & -3 & + & 2y \\ -x & - & 2y & & -x & - & 2y \\ & & 3y & = & -x & - & 3 \\ & & y & = & -\frac{1}{3}x & - & 1 \end{array}$$



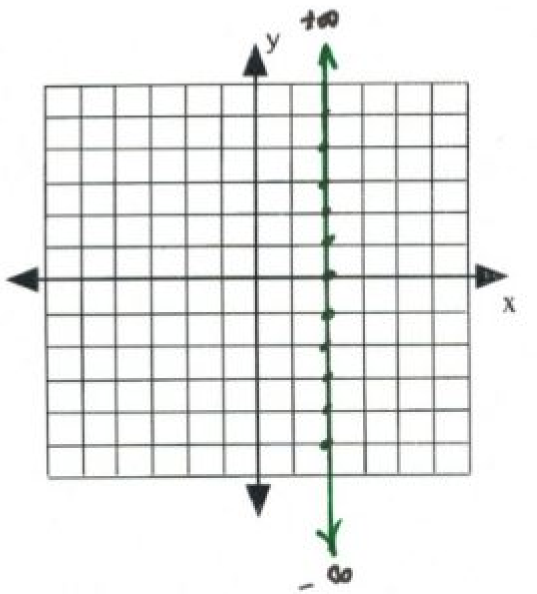
32. $$\begin{array}{rcrcrcrcr} 3x & - & y & = & 4 & + & x & - & 2y \\ -3x & + & 2y & & & - & 3x & + & 2y \\ & & y & = & -2x & + & 4 & & \end{array}$$



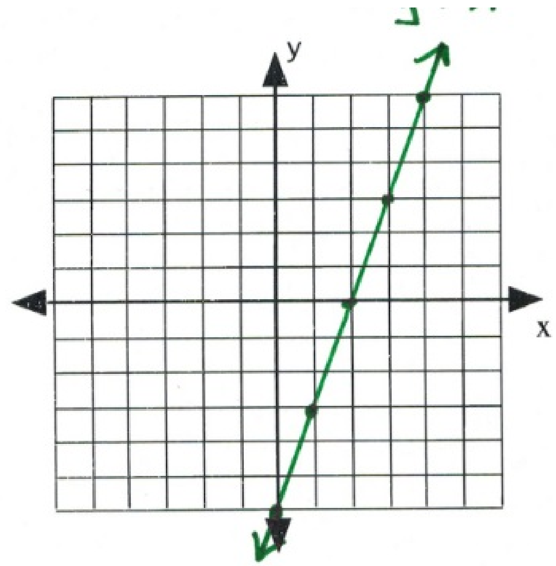
33. $$\begin{array}{rcrcrcr} 4x & + & 3y & = & 5(x & + & y) \\ 4x & + & 3y & = & 5x & + & 5y \\ -4x & - & 5y & & -4x & - & 5y \\ & & \dfrac{-2y}{-2} & = & \dfrac{x}{-2} & & \\ & & y & = & -\dfrac{1}{2}x & & \end{array}$$



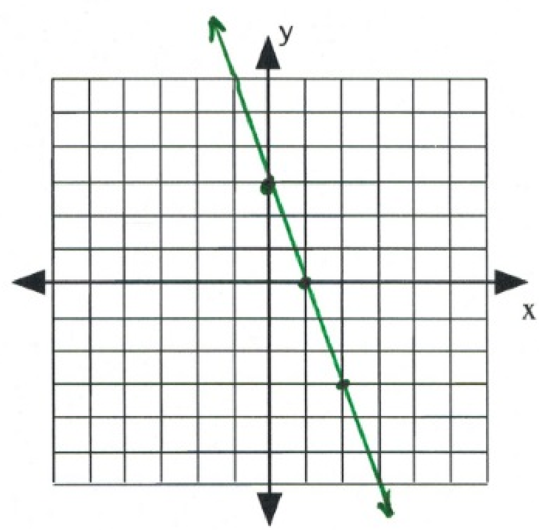
34.
$$\begin{array}{rcrcrcrcr} 3x & + & 4y & = & 12 & - & 2y \\ -3x & + & 2y & & -3x & + & 2y \\ & & \dfrac{6y}{6} & = & \dfrac{-3x}{6} & + & \dfrac{12}{6} \\ & & y & = & -\dfrac{1}{2}x & + & 2 \end{array}$$



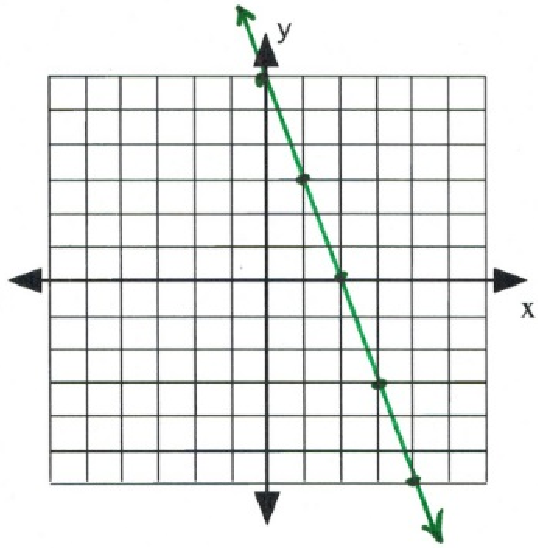
35.
$$\begin{array}{rcrcrcr} 2x & - & y & = & 2 & - & y \\ & + & y & & & + & y \\ & & 2x & = & 2 & & \\ & & x & = & 1 & & \end{array}$$



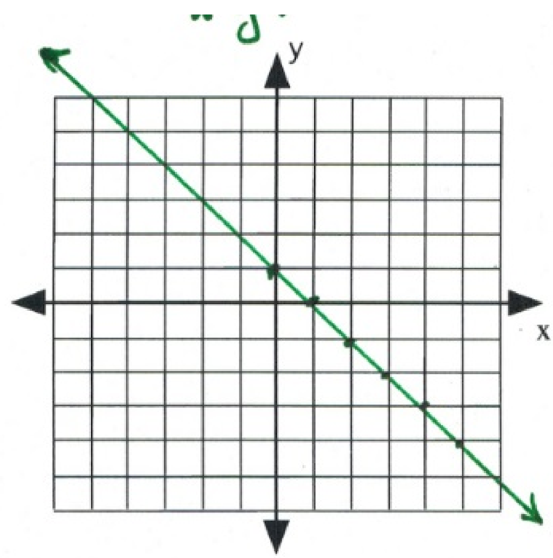
36.
$$\begin{array}{rcrcrcrcl} 7x & + & 3y & = & 2(2x & + & 2y) & + & 6 \\ 7x & + & 3y & = & 4x & + & 4y & + & 6 \\ -7x & - & 4y & & -7x & - & 4y & & \\ & & -y & = & -3x & + & 6 & & \\ & & y & = & 3x & - & 6 & & \end{array}$$



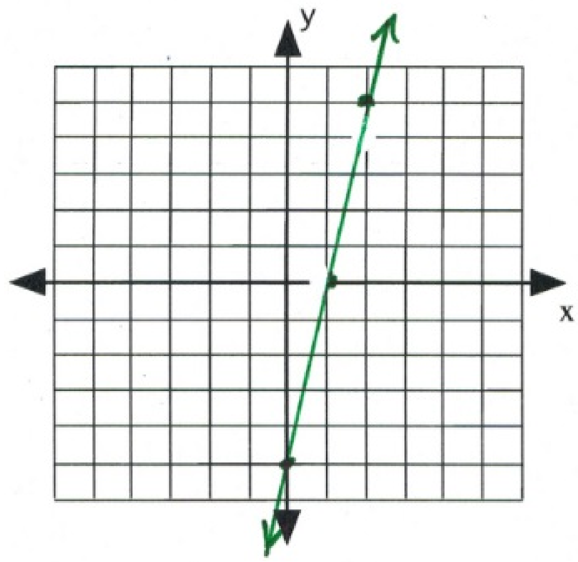
37.
$$\begin{array}{rcrcrcl} x & + & y & = & -2x & + & 3 \\ -x & & & & -\ x & & \\ & & y & = & -3x & + & 3 \end{array}$$



$$\begin{array}{rrrrrrr} & 3x & + & 4y & = & 3y & + & 6 \\ 38. & -3x & - & 3y & & -3y & - & 3x \\ & & & y & = & -3x & + & 6 \end{array}$$



$$\begin{array}{rrrrrrrrr} & 2(x & + & y) & = & -3(x & + & y) & + & 5 \\ & 2x & + & 2y & = & -3x & - & 3y & + & 5 \\ 39. & -2x & + & 3y & & -2x & + & 3y & & \\ & & & 5y & = & -5x & + & 5 & & \\ & & & y & = & -x & + & 1 & & \end{array}$$



40.
$$\begin{array}{rcrcrcrcr}
9x & - & y & = & 4x & + & 5 \\
-9x & & & & -9x & & \\
& & -y & = & -5x & + & 5 \\
& & y & = & 5x & - & 5
\end{array}$$

Answer Key 3.5

1. $$\begin{array}{rrrrrrr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 3 & = & \frac{2}{3}(x & - & 2) \\ y & - & 3 & = & \frac{2}{3}x & - & \frac{4}{3} \\ & + & 3 & & & + & 3 \\ & & y & = & \frac{2}{3}x & + & \frac{5}{3} \end{array}$$
2. $$\begin{array}{rrrrrrr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 2 & = & 4(x & - & 1) \\ y & - & 2 & = & 4x & - & 4 \\ & + & 2 & & & + & 2 \\ & & y & = & 4x & - & 2 \end{array}$$
3. $$\begin{array}{rrrrrrr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 2 & = & \frac{1}{2}(x & - & 2) \\ y & - & 2 & = & \frac{1}{2}x & - & 1 \\ & + & 2 & & & + & 2 \\ & & y & = & \frac{1}{2}x & + & 1 \end{array}$$
4. $$\begin{array}{rrrrrrr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 1 & = & -\frac{1}{2}(x & - & 2) \\ y & - & 1 & = & -\frac{1}{2}x & + & 1 \\ & + & 1 & & & + & 1 \\ & & y & = & -\frac{1}{2}x & + & 2 \end{array}$$
5. $$\begin{array}{rrrrrrr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & -5 & = & 9(x & - & -1) \\ y & + & 5 & = & 9x & + & 9 \\ & - & 5 & & & - & 5 \\ & & y & = & 9x & + & 4 \end{array}$$

6. $$\begin{array}{rcrcrcr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & -2 & = & -2(x & - & 2) \\ y & + & 2 & = & -2x & + & 4 \\ & - & 2 & & & - & 2 \\ & & y & = & -2x & + & 2 \end{array}$$

7. $$\begin{array}{rcrcrcr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 1 & = & \frac{3}{4}(x & - & -4) \\ y & - & 1 & = & \frac{3}{4}x & + & 3 \\ & + & 1 & & & + & 1 \\ & & y & = & \frac{3}{4}x & + & 4 \end{array}$$

8. $$\begin{array}{rcrcrcr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & -3 & = & -2(x & - & 4) \\ y & + & 3 & = & -2x & + & 8 \\ & - & 3 & & & - & 3 \\ & & y & = & -2x & + & 5 \end{array}$$

9. $$\begin{array}{rcrcrcr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & -2 & = & -3(x & - & 0) \\ y & + & 2 & = & -3x & & \\ & - & 2 & & & - & 2 \\ & & y & = & -3x & - & 2 \end{array}$$

10. $$\begin{array}{rcrcrcr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 1 & = & 4(x & - & -1) \\ y & - & 1 & = & 4x & + & 4 \\ & + & 1 & & & + & 1 \\ & & y & = & 4x & + & 5 \end{array}$$

11. $$\begin{array}{rcrcrcr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & -5 & = & -\frac{1}{4}(x & - & 0) \\ y & + & 5 & = & -\frac{1}{4}x & & \\ & - & 5 & & & - & 5 \\ & & y & = & -\frac{1}{4}x & - & 5 \end{array}$$

12. $$\begin{array}{rrrrlrl} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 2 & = & -\frac{5}{4}(x & - & 0) \\ y & - & 2 & = & -\frac{5}{4}x & & \\ & + & 2 & & & + & 2 \\ & & y & = & -\frac{5}{4}x & + & 2 \end{array}$$

13. $$\begin{array}{rrrrlrrrr} y & - & y_1 & = & m(x & - & x_1) & & \\ y & - & -5 & = & 2(x & - & -1) & & \\ y & + & 5 & = & 2x & + & 2 & & \\ -y & - & 5 & & -y & - & 5 & & \\ & & 0 & = & 2x & - & y & - & 3 \end{array}$$

14. $$\begin{array}{rrrrlrrrrr} y & - & y_1 & = & m(x & - & x_1) & & & \\ y & - & -2 & = & -2(x & - & 2) & & & \\ y & + & 2 & = & -2x & + & 4 & & & \\ -y & - & 2 & & -y & - & 2 & & & \\ & & (0 & = & -2x & - & y & + & 2) & (-1) \\ & & 0 & = & 2x & + & y & - & 2 & \end{array}$$

15. $$\begin{array}{rrrrlrrrrr} y & - & y_1 & = & m(x & - & x_1) & & & \\ y & - & -1 & = & -\frac{3}{5}(x & - & 5) & & & \\ y & + & 1 & = & -\frac{3}{5}x & + & 3 & & & \\ -y & - & 1 & & -y & - & 1 & & & \\ & & (0 & = & -\frac{3}{5}x & - & y & + & 2) & (-5) \\ & & 0 & = & 3x & + & 5y & - & 10 & \end{array}$$

16. $$\begin{array}{rrrrlrrrrr} y & - & y_1 & = & m(x & - & x_1) & & & \\ y & - & -2 & = & -\frac{2}{3}(x & - & -2) & & & \\ y & + & 2 & = & -\frac{2}{3}x & - & \frac{4}{3} & & & \\ -y & - & 2 & & -y & - & 2 & & & \\ & & (0 & = & -\frac{2}{3}x & - & y & - & \frac{10}{3}) & (-3) \\ & & 0 & = & 2x & + & 3y & + & 10 & \end{array}$$

17.

$$\begin{array}{rcrcrcrcrcrl}
y & - & y_1 & = & m(x & - & x_1) & & & & & \\
y & - & 1 & = & \frac{1}{2}(x & - & -4) & & & & & \\
y & - & 1 & = & \frac{1}{2}x & + & 2 & & & & & \\
-y & + & 1 & & -y & + & 1 & & & & & \\
 & & (0 & = & \frac{1}{2}x & - & y & + & 3) & & (2) & \\
 & & 0 & = & x & - & 2y & + & 6 & & &
\end{array}$$

18.

$$\begin{array}{rcrcrcrcrcrl}
y & - & y_1 & = & m(x & - & x_1) & & & & & \\
y & - & -3 & = & -\frac{7}{4}(x & - & 4) & & & & & \\
y & + & 3 & = & -\frac{7}{4}x & + & 7 & & & & & \\
-y & - & 3 & & -y & - & 3 & & & & & \\
 & & (0 & = & -\frac{7}{4}x & - & y & + & 4) & & (-4) & \\
 & & 0 & = & 7x & + & 4y & - & 16 & & &
\end{array}$$

19.

$$\begin{array}{rcrcrcrcrcrl}
y & - & y_1 & = & m(x & - & x_1) & & & & & \\
y & - & -2 & = & -\frac{3}{2}(x & - & 4) & & & & & \\
y & + & 2 & = & -\frac{3}{2}x & + & 6 & & & & & \\
-y & - & 2 & & -y & - & 2 & & & & & \\
 & & (0 & = & -\frac{3}{2}x & - & y & + & 4) & & (-2) & \\
 & & 0 & = & 3x & + & 2y & - & 8 & & &
\end{array}$$

20. $$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= -\frac{5}{2}(x - -2) \\ y &= -\frac{5}{2}x - 5 \\ -y \quad & \quad -y \\ (0 &= -\frac{5}{2}x - y + 5) \quad (-2) \\ 0 &= 5x + 2y + 10 \end{aligned}$$

21. $$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - -3 &= -\frac{2}{5}(x - -5) \\ y + 3 &= -\frac{2}{5}x - 2 \\ -y - 3 \quad & \quad -y - 3 \\ (0 &= -\frac{2}{5}x - y - 5) \quad (-5) \\ 0 &= 2x + 5y + 25 \end{aligned}$$

22. $$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= \frac{7}{3}(x - 3) \\ y - 3 &= \frac{7}{3}x - 7 \\ -y + 3 \quad & \quad -y + 3 \\ (0 &= \frac{7}{3}x - y - 4) \quad (3) \\ 0 &= 7x - 3y - 12 \end{aligned}$$

23. $$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - -2 &= 1(x - 2) \\ y + 2 &= x - 2 \\ -y - 2 \quad & \quad -y - 2 \\ 0 &= x - y - 4 \end{aligned}$$

24.
$$\begin{array}{rcrcrcrcrcr}
y & - & y_1 & = & m(x & - & x_1) \\
y & - & 4 & = & -\frac{1}{3}(x & - & -3) \\
y & - & 4 & = & -\frac{1}{3}x & - & 1 \\
-y & + & 4 & & -y & + & 4 \\
 & & (0 & = & -\frac{1}{3}x & - & y & + & 3) & (-3) \\
 & & 0 & = & x & + & 3y & - & 9
\end{array}$$

25.
$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{1-3}{-3--4} \Rightarrow \frac{-2}{1} \Rightarrow -2$$

$$\begin{array}{rcrcrcr}
y & - & y_1 & = & m(x & - & x_1) \\
y & - & 1 & = & -2(x & - & -3) \\
y & - & 1 & = & -2x & - & 6 \\
 & + & 1 & & & + & 1 \\
 & & y & = & -2x & - & 5
\end{array}$$

26.
$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{-3-3}{-3-1} \Rightarrow \frac{-6}{-4} \Rightarrow \frac{3}{2}$$

$$\begin{array}{rcrcrcr}
y & - & y_1 & = & m(x & - & x_1) \\
y & - & 3 & = & \frac{3}{2}(x & - & 1) \\
y & - & 3 & = & \frac{3}{2}x & - & \frac{3}{2} \\
 & + & 3 & & & + & 3 \\
 & & y & = & \frac{3}{2}x & + & \frac{3}{2}
\end{array}$$

27.
$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{0-1}{-3-5} \Rightarrow \frac{-1}{-8} \Rightarrow \frac{1}{8}$$

$$\begin{array}{rcrcrcr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 0 & = & \frac{1}{8}(x & - & -3) \\ & & y & = & \frac{1}{8}x & + & \frac{3}{8} \end{array}$$

28.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{4-5}{4--4} \Rightarrow \frac{-1}{8}$$

$$\begin{array}{rcrcrcr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 4 & = & -\frac{1}{8}(x & - & 4) \\ y & - & 4 & = & -\frac{1}{8}x & + & \frac{1}{2} \\ & + & 4 & & & + & 4 \\ & & y & = & -\frac{1}{8}x & + & \frac{9}{2} \end{array}$$

29.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{4--2}{0--4} \Rightarrow \frac{6}{4} \Rightarrow \frac{3}{2}$$

$$\begin{array}{rcrcrcr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 4 & = & \frac{3}{2}(x & - & 0) \\ y & - & 4 & = & \frac{3}{2}x & & \\ & + & 4 & & & + & 4 \\ & & y & = & \frac{3}{2}x & + & 4 \end{array}$$

30.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{4-1}{4--4} \Rightarrow \frac{3}{8}$$

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 4 &= \frac{3}{8}(x - 4) \\
y - 4 &= \frac{3}{8}x - \frac{3}{2} \\
+4 & \qquad\quad +4 \\
y &= \frac{3}{8}x + \frac{5}{2}
\end{aligned}
$$

31.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{3-5}{-5-3} \Rightarrow \frac{-2}{-8} \Rightarrow \frac{1}{4}$$

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 3 &= \frac{1}{4}(x - -5) \\
y - 3 &= \frac{1}{4}x - \frac{5}{4} \\
+3 & \qquad\quad +3 \\
y &= \frac{1}{4}x + \frac{17}{4}
\end{aligned}
$$

32.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{0 - -4}{-5 - -1} \Rightarrow \frac{4}{-4} \Rightarrow -1$$

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 0 &= -1(x - -5) \\
y &= -x - 5
\end{aligned}
$$

33.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{5 - -3}{-4-3} \Rightarrow \frac{8}{-7} \Rightarrow -\frac{8}{7}$$

$$\begin{array}{rcrcrcrcrcr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 5 & = & -\frac{8}{7}(x & - & -4) \\ \\ y & - & 5 & = & -\frac{8}{7}x & - & \frac{32}{7} \\ \\ -y & + & 5 & & -y & + & 5 \\ & & (0 & = & -\frac{8}{7}x & - & y & + & \frac{3}{7}) & & (-7) \\ \\ & & 0 & = & 8x & + & 7y & - & 3 \end{array}$$

34.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{-4 - -5}{-5 - -1} \Rightarrow \frac{1}{-4} \Rightarrow -\frac{1}{4}$$

$$\begin{array}{rcrcrcrcrcr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & -5 & = & -\frac{1}{4}(x & - & -1) \\ \\ y & + & 5 & = & -\frac{1}{4}(x & + & 1) \\ \\ y & + & 5 & = & -\frac{1}{4}x & - & \frac{1}{4} \\ \\ -y & - & 5 & & -y & - & 5 \\ & & (0 & = & -\frac{1}{4}x & - & y & - & \frac{21}{4}) & & (-4) \\ \\ & & 0 & = & x & + & 4y & + & 21 \end{array}$$

35.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{4 - -3}{-2 - 3} \Rightarrow \frac{7}{-5} \Rightarrow -\frac{7}{5}$$

$$\begin{array}{rcrcrcrcrcl} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 4 & = & -\frac{7}{5}(x & - & -2) \\ y & - & 4 & = & -\frac{7}{5}x & - & \frac{14}{5} \\ -y & + & 4 & & -y & + & 4 \\ & & (0 & = & -\frac{7}{5}x & - & y & + & \frac{6}{5}) & & (-5) \\ & & 0 & = & 7x & + & 5y & - & 6 \end{array}$$

36.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{-4 - -7}{-3 - -6} \Rightarrow \frac{3}{3} \Rightarrow 1$$

$$\begin{array}{rcrcrcrcl} y & - & y_1 & = & m(x & - & x_1) \\ y & - & -4 & = & 1(x & - & -3) \\ y & + & 4 & = & x & + & 3 \\ -y & - & 4 & & -y & - & 4 \\ & & 0 & = & x & - & y & - & 1 \end{array}$$

37.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{-2 - 1}{-1 - -5} \Rightarrow \frac{-3}{4}$$

$$\begin{array}{rcrcrcrcrcl} y & - & y_1 & = & m(x & - & x_1) \\ y & - & -2 & = & -\frac{3}{4}(x & - & -1) \\ y & + & 2 & = & -\frac{3}{4}x & - & \frac{3}{4} \\ -y & - & 2 & & -y & - & 2 \\ & & (0 & = & -\frac{3}{4}x & - & y & - & \frac{11}{4}) & & (-4) \\ & & 0 & = & 3x & + & 4y & + & 11 \end{array}$$

38.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{-2 - -1}{5 - -5} \Rightarrow \frac{-1}{10}$$

$$\begin{array}{rcrcl}
y - y_1 & = & m(x - x_1) \\
y - -1 & = & -\frac{1}{10}(x - -5) \\
y + 1 & = & -\frac{1}{10}x - \frac{1}{2} \\
-y - 1 & & -y - 1 \\
(0 & = & -\frac{1}{10}x - y - \frac{3}{2})\quad(-10) \\
0 & = & x + 10y + 15
\end{array}$$

39.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{-3-5}{2--5} \Rightarrow \frac{-8}{7}$$

$$\begin{array}{rcl}
y - y_1 & = & m(x - x_1) \\
y - -3 & = & -\frac{8}{7}(x - 2) \\
y + 3 & = & -\frac{8}{7}x + \frac{16}{7} \\
-y - 3 & & -y - 3 \\
(0 & = & -\frac{8}{7}x - y - \frac{5}{7})\quad(-7) \\
0 & = & 8x + 7y + 5
\end{array}$$

40.

$$m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{-4--1}{-5-1} \Rightarrow \frac{-3}{-6} \Rightarrow \frac{1}{2}$$

$$\begin{array}{rcl}
y - y_1 & = & m(x - x_1) \\
y - -1 & = & \frac{1}{2}(x - 1) \\
y + 1 & = & \frac{1}{2}x - \frac{1}{2} \\
-y - 1 & & -y - 1 \\
(0 & = & \frac{1}{2}x - y - \frac{3}{2})\quad(2) \\
0 & = & x - 2y - 3
\end{array}$$

Answer Key 3.6

1. $m = 2$
2. $m = -\dfrac{2}{3}$
3. $m = 4$
4. $m = -10$
5. $$\begin{array}{rrrrrrr} x & - & y & = & 4 & & \\ -x & & & & -x & & \\ & & (-y & = & -x & + & 4) \quad (-1) \\ & & y & = & x & - & 4 \\ & & m & = & 1 & & \end{array}$$
6. $$\begin{array}{rrrrrrr} 6x & - & 5y & = & 20 & & \\ -6x & & & & -6x & & \\ & & \dfrac{-5y}{-5} & = & \dfrac{-6x}{-5} & + & \dfrac{20}{-5} \\ & & y & = & \dfrac{6}{5}x & - & 4 \\ & & m & = & \dfrac{6}{5} & & \end{array}$$
7. $$\begin{array}{rcl} y & = & \dfrac{1}{3}x \\ \therefore m & = & \dfrac{1}{3} \\ m_{\perp} & = & -1 \div \dfrac{1}{3} \quad \text{or} \\ m_{\perp} & = & -3 \end{array}$$
8. $$\begin{array}{rcl} m & = & -\dfrac{1}{2} \\ m_{\perp} & = & -1 \div -\dfrac{1}{2} \\ m_{\perp} & = & -1 \cdot -\dfrac{2}{1} = 2 \end{array}$$
9. $$\begin{array}{rcl} m & = & -\dfrac{1}{3} \\ m_{\perp} & = & -1 \div -\dfrac{1}{3} \\ m_{\perp} & = & -1 \cdot -\dfrac{3}{1} = 3 \end{array}$$

10. $$\begin{aligned} m &= \frac{4}{5} \\ m_{\perp} &= -1 \div \frac{4}{5} \\ m_{\perp} &= -1 \cdot \frac{5}{4} = -\frac{5}{4} \end{aligned}$$

11. $$\begin{array}{rrrrrrr} x & - & 3y & = & -6 & & \\ -x & & & & -x & & \\ & & \dfrac{-3y}{-3} & = & \dfrac{-x}{-3} & - & \dfrac{6}{-3} \\ & & y & = & \frac{1}{3}x & + & 2 \\ & & m_{\perp} & = & -1 & \div & \frac{1}{3} \\ & & m_{\perp} & = & -3 & & \end{array}$$

12. $$\begin{array}{rrrrrrr} 3x & - & y & = & -3 & & \\ -3x & & & & -3x & & \\ & & -y & = & -3x & - & 3 \\ & & y & = & 3x & + & 3 \\ & & m_{\perp} & = & -1 & \div & 3 \\ & & m_{\perp} & = & -\frac{1}{3} & & \end{array}$$

13. $$m = \frac{2}{5}$$

$$\begin{array}{rrrrrrr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 4 & = & \frac{2}{5}(x & - & 1) \\ y & - & 4 & = & \frac{2}{5}x & - & \frac{2}{5} \\ & + & 4 & & & + & 4 \\ & & y & = & \frac{2}{5}x & + & \frac{18}{5} \end{array}$$

14. $$m = -3$$

$$\begin{array}{rrrrrrr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 2 & = & -3(x & - & 5) \\ y & - & 2 & = & -3x & + & 15 \\ & + & 2 & & & + & 2 \\ & & y & = & -3x & + & 17 \end{array}$$

15. $m = \frac{1}{2}$

$$\begin{array}{rrrrrrr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 4 & = & \frac{1}{2}(x & - & 3) \\ y & - & 4 & = & \frac{1}{2}x & - & \frac{3}{2} \\ & + & 4 & & & + & 4 \\ & & y & = & \frac{1}{2}x & + & \frac{5}{2} \end{array}$$

16. $m = \frac{4}{3}$

$$\begin{array}{rrrrrrr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & -1 & = & \frac{4}{3}(x & - & 1) \\ y & + & 1 & = & \frac{4}{3}x & - & \frac{4}{3} \\ & - & 1 & & & - & 1 \\ & & y & = & \frac{4}{3}x & - & \frac{7}{3} \end{array}$$

17. $m = -\frac{3}{5}$

$$\begin{array}{rrrrrrr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 3 & = & -\frac{3}{5}(x & - & 2) \\ y & - & 3 & = & -\frac{3}{5}x & + & \frac{6}{5} \\ & + & 3 & & & + & 3 \\ & & y & = & -\frac{3}{5}x & + & \frac{21}{5} \end{array}$$

18. $$m = \frac{1}{3}$$

$$\begin{array}{rrrrrrr} y & - & y_1 & = & m(x & - & x_1) \\ y & - & 3 & = & \frac{1}{3}(x & - & -1) \\ y & - & 3 & = & \frac{1}{3}x & + & \frac{1}{3} \\ & + & 3 & & & + & 3 \\ & & y & = & \frac{1}{3}x & + & \frac{10}{3} \end{array}$$

19. $$\begin{array}{rrrrrrrrr} -x & + & y & = & 1 & & & & \\ +x & & & & +x & & & & \\ & & y & = & x & + & 1 & & \\ & & \therefore m & = & 1 & & & & \\ y & - & y_1 & = & m(x & - & x_1) & & \\ y & - & -5 & = & 1(x & - & 1) & & \\ y & + & 5 & = & x & - & 1 & & \\ -y & - & 5 & & -y & - & 5 & & \\ & & 0 & = & x & - & y & - & 6 \end{array}$$

20. $$\begin{array}{rrrrrrrr} -x & + & 2y & = & 2 & & & \\ +x & & & & +x & & & \\ & & 2y & = & x & + & 2 & \\ & \text{or} & y & = & \frac{1}{2}x & + & 1 & \\ & & \therefore m & = & -2 & & & \\ y & - & y_1 & = & m(x & - & x_1) & \\ y & - & -2 & = & -2(x & - & 1) & \\ y & + & 2 & = & -2x & + & 2 & \\ -y & - & 2 & & -y & - & 2 & \\ & & (0 & = & -2x & - & y) & (-1) \\ & & 0 & = & 2x & + & y & \end{array}$$

21.

$$
\begin{array}{rrrrrrrrrr}
5x & + & y & = & -3 & & & & & \\
-5x & & & & -5x & & & & & \\
 & & y & = & -5x & - & 3 & & & \\
 & & \therefore m & = & -5 & & & & & \\
 & & & & & & & & & \\
y & - & y_1 & = & m(x & - & x_1) & & & \\
y & - & 2 & = & -5(x & - & 5) & & & \\
y & - & 2 & = & -5x & + & 25 & & & \\
-y & + & 2 & & -y & + & 2 & & & \\
 & & (0 & = & -5x & - & y & + & 27) & (-1) \\
 & & 0 & = & 5x & + & y & - & 27 &
\end{array}
$$

22.

$$
\begin{array}{rrrrrrrrrr}
-x & + & y & = & 1 & & & & & \\
+x & & & & +x & & & & & \\
 & & y & = & x & + & 1 & & & \\
 & & \therefore m & = & -1 & & & & & \\
 & & & & & & & & & \\
y & - & y_1 & = & m(x & - & x_1) & & & \\
y & - & 3 & = & -1(x & - & 1) & & & \\
y & - & 3 & = & -x & + & 1 & & & \\
-y & + & 3 & & -y & + & 3 & & & \\
 & & (0 & = & -x & - & y & + & 4) & (-1) \\
 & & 0 & = & x & + & y & - & 4 &
\end{array}
$$

23.

$$
\begin{array}{rrrrrrrrr}
-4x & + & y & = & 0 & & & & \\
+4x & & & & +4x & & & & \\
 & & y & = & 4x & & & & \\
 & & \therefore m & = & 4 & & & & \\
 & & & & & & & & \\
y & - & y_1 & = & m(x & - & x_1) & & \\
y & - & 2 & = & 4(x & - & 4) & & \\
y & - & 2 & = & 4x & - & 16 & & \\
-y & + & 2 & & -y & + & 2 & & \\
 & & 0 & = & 4x & - & y & - & 14
\end{array}
$$

24.
$$\begin{array}{rrrrrrrrrr}
3x & + & 7y & = & 0 \\
-3x & & & & -3x \\
& & 7y & = & -3x \\
& \text{or} & y & = & -\frac{3}{7}x \\
& & \therefore m & = & \frac{7}{3} \\
y & - & y_1 & = & m(x & - & x_1) \\
y & - & -5 & = & \frac{7}{3}(x & - & -3) \\
y & + & 5 & = & \frac{7}{3}x & + & 7 \\
-y & - & 5 & & -y & - & 5 \\
& & (0 & = & \frac{7}{3}x & - & y & + & 2) & (3) \\
& & 0 & = & 7x & - & 3y & + & 6
\end{array}$$

25. $y = -3$
26. $x = -5$
27. $x = -3$
28. $y = 0$
29. $y = -1$
30. $x = 2$
31. $x = -2$
32. $y = -4$
33. $y = 3$
34. $x = -3$
35. $x = 5$
36. $y = -1$

Answer Key 3.7

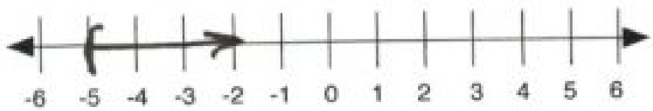
1. $2x+5=25$
2. $4x+12=36$
3. $3x-8=22$
4. $6x-8=22$
5. $x-8=\dfrac{x}{2}$
6. $x-4=\dfrac{x}{2}$
7. $x+x+1+x+2=21$
8. $x+x+2-(x+4)=5$
9. $$\begin{array}{rcrcr} 5 & + & 3x & = & 17 \\ -5 & & & & -5 \\ & & 3x & = & 12 \\ \\ & & x & = & \dfrac{12}{3} \text{ or } 4 \end{array}$$
10. $$\begin{array}{rcrcr} 3x & - & 5 & = & 10 \\ & + & 5 & & +5 \\ & & 3x & = & 15 \\ \\ & & x & = & \dfrac{15}{3} \text{ or } 5 \end{array}$$
11. $$\begin{array}{rcrcrcr} 60 & + & 9x & = & 10x & - & 2 \\ +2 & - & 9x & & -9x & + & 2 \\ & & 62 & = & x & & \end{array}$$
12. $$\begin{array}{rcrcrcr} 7x & - & 11 & = & 6x & + & 5 \\ -6x & + & 11 & & -6x & + & 11 \\ & & x & = & 16 & & \end{array}$$
13. $$\begin{array}{rcrcrcrcrcr} x & + & x & + & 1 & + & x & + & 2 & = & 108 \\ & & & & & & 3x & + & 3 & = & 108 \\ & & & & & & & - & 3 & & -3 \\ & & & & & & & & 3x & = & 105 \\ \\ & & & & & & & & x & = & \dfrac{105}{3} \text{ or } 35 \\ \\ & & & & & & & & & \therefore & 35, 36, 37 \end{array}$$

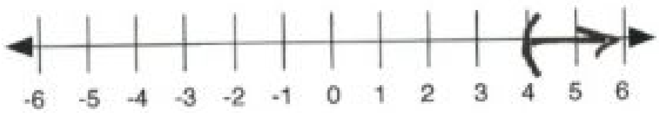
14.
$$\begin{aligned} x + x + 1 + x + 2 &= -126 \\ 3x + 3 &= -126 \\ -3 &\quad -3 \\ 3x &= -129 \\ x &= \frac{-129}{3} \text{ or } -43 \\ &\therefore -43, -42, -41 \end{aligned}$$

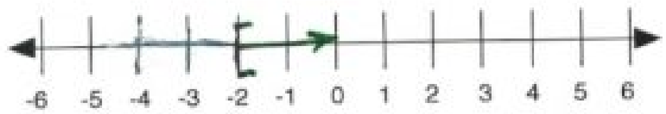
15.
$$\begin{aligned} x + 2(x + 1) + 3(x + 2) &= -76 \\ x + 2x + 2 + 3x + 6 &= -76 \\ 6x + 8 &= -76 \\ -8 &\quad -8 \\ 6x &= -84 \\ x &= \frac{-84}{6} \text{ or } -14 \\ &\therefore -14, -13, -12 \end{aligned}$$

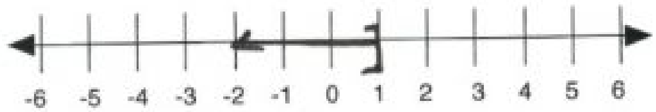
16.
$$\begin{aligned} x + 2(x + 2) + 3(x + 4) &= 70 \\ x + 2x + 4 + 3x + 12 &= 70 \\ 6x + 16 &= 70 \\ -16 &\quad -16 \\ 6x &= 54 \\ x &= \frac{54}{6} \text{ or } 9 \\ &\therefore 9, 11, 13 \end{aligned}$$

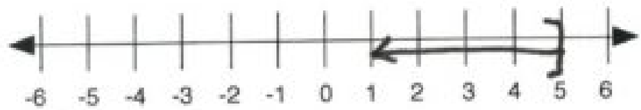
Answer Key 4.1

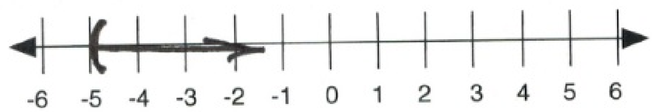

1. $(-5, \infty)$

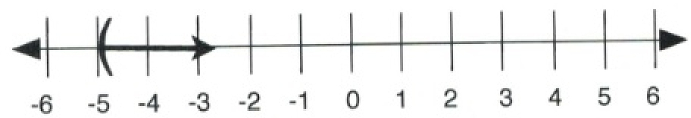

2. $(4, \infty)$

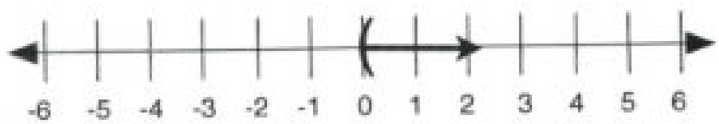

3. $(-2, \infty)$

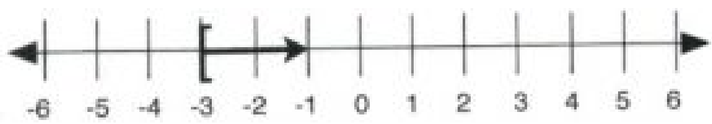

4. $(-\infty, 1]$

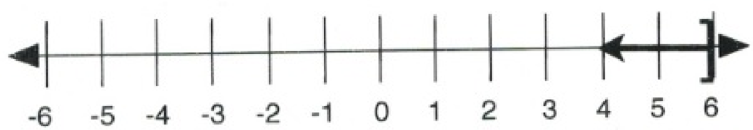

5. $(-\infty, 5]$

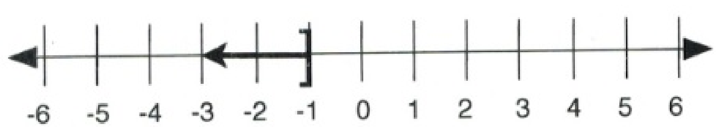

6. $(-5, \infty)$

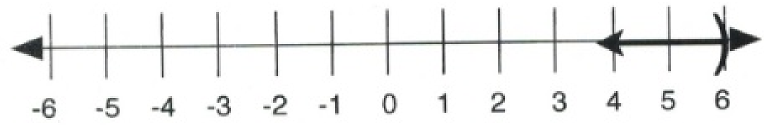

7. $x > -5$ $\quad (-5, \infty)$


8. $x > 0$ $\quad (0, \infty)$


9. $x \geq -3$ $\quad [-3, \infty)$


10. $x \leq 6$ $\quad (-\infty, 6]$

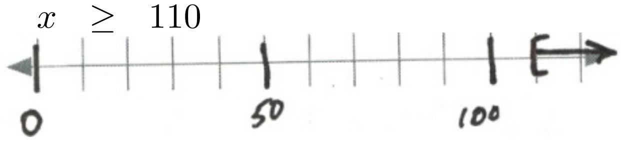

11. $x \leq -1$ $\quad (-\infty, -1]$



12. $x < 6$ $\quad (-\infty, 6)$

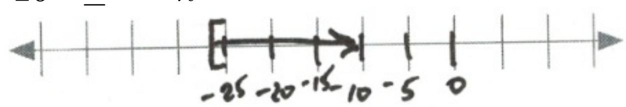
13. $\left(\frac{x}{11} \geq 10\right)(11)$Interval notation: $[110, \infty)$

$$x \geq 110$$



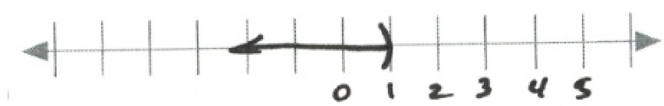
14. $\left(-2 \leq \frac{n}{13}\right)(13)$Interval notation: $[-26, \infty)$

$$-26 \leq n$$



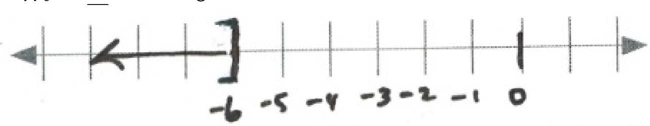
15. $2 + r < 3$Interval notation: $(-\infty, 1)$

$$\begin{aligned} -2 \quad &\quad -2 \\ r &< 1 \end{aligned}$$



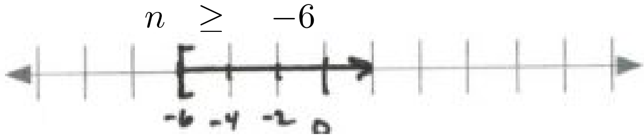
16. $\left(\frac{m}{5} \leq -\frac{6}{5}\right)(5)$Interval notation: $(-\infty, -6]$

$$m \leq -6$$



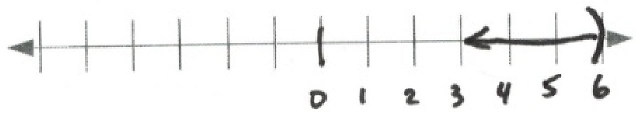
17. $8 + \frac{n}{3} \geq 6$ Interval notation: $[-6, \infty)$

$$\begin{aligned} -8 \quad &\quad -8 \\ \left(\frac{n}{3} \geq -2\right) &(3) \\ n &\geq -6 \end{aligned}$$

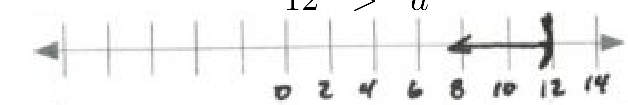


18. $$\begin{array}{lcccc} 11 & > & 8 & + & \frac{x}{2} \\ -8 & & -8 & & \\ (3 & > & \frac{x}{2}) & (2) & \end{array}$$ Interval notation: $(-\infty, 6)$

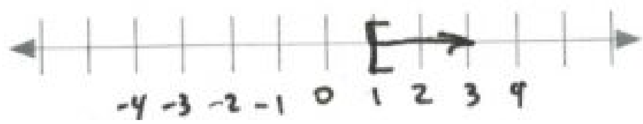
$$6 > x$$



19. $$\left(2 > \frac{(a-2)}{5}\right)(5) \quad \begin{array}{ccccc} 10 & > & a & - & 2 \\ +2 & & & + & 2 \\ 12 & > & a & & \end{array}$$ Interval notation: $(-\infty, 12)$

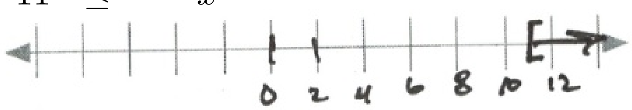


20. $$\left(\frac{(v-9)}{-4} \leq 2\right)(-4) \quad \begin{array}{ccccc} v & - & 9 & \geq & -8 \\ & + & 9 & & +9 \\ & & v & \geq & 1 \end{array}$$ Interval notation: $[1, \infty)$

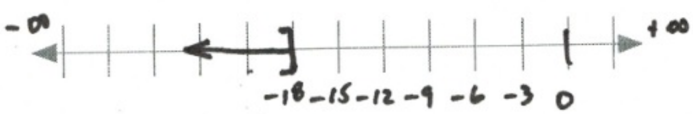


21. $$\begin{array}{ccccc} -47 & \geq & 8 & - & 5x \\ -8 & & -8 & & \\ \frac{-55}{-5} & \geq & \frac{-5x}{-5} & & \end{array}$$ Interval notation: $[11, \infty)$

$$11 \leq x$$

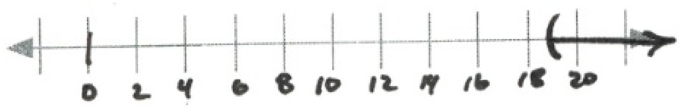


22. $$\left(\frac{(6+x)}{12} \leq -1\right)(12) \quad \begin{array}{ccccc} 6 & + & x & \leq & -12 \\ -6 & & & & -6 \\ & & x & \leq & -18 \end{array}$$ Interval notation: $(-\infty, -18]$

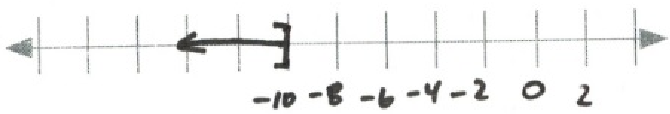


23. $$\frac{-2}{-2}(3 + k) < \frac{-44}{-2}$$ Interval notation: $(19, \infty)$

$$\begin{array}{ccccc} 3 & + & k & > & 22 \\ -3 & & & & -3 \\ & & k & > & 19 \end{array}$$

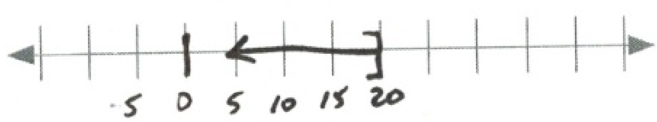


24. $\begin{array}{rcrcl} -7n & - & 10 & \geq & 60 \\ & + & 10 & & +10 \\ & & \dfrac{-7n}{-7} & \geq & \dfrac{70}{-7} \\ & & n & \leq & -10 \end{array}$ Interval notation: $(-\infty, -10]$

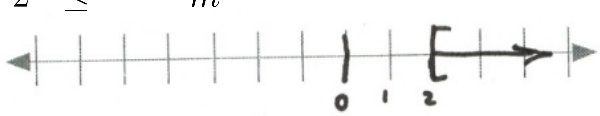


25. $\begin{array}{rcl} \dfrac{18}{-2} & < & \dfrac{-2}{-2}(-8 + p) \\ -9 & > & -8 + p \\ +8 & & +8 \\ -1 & > & p \end{array}$ Interval notation: $(-\infty, -1)$

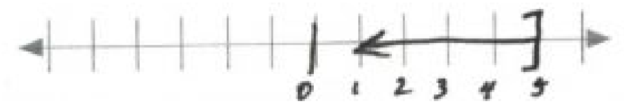
26. $\left(5 > \dfrac{x}{5} + 1\right)(5)$ $\begin{array}{rcl} 25 & \geq & x + 5 \\ -5 & & -5 \\ 20 & \geq & x \end{array}$ Interval notation: $(-\infty, 20]$



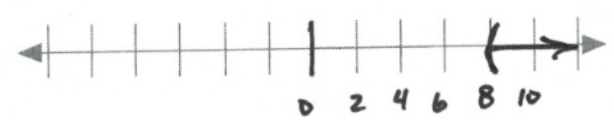
27. $\begin{array}{rcl} \dfrac{24}{-6} & \geq & \dfrac{-6}{-6}(m - 6) \\ -4 & \leq & m - 6 \\ +6 & & + 6 \\ 2 & \leq & m \end{array}$ Interval notation: $[2, \infty)$



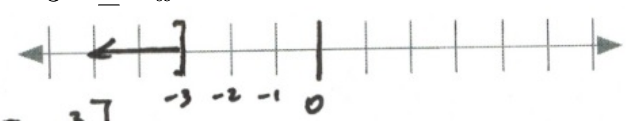
28. $\begin{array}{rcl} \dfrac{-8}{-8}(n - 5) & \geq & \dfrac{0}{-8} \\ n - 5 & \leq & 0 \\ + 5 & & +5 \\ n & \leq & 5 \end{array}$ Interval notation: $(-\infty, 5]$



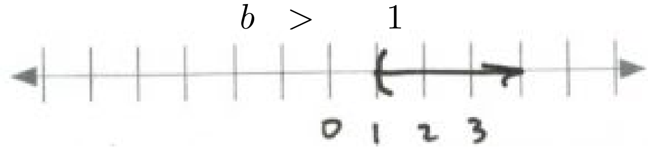
29. $$\begin{aligned} -r - 5(r - 6) &< -18 \\ -r - 5r + 30 &< -18 \\ -30 \quad & \quad -30 \\ \frac{-6r}{-6} &< \frac{-48}{-6} \\ r &> 8 \end{aligned}$$ Interval notation: $(8, \infty)$



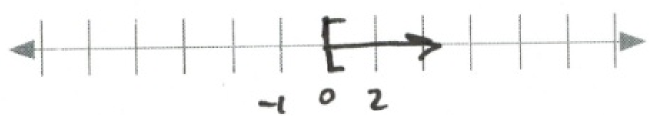
30. $$\begin{aligned} \frac{-60}{-4} &\geq \frac{-4}{-4}(-6x - 3) \\ 15 &\leq -6x - 3 \\ +3 \quad & \quad +3 \\ \frac{18}{-6} &\leq \frac{-6x}{-6} \\ -3 &\geq x \end{aligned}$$ Interval notation: $(-\infty, -3]$



31. $$\begin{aligned} \frac{24 + 4b}{4} &< \frac{4}{4}(1 + 6b) \\ 6 + b &< 1 + 6b \\ -1 - b \quad & \quad -1 - b \\ \frac{5}{5} &< \frac{5b}{5} \\ b &> 1 \end{aligned}$$ Interval notation: $(1, \infty)$



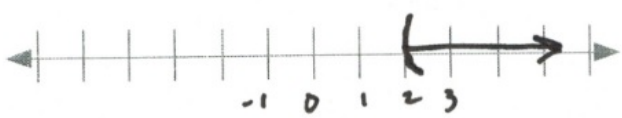
32. $$\begin{aligned} -8(2 - 2n) &\geq -16 + n \\ -16 + 16n &\geq -16 + n \\ +16 - n \quad & \quad +16 - n \\ 15n &\geq 0 \\ n &\geq 0 \end{aligned}$$ Interval notation: $[0, \infty)$



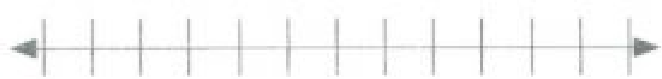
33.
$$\frac{-5v-5}{-5} < \frac{-5}{-5}(4v + 1)$$
Interval notation: $(-\infty, 0)$

$$\begin{array}{rcrcrcr} v & + & 1 & > & 4v & + & 1 \\ -v & - & 1 & & -v & - & 1 \\ & & \dfrac{0}{3} & > & \dfrac{3v}{3} & & \\ & & v & < & 0 & & \end{array}$$

34.
$$\begin{array}{rcrcrcrcr} -36 & + & 6x & > & -8(x & + & 2) & + & 4x \\ -36 & + & 6x & > & -8x & - & 16 & + & 4x \\ +16 & - & 6x & & -6x & + & 16 & & \\ & & \dfrac{-20}{-10} & > & \dfrac{-10x}{-10} & & & & \\ & & x & > & 2 & & & & \end{array}$$
Interval notation: $(2, \infty)$

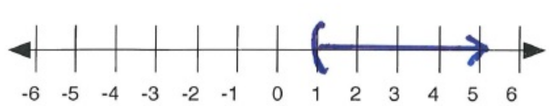


35.
$$\begin{array}{rcrcrcrcr} 4 & + & 2(a & + & 5) & < & -2(-a & - & 4) \\ 4 & + & 2a & + & 10 & < & 2a & + & 8 \\ -4 & - & 2a & - & 10 & & -2a & - & 10-4 \\ & & 0 & < & -6 & & & & \end{array}$$
False. No solution.



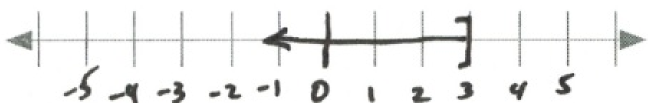
36.
$$\begin{array}{rcrcrcrcrcrcr} 3(n & + & 3) & + & 7(8 & - & 8n) & < & 5n & + & 5 & + & 2 \\ 3n & + & 9 & + & 56 & - & 56n & < & 5n & + & 7 & & \\ & & & & -53n & + & 65 & < & 5n & + & 7 & & \\ & & & & -5n & - & 65 & & -5n & - & 65 & & \\ & & & & & & -58n & < & -58 & & & & \\ & & & & & & n & > & 1 & & & & \end{array}$$

Interval notation: $(1, \infty)$



37. $\begin{array}{rcrcrcr} -(k & - & 2) & > & -k & - & 20 \\ -k & + & 2 & > & -k & - & 20 \\ +k & - & 2 & & +k & - & 2 \\ & & 0 & > & -22 & & \end{array}$ Always true. Solution is all real numbers: $(-\infty, \infty)$

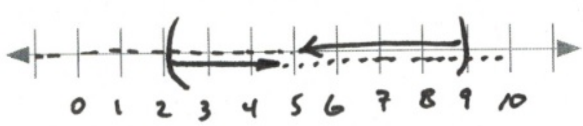
38. $\begin{array}{rcrcrcrcr} -(4 & - & 5p) & + & 3 & \geq & -2(8 & - & 5p) \\ -4 & + & 5p & + & 3 & \geq & -16 & + & 10p \\ & & -1 & + & 5p & \geq & -16 & + & 10p \\ & & +1 & - & 10p & & +1 & - & 10p \\ & & & & \dfrac{-5p}{-5} & \geq & \dfrac{-15}{-5} & & \\ & & & & p & \leq & 3 & & \end{array}$ Interval notation: $(-\infty, 3]$



Answer Key 4.2

1. $$(3)\quad (\frac{n}{3} < 3) \quad \text{or} \quad \frac{-5n}{-5} < \frac{-10}{-5} \text{Interval notation: } (-\infty, \infty)$$

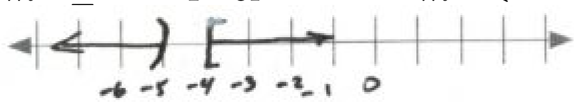
$$n < 9 \quad \text{or} \quad n > 2$$



2. $$\frac{6m}{6} \geq \frac{-24}{6} \quad \text{or} \quad m - 7 < -12 \text{Interval notation: } (-\infty, -5) \cup [-4, \infty)$$

$$+7 < +7$$

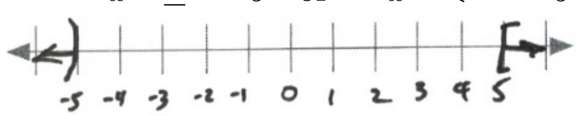
$$m \geq -4 \quad \text{or} \quad m < -5$$



3. $$x + 7 \geq 12 \quad \text{or} \quad \frac{9x}{9} < \frac{-45}{9} \text{Interval notation: } (-\infty, -5) \cup [5, \infty)$$

$$-7 \qquad -7$$

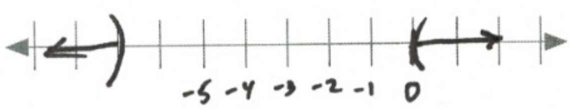
$$x \geq 5 \quad \text{or} \quad x < -5$$



4. $$\frac{10r}{10} > \frac{0}{10} \quad \text{or} \quad r - 5 < -12 \text{Interval notation: } (-\infty, -7) \cup (0, \infty)$$

$$+5 \qquad +5$$

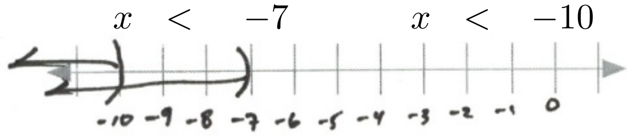
$$r > 0 \quad \text{or} \quad r < -7$$



5. $$x - 6 < -13 \quad \text{or} \quad \frac{6x}{6} < \frac{-60}{6} \text{Interval notation: } (-\infty, -7)$$

$$+6 \qquad +6$$

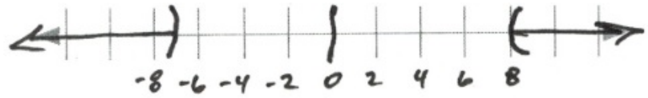
$$x < -7 \qquad x < -10$$



6. $$9 + n < 2 \quad \text{or} \quad \frac{5n}{5} > \frac{40}{5} \text{Interval notation: } (-\infty, -7) \cup (8, \infty)$$

$$-9 \qquad -9$$

$$n < -7 \quad \text{or} \quad n > 8$$



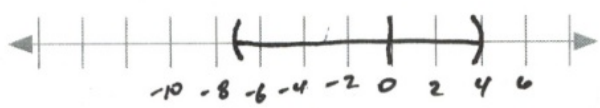
7. $$\begin{array}{rcccccccl} (8)\ (\frac{v}{8} & > & -1) & \text{and} & v & - & 2 & < & 1 \\ & & & & & + & 2 & & +2 \\ v & > & -8 & \text{and} & & & v & < & 3 \end{array}$$ Interval notation: $(-8, 3)$

$$-8 < v < 3$$

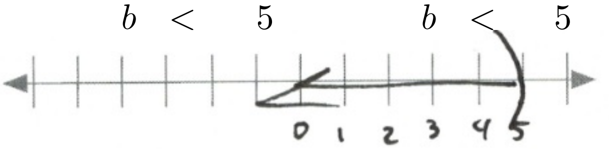
8. $$\frac{-9x}{-9} < \frac{63}{-9} \quad \text{and} \quad (\frac{x}{4} < 1)\ (4)$$ Interval notation: $(-7, 4)$

$$x > -7 \quad \text{and} \quad x < 4$$

$$-7 < x < 4$$



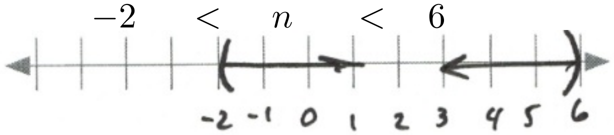
9. $$\begin{array}{rcccccccc} -8 & + & b & < & -3 & \text{and} & \frac{4b}{4} & < & \frac{20}{4} \\ +8 & & & & +8 & & & & \\ & & b & < & 5 & & b & < & 5 \end{array}$$ Interval notation: $(-\infty, 5)$



10. $$\frac{-6n}{-6} < \frac{12}{-6} \quad \text{and} \quad (\frac{n}{3} < 2)\ (3)$$ Interval notation: $(-2, 6)$

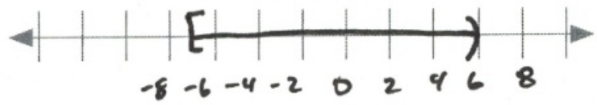
$$n > -2 \quad \text{and} \quad n < 6$$

$$-2 < n < 6$$

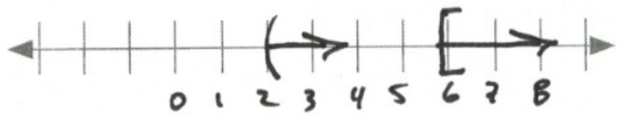


11. $$\begin{array}{rccccccc} a & + & 10 & \geq & 3 & \text{and} & \frac{8a}{8} & < & \frac{48}{8} \\ & - & 10 & & -10 & & & & \\ & & a & \geq & -7 & \text{and} & a & < & 6 \end{array}$$ Interval notation: $[-7, 6)$

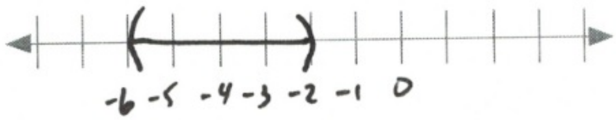
$$-7 \leq a < 6$$



12. $$\begin{array}{rcrcrcrcr} -6 & + & v & \geq & 0 & \text{and} & \frac{2v}{2} & > & \frac{4}{2} \\ +6 & & & & +6 & & & & \\ & & v & \geq & 6 & \text{and} & v & > & 2 \end{array}$$ Interval notation: $[6, \infty)$

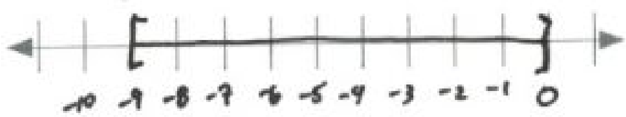


13. $$\begin{array}{rcrcrcr} 3 & < & 9 & + & x & < & 7 \\ -9 & & -9 & & & & -9 \\ -6 & < & & & x & < & -2 \end{array}$$ Interval notation: $(-6, -2)$

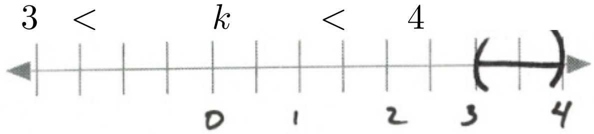


14. $$\left(0 \geq \frac{x}{9} \geq -1\right)(9)$$ Interval notation: $[-9, 0]$

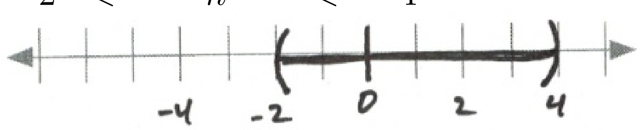
$$0 \geq x \geq -9$$



15. $$\begin{array}{rcrcrcr} 11 & < & 8 & + & k & < & 12 \\ -8 & & -8 & & & & -8 \\ 3 & < & & & k & < & 4 \end{array}$$ Interval notation: $(3, 4)$



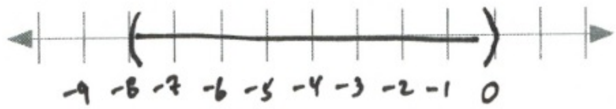
16. $$\begin{array}{rcrcrcr} -11 & < & n & - & 9 & < & -5 \\ +9 & & & + & 9 & & +9 \\ -2 & < & & n & & < & 4 \end{array}$$ Interval notation: $(-2, 4)$



17. $$\begin{array}{rcrcrcr} -3 & < & x & - & 1 & < & 1 \\ +1 & & & + & 1 & & +1 \\ -2 & < & & x & & < & 2 \end{array}$$ Interval notation: $(-2, 2)$

18. $\left(-1 < \frac{p}{8} < 0\right)$ (8)Interval notation: $(-8, 0)$

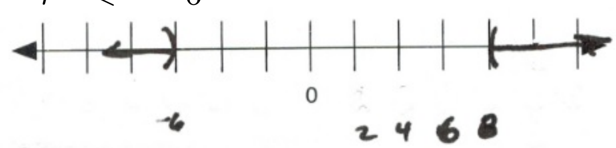
$$-8 < p < 0$$



19.
$$\begin{array}{ccccccc}
-4 & < & 8 & - & 3m & < & 11 \\
-8 & & -8 & & & & -8 \\
\frac{-12}{-3} & < & & \frac{-3m}{-3} & & < & \frac{3}{-3} \\
4 & > & & m & & > & -1
\end{array}$$

Interval notation: $(-1, 4)$

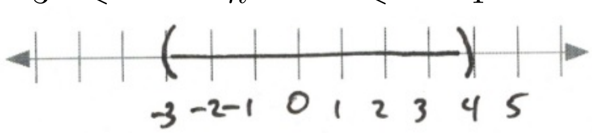
20.
$$\begin{array}{ccccccccccc}
3 & + & 7r & > & 59 & \text{or} & -6r & - & 3 & > & 33 \\
-3 & & & & -3 & & & + & 3 & > & +3 \\
& & \frac{7r}{7} & > & \frac{56}{7} & \text{or} & & & \frac{-6r}{-6} & > & \frac{36}{-6} \\
& & r & > & 8 & \text{or} & & & r & < & -6
\end{array}$$



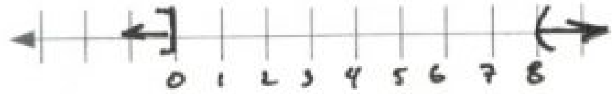
Interval notation: $(-\infty, -6) \cup (8, \infty)$

21.
$$\begin{array}{ccccccc}
-16 & < & 2n & - & 10 & < & -2 \\
+10 & & & + & 10 & & +10 \\
\frac{-6}{2} & < & & \frac{2n}{2} & & < & \frac{8}{2} \\
-3 & < & & n & & < & 4
\end{array}$$

Interval notation: $(-3, 4)$



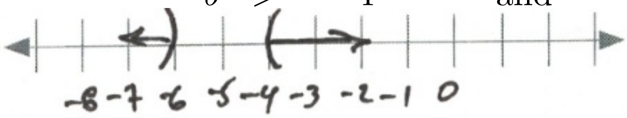
22.
$$\begin{array}{ccccccccccc}
-6 & - & 8x & \geq & -6 & \text{or} & 2 & + & 10x & > & 82 \\
+6 & & & & +6 & & -2 & & & > & -2 \\
& & \frac{-8x}{-8} & \geq & \frac{0}{-8} & & & & \frac{10x}{10} & > & \frac{80}{10} \\
& & x & \leq & 0 & \text{or} & & & x & > & 8
\end{array}$$



Interval notation: $(-\infty, 0] \cup (8, \infty)$

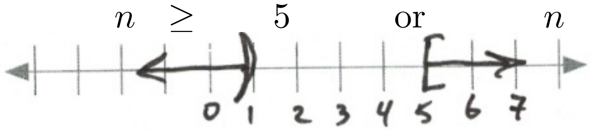
23.
$$\begin{array}{rrrrrrrrrrr}
-5b & + & 10 & < & 30 & \text{and} & 7b & + & 2 & < & -40 \\
 & & -10 & & -10 & & & - & 2 & & -2 \\
 & & \dfrac{-5b}{-5} & < & \dfrac{20}{-5} & & & & \dfrac{7b}{7} & < & \dfrac{-42}{7} \\
 & & b & > & -4 & \text{and} & & & b & < & -6
\end{array}$$

$\therefore$ No solution



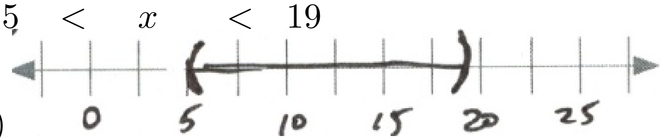
24.
$$\begin{array}{rrrrrrrrrrr}
n & + & 10 & \geq & 15 & \text{or} & 4n & - & 5 & < & -1 \\
 & - & 10 & & -10 & & & + & 5 & & +5 \\
 & & n & \geq & 5 & & & & \dfrac{4n}{4} & < & \dfrac{4}{4} \\
 & & n & \geq & 5 & \text{or} & & & n & < & 1
\end{array}$$

Interval notation: $(-\infty, 1) \cup [5, \infty)$



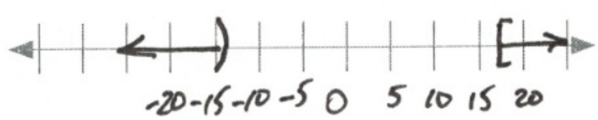
25.
$$\begin{array}{rrrrrrrrrrrrr}
3x & - & 9 & < & 2x & + & 10 & \text{and} & 5 & + & 7x & < & 10x & - & 10 \\
-2x & + & 9 & & -2x & + & 9 & & -5 & - & 10x & & -10x & - & 5 \\
 & & x & < & 19 & & & & & & \dfrac{-3x}{-3} & < & \dfrac{-15}{-3} \\
 & & x & < & 19 & & & \text{and} & & & x & > & 5
\end{array}$$

$$5 < x < 19$$



Interval notation: $(5, 19)$

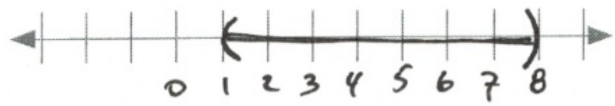
26.
$$\begin{array}{rrrrrrrrrrrrr}
4n & + & 8 & < & 3n & - & 6 & \text{or} & 10n & - & 8 & \geq & 9 & + & 9n \\
-3n & - & 8 & & -3n & - & 8 & & -9n & + & 8 & & +8 & - & 9n \\
 & & n & < & -14 & & & \text{or} & & & n & \geq & 17
\end{array}$$



Interval notation: $(-\infty, -14) \cup [17, \infty)$

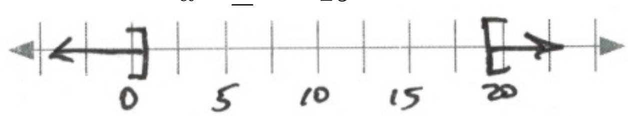
27.
$$\begin{array}{rcrcrcrcrcrcrcr}
-8 & - & 6v & < & 8 & - & 8v & \text{and} & 7v & + & 9 & < & 6 & + & 10v \\
+8 & + & 8v & & +8 & + & 8v & & -10v & - & 9 & & -9 & - & 10v \\
 & & \dfrac{2v}{2} & < & \dfrac{16}{2} & & & & & & \dfrac{-3v}{-3} & < & \dfrac{-3}{-3} & & \\
 & & v & < & 8 & & & \text{and} & & & v & > & 1 & &
\end{array}$$

$$1 < v < 8$$



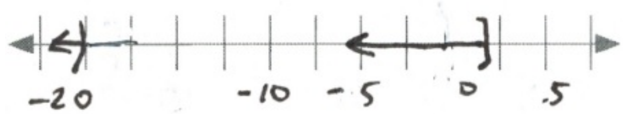
Interval notation: $(1,8)$

28.
$$\begin{array}{rcrcrcrcrcrcrcr}
5 & - & 2a & \geq & 2a & + & 1 & \text{or} & 10a & - & 10 & \geq & 9a & + & 9 \\
-5 & - & 2a & & -2a & - & 5 & & -9a & + & 10 & & -9a & + & 10 \\
 & & \dfrac{-4a}{-4} & \geq & \dfrac{-4}{-4} & & & & & & a & \geq & 19 & & \\
 & & a & \leq & 1 & & & \text{or} & & & a & \geq & 19 & &
\end{array}$$



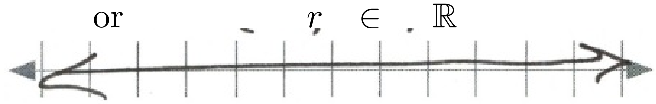
Interval notation: $(-\infty,1] \cup [19,\infty)$

29.
$$\begin{array}{rcrcrcrcrcrcrcr}
1 & + & 5k & \geq & 7k & - & 3 & \text{or} & k & - & 10 & > & 2k & + & 10 \\
-1 & - & 7k & & -7k & - & 1 & & -2k & + & 10 & & -2k & + & 10 \\
 & & \dfrac{-2k}{-2} & \geq & \dfrac{-4}{-2} & & & & & & -k & > & 20 & & \\
 & & k & \leq & 2 & & & \text{or} & & & k & < & -20 & &
\end{array}$$



Interval notation: $(-\infty,2]$

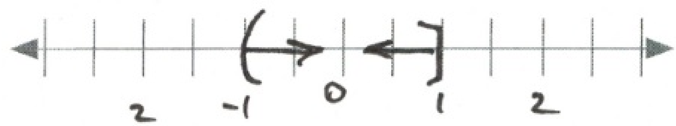
30.
$$\begin{array}{rcrcrcrcrcrcrcr}
8 & - & 10r & < & 8 & + & 4r & \text{or} & -6 & + & 8r & < & 2 & + & 8r \\
-8 & - & 4r & & -8 & - & 4r & & +6 & - & 8r & & +6 & - & 8r \\
 & & \dfrac{-14r}{-14} & < & \dfrac{0}{-14} & & & & & & 0 & < & 8 & \leftarrow & \text{This is always true} \\
 & & r & > & 0 & & & \text{or} & & & r & \in & \mathbb{R} & &
\end{array}$$



Interval notation: $(-\infty,\infty)$

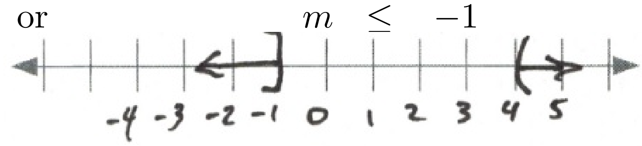
31.
$$\begin{array}{rcrcrl}
2x + 9 & \geq & 10x + 1 & \text{and} & 3x - 2 & < \; 7x + 2 \\
-10x - 9 & & -10x - 9 & & -7x + 2 & \;\; -7x + 2 \\
\dfrac{-8x}{-8} & \geq & \dfrac{-8}{-8} & & \dfrac{-4x}{-4} & < \; \dfrac{4}{-4} \\
x & \leq & 1 & \text{and} & x & > \; -1
\end{array}$$

$$-1 < x \leq 1$$



Interval notation: $(-1, 1]$

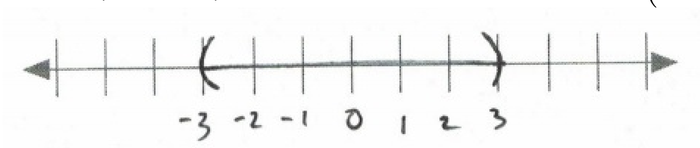
32.
$$\begin{array}{rcrcrl}
-9m + 2 & < & -10 - 6m & \text{or} & -m + 5 & \geq \; 10 + 4m \\
+6m - 2 & & -2 + 6m & & -4m - 5 & \;\; -5 - 4m \\
\dfrac{-3m}{-3} & < & \dfrac{-12}{-3} & & \dfrac{-5m}{-5} & \geq \; \dfrac{5}{-5} \\
m & > & 4 & \text{or} & m & \leq \; -1
\end{array}$$



Interval notation: $(-\infty, -1] \cup (4, \infty)$

Answer Key 4.3

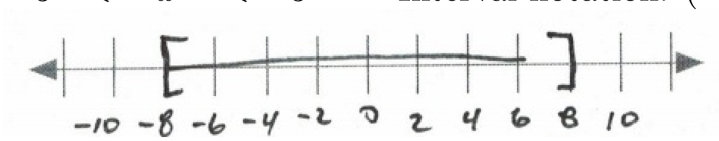
1. $-3 < x < 3$ Interval notation: $(-3, 3)$



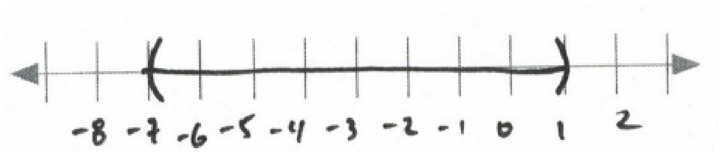
2. $-8 \leq x \leq 8$ Interval notation: $[-8, 8]$

3. $\frac{-6}{2} < \frac{2x}{2} < \frac{6}{2}$

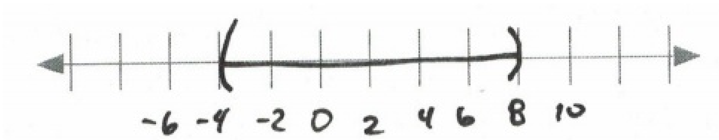
$-3 < x < 3$ Interval notation: $(-3, 3)$



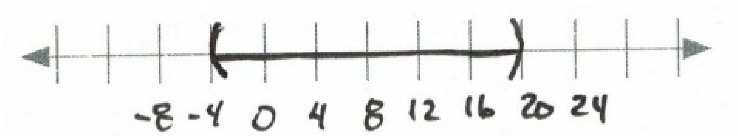
4. $$\begin{array}{rcrcrcr} -4 & < & x & + & 3 & < & 4 \\ -3 & & & - & 3 & & -3 \\ -7 & < & & x & & < & 1 \end{array}$$ Interval notation: $(-7, 1)$



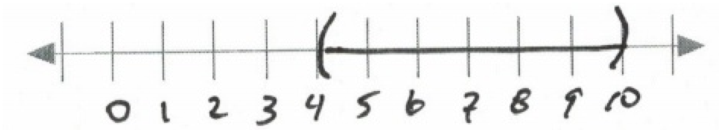
5. $$\begin{array}{rcrcrcr} -6 & < & x & - & 2 & < & 6 \\ +2 & & & + & 2 & & +2 \\ -4 & < & & x & & < & 8 \end{array}$$ Interval notation: $(-4, 8)$



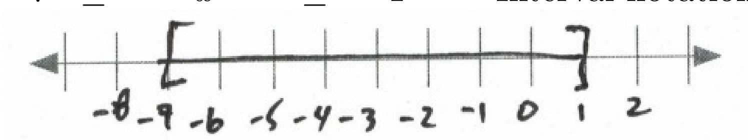
6. $$\begin{array}{rcrcrcr} -12 & < & x & - & 8 & < & 12 \\ +8 & & & + & 8 & & +8 \\ -4 & < & & x & & < & 20 \end{array}$$ Interval notation: $(-4, 20)$



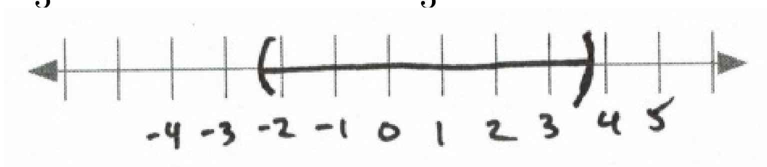
7. $$\begin{array}{rcrcrcr} -3 & < & x & - & 7 & < & 3 \\ +7 & & & + & 7 & & +7 \\ 4 & < & & x & & < & 10 \end{array}$$ Interval notation: $(4, 10)$



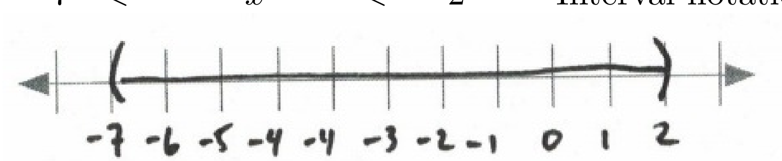
8. $$\begin{array}{ccccccc} -4 & \leq & x & + & 3 & \leq & 4 \\ -3 & & & - & 3 & & -3 \\ -7 & \leq & & x & & \leq & 1 \end{array} \quad \text{Interval notation: } [-7, 1]$$



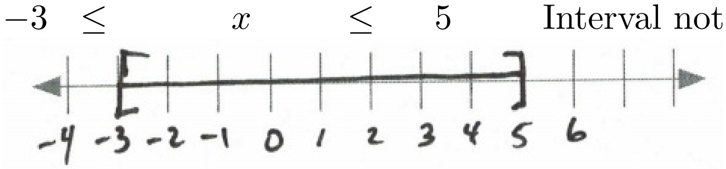
9. $$\begin{array}{ccccccc} -9 & < & 3x & - & 2 & < & 9 \\ +2 & & & + & 2 & & +2 \\ \frac{-7}{3} & < & & \frac{3x}{3} & & < & \frac{11}{3} \\ & & & & & & \\ -\frac{7}{3} & < & & x & & < & \frac{11}{3} \end{array} \quad \text{Interval notation: } (-\frac{7}{3}, \frac{11}{3})$$



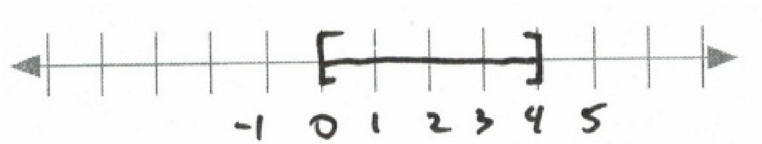
10. $$\begin{array}{ccccccc} -9 & < & 2x & + & 5 & < & 9 \\ -5 & & & - & 5 & & -5 \\ \frac{-14}{2} & < & & \frac{2x}{2} & & < & \frac{4}{2} \\ & & & & & & \\ -7 & < & & x & & < & 2 \end{array} \quad \text{Interval notation: } (-7, 2)$$



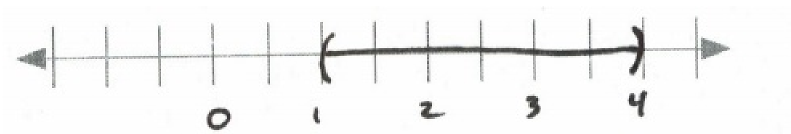
11. $$\begin{array}{ccccccc} 1 & + & 2|x & - & 1| & \leq & 9 \\ -1 & & & & & & -1 \\ & & \frac{2}{2}|x & - & 1| & \leq & \frac{8}{2} \\ & & & & & & \\ & & |x & - & 1| & \leq & 4 \\ -4 & \leq & x & - & 1 & \leq & 4 \\ +1 & & & + & 1 & \leq & +1 \\ -3 & \leq & & x & & \leq & 5 \end{array} \quad \text{Interval notation: } [-3, 5]$$



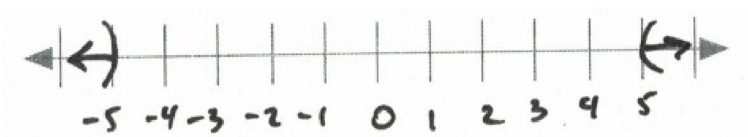
12.
$$\begin{array}{rcrcrcr} 10 & - & 3|x & - & 2| & \geq & 4 \\ -10 & & & & & & -10 \\ & & \dfrac{-3}{-3}|x & - & 2| & \geq & \dfrac{-6}{-3} \\ & & |x & - & 2| & \leq & 2 \\ -2 & \leq & x & - & 2 & \leq & 2 \\ +2 & & & + & 2 & & +2 \\ 0 & \leq & & x & & \leq & 4 \end{array} \quad \text{Interval notation: } [0,4]$$



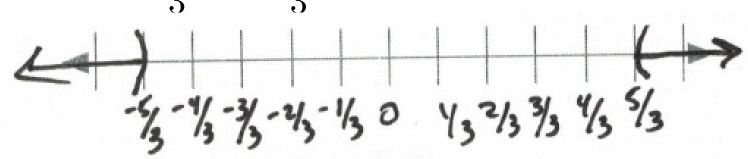
13.
$$\begin{array}{rcrcrcrl} 6 & - & |2x & - & 5| & > & 3 & \\ -6 & & & & & & -6 & \\ & & (-|2x & - & 5| & > & -3) & (-1) \\ & & |2x & - & 5| & < & 3 & \\ -3 & < & 2x & - & 5 & < & 3 & \\ +5 & & & + & 5 & & +5 & \\ \dfrac{2}{2} & < & & \dfrac{2x}{2} & & < & \dfrac{8}{2} & \\ 1 & < & & x & & < & 4 & \end{array} \quad \text{Interval notation: } (1,4)$$



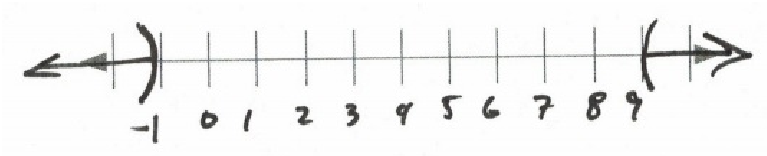
14. $x < -5$ or $5 < x$ Interval notation: $(-\infty,-5)\cup(5,\infty)$



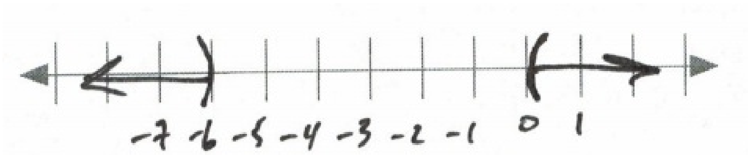
15.
$$\begin{array}{ccccccc} \dfrac{3x}{3} & < & \dfrac{-5}{3} & \text{or} & \dfrac{5}{3} & < & \dfrac{3x}{3} \\ x & < & -\dfrac{5}{3} & \text{or} & \dfrac{5}{3} & < & x \end{array} \quad \text{Interval notation: } (-\infty,-\frac{5}{3})\cup(\frac{5}{3},\infty)$$



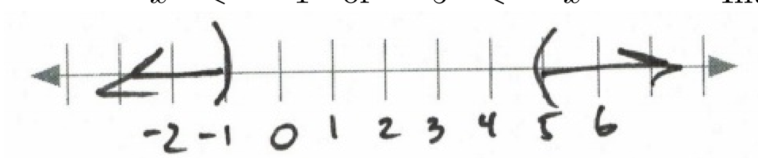
16.
$$\begin{array}{cccccccccc} x & - & 4 & < & -5 & \text{or} & 5 & < & x & - & 4 \\ & + & 4 & & +4 & \text{or} & +4 & & & + & 4 \\ & & x & < & -1 & \text{or} & 9 & < & x & & \end{array} \quad \text{Interval notation: } (-\infty,-1)\cup(9,\infty)$$



17. $$\begin{array}{cccccccccc} x & + & 3 & < & -3 & \text{or} & 3 & < & x & + & 3 \\ & - & 3 & & -3 & \text{or} & -3 & & & - & 3 \\ & & x & < & -6 & \text{or} & 0 & < & x & & \end{array}$$ Interval notation: $(-\infty, -6) \cup (0, \infty)$

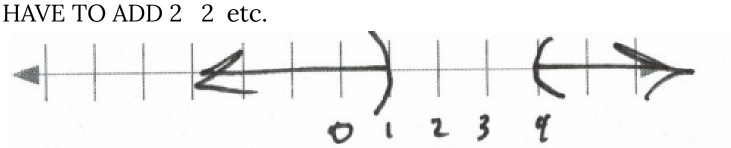


18. $$\begin{array}{ccccccccccc} 2x & - & 4 & < & -6 & \text{or} & 6 & < & 2x & - & 4 \\ & + & 4 & & +4 & & +4 & & & + & 4 \\ & & \frac{2x}{2} & < & \frac{-2}{2} & & \frac{10}{2} & < & \frac{2x}{2} & & \\ & & x & < & -1 & \text{or} & 5 & < & x & & \end{array}$$ Interval notation: $(-\infty, -1) \cup (5, \infty)$



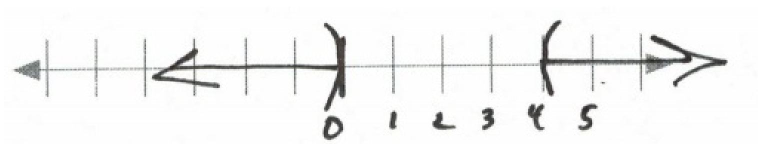
19. $$\begin{array}{ccccccccccc} x & - & 5 & < & -3 & \text{or} & 3 & < & x & - & 5 \\ & + & 5 & & +5 & & +5 & & & + & 5 \\ & & x & < & 2 & \text{or} & 8 & < & x & & \end{array}$$ Interval notation: $(-\infty, 2) \cup (8, \infty)$

HAVE TO ADD 2 2 etc.



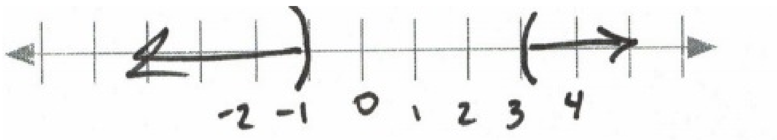
20. $$\begin{array}{cccccc} 3 & -|2 & - & x| & < & 1 \\ -3 & & & & & -3 \\ & (-|2 & - & x| & < & -2) \quad (-1) \\ & |2 & - & x| & > & 2 \end{array}$$

$$\begin{array}{ccccccccccc} 2 & - & x & < & -2 & \text{or} & 2 & < & 2 & - & x \\ -2 & & & & -2 & & -2 & & -2 & & \\ & & -x & < & -4 & \text{or} & 0 & < & -x & & \\ & & x & > & 4 & \text{or} & 0 & > & x & & \end{array}$$ Interval notation: $(-\infty, 0) \cup (4, \infty)$



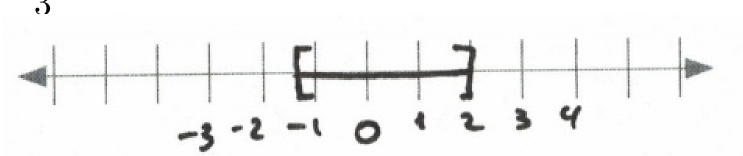
21.
$$\begin{array}{rcrcrcr} 4 & + & 3|x & - & 1| & < & 10 \\ -4 & & & & & & -4 \\ & & \frac{3}{3}|x & - & 1| & < & \frac{6}{3} \\ & & |x & - & 1| & < & 2 \end{array}$$

$$\begin{array}{rcrcrcrcrcr} x & - & 1 & < & -2 & \text{or} & 2 & < & x & - & 1 \\ & + & 1 & & +1 & & +1 & & & + & 1 \\ & & x & < & -1 & \text{or} & 3 & < & x & & \end{array}$$ Interval notation: $(-\infty, -1) \cup (3, \infty)$



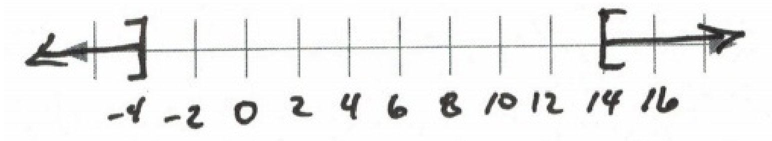
22.
$$\begin{array}{rcrcrcr} 3 & - & 2|3x & - & 1| & \geq & -7 \\ -3 & & & & & & -3 \\ & & \frac{-2}{-2}|3x & - & 1| & \geq & \frac{-10}{-2} \\ & & |3x & - & 1| & \leq & 5 \end{array}$$

$$\begin{array}{rcrcrcr} -5 & \leq & 3x & - & 1 & \leq & 5 \\ +1 & & & + & 1 & & +1 \\ \frac{-4}{3} & \leq & & \frac{3x}{3} & & \leq & \frac{6}{3} \\ -\frac{4}{3} & \leq & & x & & \leq & 2 \end{array}$$ Interval notation: $[-\frac{4}{3}, 2]$



23.
$$\begin{array}{rcrcrcr} 3 & - & 2|x & - & 5| & \leq & -15 \\ -3 & & & & & & -3 \\ & & \frac{-2}{-2}|x & - & 5| & \leq & \frac{-18}{-2} \\ & & |x & - & 5| & \geq & 9 \end{array}$$

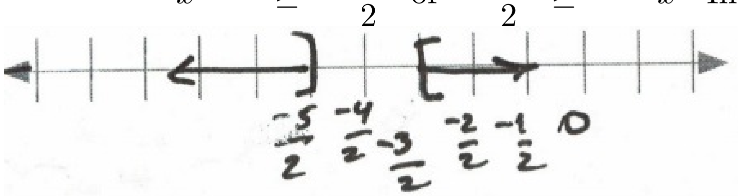
$$\begin{array}{rcrcrcrcrcr} x & - & 5 & \leq & -9 & \text{or} & 9 & \leq & x & - & 5 \\ & + & 5 & & +5 & & +5 & & & + & 5 \\ & & x & \leq & -4 & \text{or} & 14 & \leq & x & & \end{array}$$ Interval notation: $(-\infty, -4] \cup [14, \infty)$



24.
$$\begin{array}{rcrcrcr} 4 & - & 6|-6 & - & 3x| & \leq & -5 \\ -4 & & & & & & -4 \\ & & \dfrac{-6}{-6}|-6 & - & 3x| & \leq & \dfrac{-9}{-6} \\ & & |-6 & - & 3x| & \geq & \dfrac{3}{2} \end{array}$$

$$\begin{array}{rcrcrcrcrcr} -6 & - & 3x & \leq & -\dfrac{3}{2} & \text{or} & \dfrac{3}{2} & \leq & -6 & - & 3x \\ +6 & & & & +6 & & +6 & & +6 & & \\ & & \dfrac{-3x}{-3} & \leq & \dfrac{\frac{9}{2}}{-3} & & \dfrac{\frac{15}{2}}{-3} & \leq & \dfrac{-3x}{-3} & & \\ & & x & \geq & -\dfrac{3}{2} & \text{or} & -\dfrac{5}{2} & \geq & x & & \end{array}$$

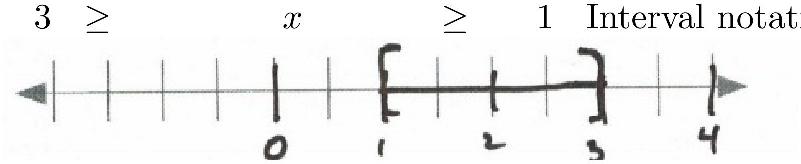
Interval notation: $(-\infty, -\frac{5}{2}] \cup [-\frac{3}{2}, \infty)$



25.
$$\begin{array}{rcrcrcr} -2 & - & 3|4 & - & 2x| & \geq & -8 \\ +2 & & & & & & +2 \\ & & \dfrac{-3}{-3}|4 & - & 2x| & \geq & \dfrac{-6}{-3} \\ & & |4 & - & 2x| & \leq & 2 \end{array}$$

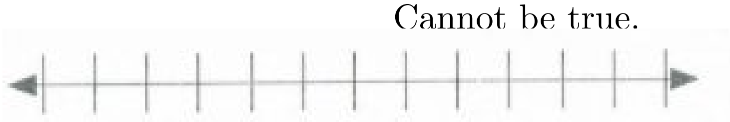
$$\begin{array}{rcrcrcr} -2 & \leq & 4 & - & 2x & \leq & 2 \\ -4 & & -4 & & & & -4 \\ \dfrac{-6}{-2} & \leq & & \dfrac{-2x}{-2} & & \leq & \dfrac{-2}{-2} \\ 3 & \geq & & x & & \geq & 1 \end{array}$$

Interval notation: $[1, 3]$



26.

$$\begin{array}{rrrrrcr}
-3 & - & 2|4x & - & 5| & \geq & 1 \\
+3 & & & & & & +3 \\
& & \dfrac{-2}{-2}|4x & - & 5| & \geq & \dfrac{4}{-2} \\
& & |4x & - & 5| & \leq & -2 \\
& & & & & \uparrow & \\
& & & & & \text{Cannot be true.} & \qquad \text{No solution.}
\end{array}$$

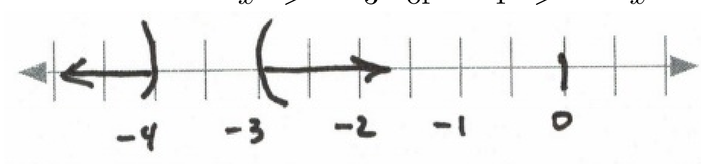


27.

$$\begin{array}{rrrrrcr}
4 & - & 5|-2x & - & 7| & < & -1 \\
-4 & & & & & & -4 \\
& & \dfrac{-5}{-5}|-2x & - & 7| & < & \dfrac{-5}{-5} \\
& & |-2x & - & 7| & > & 1
\end{array}$$

$$\begin{array}{rrrcrcrcrrr}
-2x & - & 7 & < & -1 & \text{or} & 1 & < & -2x & - & 7 \\
& + & 7 & & +7 & & +7 & & & + & 7 \\
& & \dfrac{-2x}{-2} & < & \dfrac{6}{-2} & & \dfrac{8}{-2} & < & \dfrac{-2x}{-2} & & \\
& & x & > & -3 & \text{or} & -4 & > & x & &
\end{array}$$

Interval notation: $(-\infty, -4) \cup (-3, \infty)$

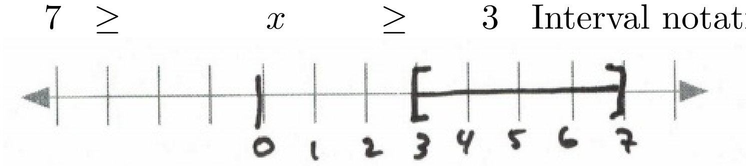


28.

$$\begin{array}{rrrrrcr}
-2 & + & 3|5 & - & x| & \leq & 4 \\
+2 & & & & & & +2 \\
& & \dfrac{3}{3}|5 & - & x| & \leq & \dfrac{6}{3} \\
& & |5 & - & x| & \leq & 2
\end{array}$$

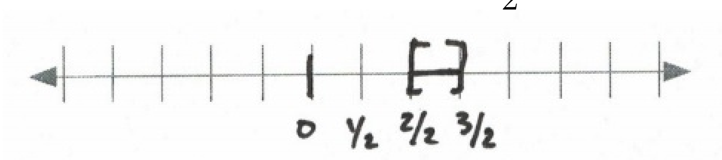
$$\begin{array}{rcrcrcr}
-2 & \leq & 5 & - & x & \leq & 2 \\
-5 & & -5 & & & & -5 \\
(-7 & \leq & & & -x & \leq & -3) \quad (-1) \\
7 & \geq & & & x & \geq & 3
\end{array}$$

Interval notation: $[3, 7]$



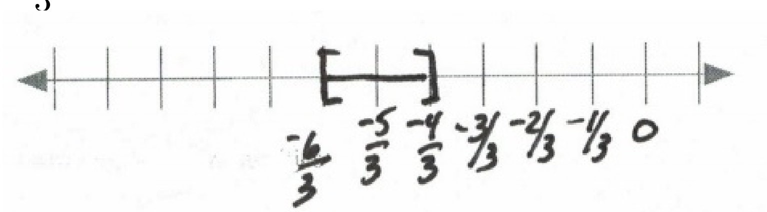
29.
$$\begin{array}{rcrcrcr}
3 & - & 2|4x & - & 5| & \geq & 1 \\
-3 & & & & & & -3 \\
& & \frac{-2}{-2}|4x & - & 5| & \geq & \frac{-2}{-2} \\
& & |4x & - & 5| & \leq & 1 \\
-1 & \leq & 4x & - & 5 & \leq & 1 \\
+5 & & & + & 5 & & +5 \\
\frac{4}{4} & \leq & & \frac{4x}{4} & & \leq & \frac{6}{4} \\
1 & \leq & & x & & \leq & \frac{3}{2}
\end{array}$$

Interval notation: $[1, \frac{3}{2}]$



30.
$$\begin{array}{rcrcrcr}
-2 & - & 3|-3x & - & 5| & \geq & -5 \\
+2 & & & & & & +2 \\
& & \frac{-3}{-3}|-3x & - & 5| & \geq & \frac{-3}{-3} \\
& & |-3x & - & 5| & \leq & 1 \\
-1 & \leq & -3x & - & 5 & \leq & 1 \\
+5 & & & + & 5 & & +5 \\
\frac{4}{-3} & \leq & & \frac{-3x}{-3} & & \leq & \frac{6}{-3} \\
-\frac{4}{3} & \geq & & x & & \geq & -2
\end{array}$$

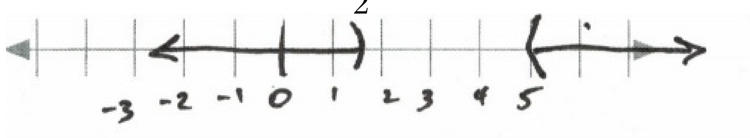
Interval notation: $[-2, -\frac{4}{3}]$



31.
$$\begin{array}{rrrrrrr} -5 & - & 2|3x & - & 6| & < & -8 \\ +5 & & & & & & +5 \\ & & \dfrac{-2}{-2}|3x & - & 6| & < & \dfrac{-3}{-2} \\ & & |3x & - & 6| & > & \dfrac{3}{2} \end{array}$$

$$\begin{array}{ccccccccccc} 3x & - & 6 & < & -\dfrac{3}{2} & \text{or} & \dfrac{3}{2} & < & 3x & - & 6 \\ & + & 6 & & +6 & & +6 & & & + & 6 \\ & & \dfrac{3x}{3} & < & \dfrac{\frac{9}{2}}{3} & & \dfrac{\frac{15}{2}}{3} & < & \dfrac{3x}{3} & & \\ & & x & < & \dfrac{3}{2} & \text{or} & 5 & < & x & & \end{array}$$

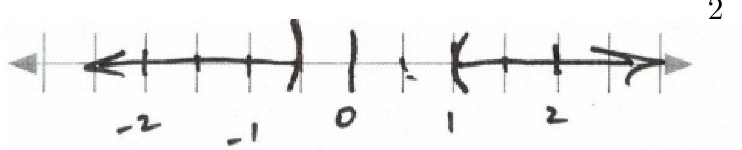
Interval notation: $(-\infty, \frac{3}{2}) \cup (5, \infty)$



32.
$$\begin{array}{rrrrrrr} 6 & - & 3|1 & - & 4x| & < & -3 \\ -6 & & & & & & -6 \\ & & \dfrac{-3}{-3}|1 & - & 4x| & < & \dfrac{-9}{-3} \\ & & |1 & - & 4x| & > & 3 \end{array}$$

$$\begin{array}{ccccccccccc} 1 & - & 4x & < & -3 & \text{or} & 1 & - & 4x & > & 3 \\ -1 & & & & -1 & & -1 & & & & -1 \\ & & -4x & < & -4 & & & & -4x & > & 2 \\ & & x & > & 1 & \text{or} & & & x & < & -\dfrac{1}{2} \end{array}$$

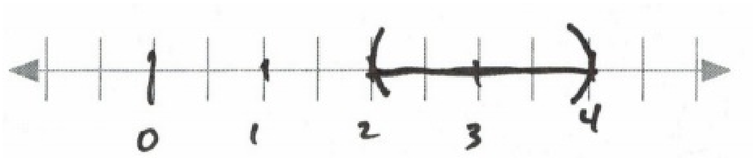
Interval notation: $(-\infty, -\frac{1}{2}) \cup (1, \infty)$



33.

$$\begin{array}{rcrcrcr}
4 & - & 4|-2x & + & 6| & > & -4 \\
-4 & & & & & & -4 \\
& & \dfrac{-4}{-4}|-2x & + & 6| & > & \dfrac{-8}{-4} \\
& & |-2x & + & 6| & < & 2 \\
-2 & < & -2x & + & 6 & < & 2 \\
-6 & & & - & 6 & & -6 \\
\dfrac{-8}{-2} & < & & \dfrac{-2x}{-2} & & < & \dfrac{-4}{-2} \\
4 & > & & x & & > & 2
\end{array}$$

Interval notation: $(2, 4)$

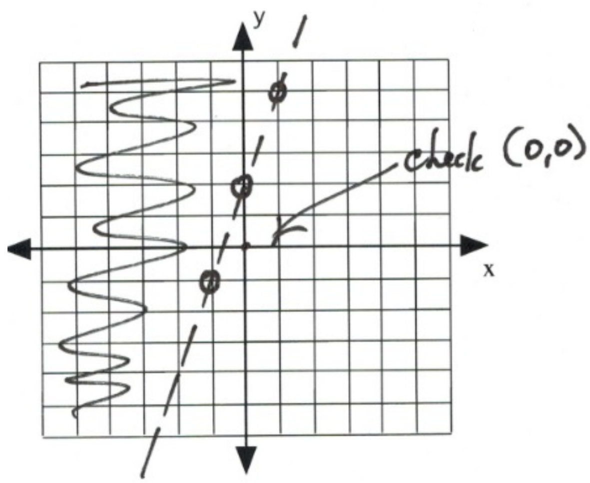


Answer Key 4.4

1.

x	y
1	5
0	2
-1	-1

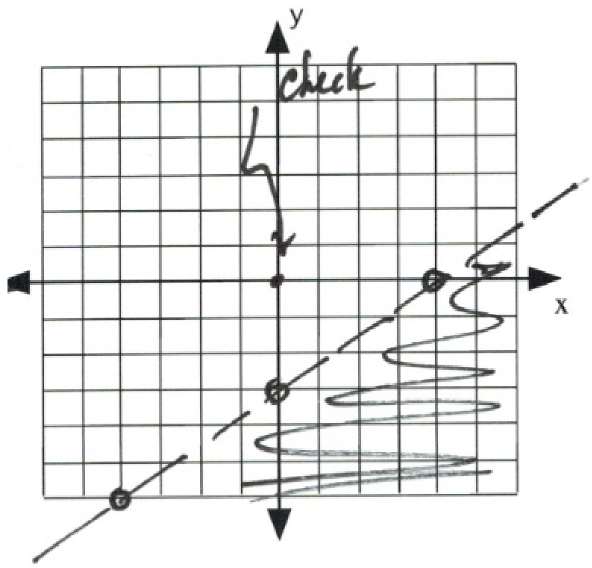
Check $(0,0)$
$0 > 3(0) + 2$
False
$\therefore$ Shade other side



2.

x	y
0	-3
4	0
-4	-6

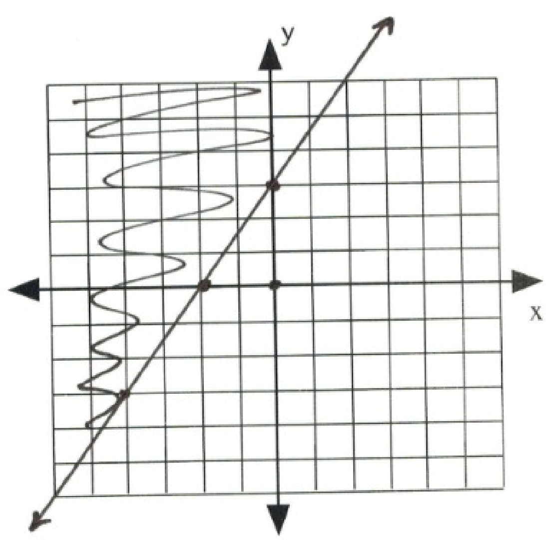
Check $(0,0)$
$3(0) - 4(0) > 12$
False
$\therefore$ Shade other side



3.

x	y
0	3
-2	0
-4	-3

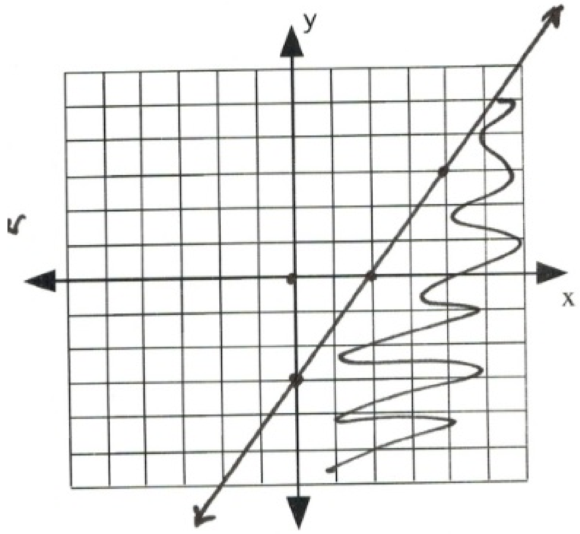
Check $(0, 0)$
$0 \geq 0 + 6$
False
$\therefore$ Shade other side



4.

x	y
0	-3
2	0
4	3

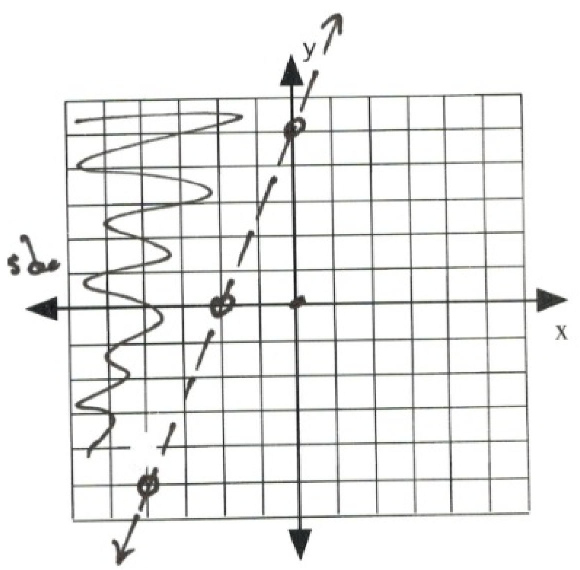
Check $(0, 0)$
$0 - 0 \geq 6$
False
$\therefore$ Shade other side



5.

x	y
0	5
-2	0
-4	-5

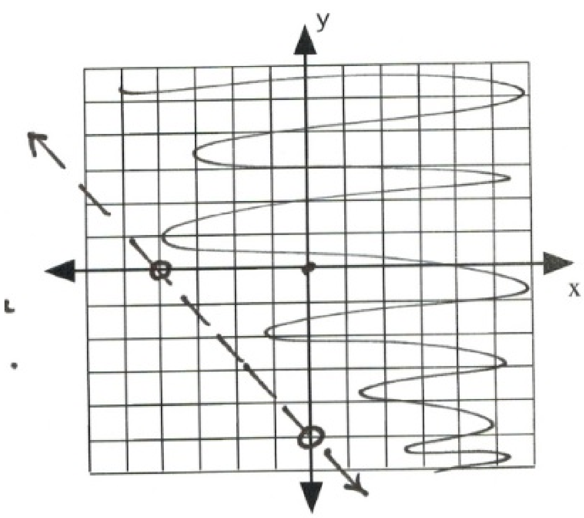
Check $(0, 0)$
$0 > 0 + 10$
False
$\therefore$ Shade other side



6.

x	y
0	-5
-4	0

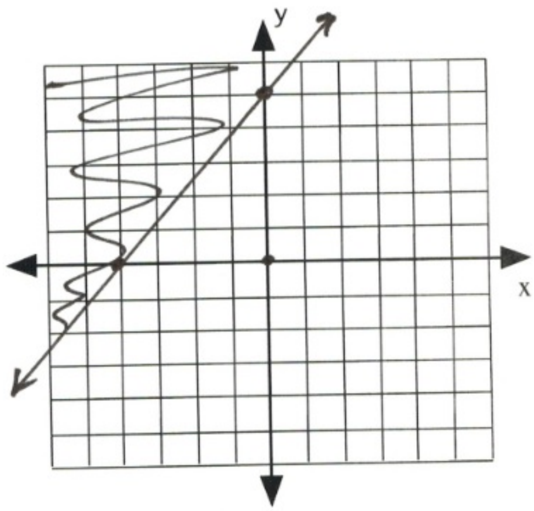
Check $(0, 0)$
$0 + 0 > -20$
True
$\therefore$ Shade the $(0, 0)$ side



7.

x	y
0	5
-4	0

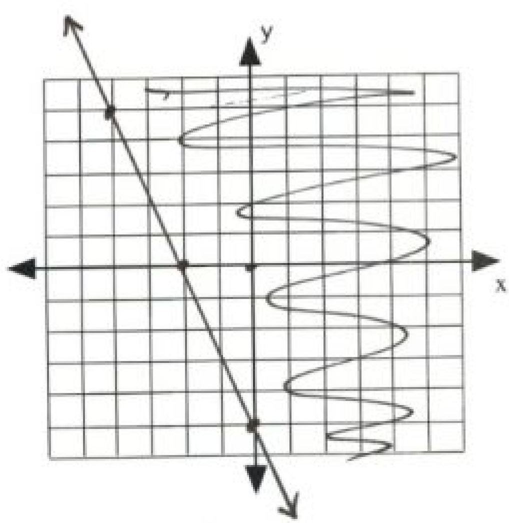
Check $(0, 0)$
$0 \geq 0 + 20$
False
$\therefore$ Shade other side



8.

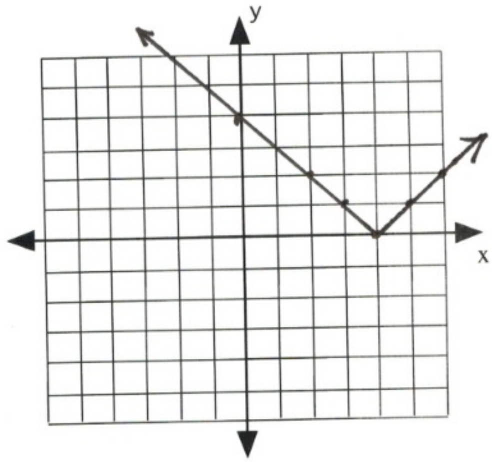
x	y
0	-5
-2	0
-4	5

Check $(0, 0)$
$0 + 0 \geq -10$
True
$\therefore$ Shade the $(0, 0)$ side



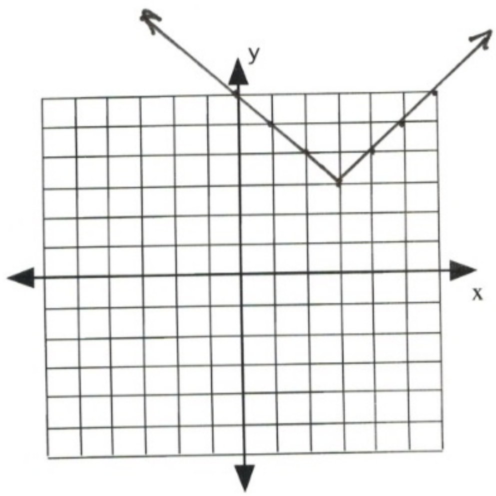
9.

x	y
6	2
5	1
4	0
3	1
2	2
0	4



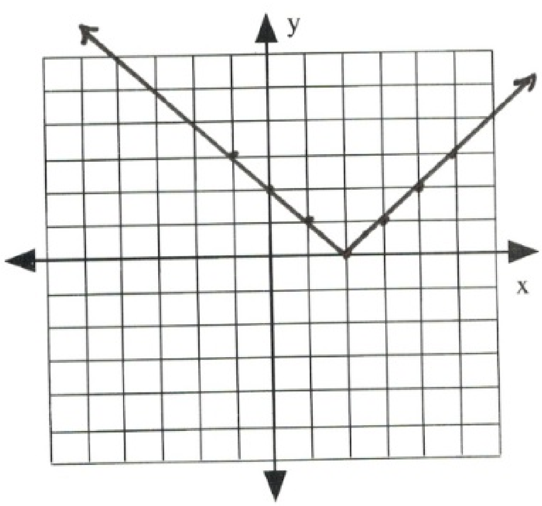
10.

x	y
6	6
5	5
4	4
3	3
2	4
1	5
0	6



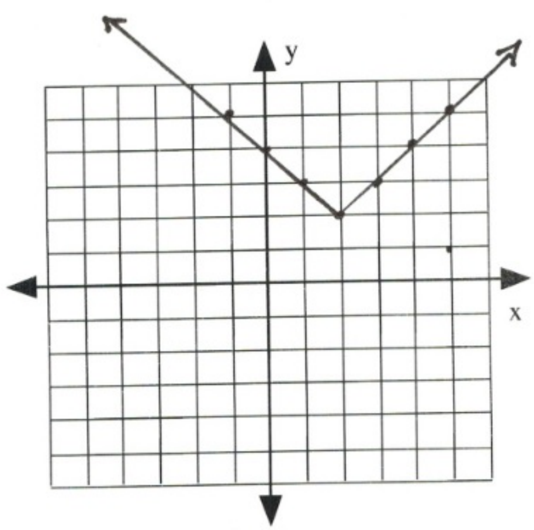
11.

x	y
5	3
4	2
3	1
2	0
1	1
0	2
−1	3



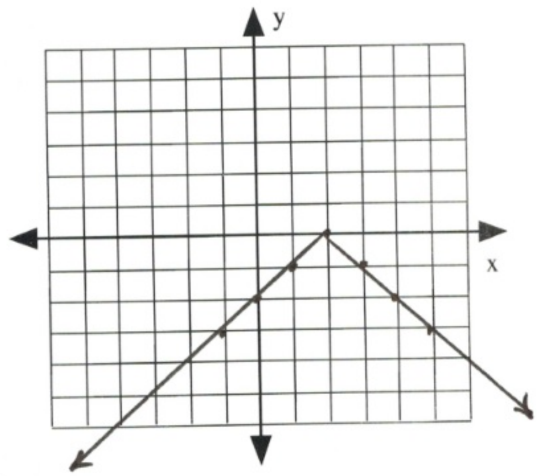
12.

x	y
5	5
4	4
3	3
2	2
1	3
0	4
−1	5



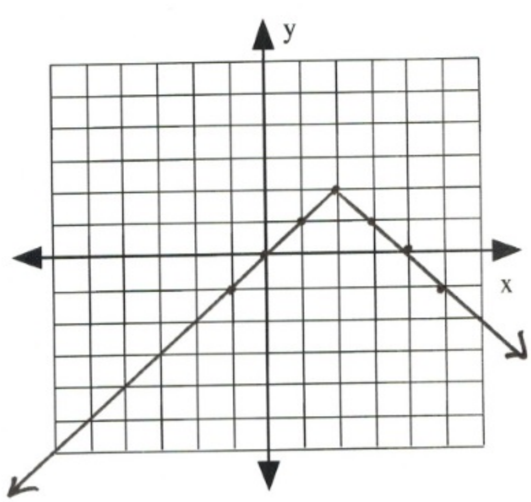
13.

x	y
5	−3
4	−2
3	−1
2	0
1	−1
0	−2
−1	−3



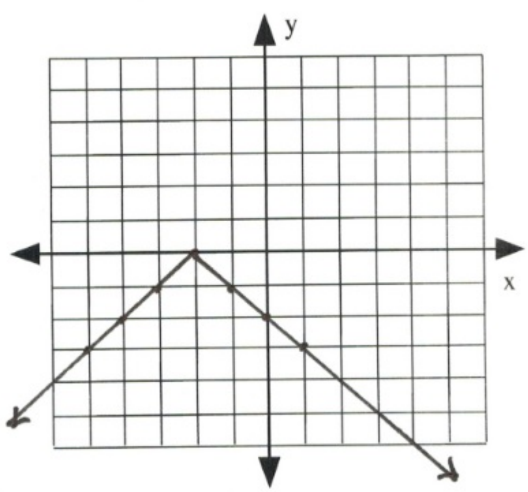
14.

x	y
5	−1
4	0
3	1
2	2
1	1
0	0
−1	−1



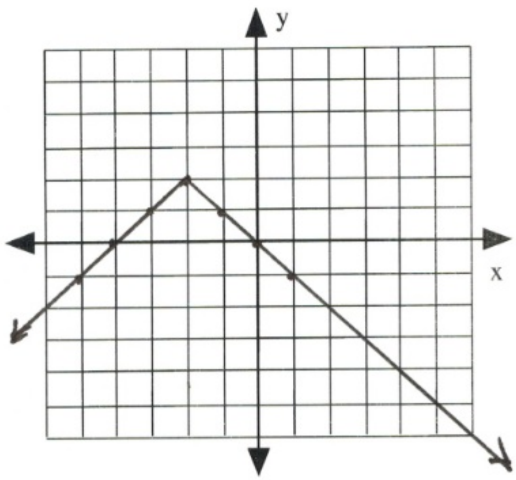
15.

x	y
1	-3
0	-2
-1	-1
-2	0
-3	-1
-4	-2
-5	-3



16.

x	y
-5	-1
-4	0
-3	1
-2	2
-1	1
0	0
1	-1


y
x

Answer Key 4.5

1. $L = 2W - 3$ and $P = 2L + 2W \Rightarrow 54 = 2(2W - 3) + 2W$
2. $L = 2W - 8$ and $P = 2L + 2W \Rightarrow 64 = 2(2W - 8) + 2W$
3. $L = 2W + 4$ and $P = 2L + 2W \Rightarrow 32 = 2(2W + 4) + 2W$
4. $A_1 = 2A_2, A_1 = 10^\circ + A_3, A_1 + A_2 + A_3 = 180^\circ \Rightarrow$
   $A_1 + \frac{A_1}{2} + A_1 - 10^\circ = 180^\circ$
5. $A_1 = \frac{1}{2}A_2, A_1 = 20^\circ + A_3, A_1 + A_2 + A_3 = 180^\circ \Rightarrow$
   $A_1 + 2A_1 + A_1 - 20^\circ = 180^\circ$
6. $A_1 + A_2 = \frac{1}{2}A_3, A_1 + A_2 + A_3 = 180^\circ \Rightarrow$
   $\frac{3}{2}A_3 = 180^\circ \qquad A_1$ and A_2?
7. $x_1 + x_2 = 140, x_1 = 5x_2 \Rightarrow 5x_2 + x_2 = 140$
8. $x_1 + x_2 = 48, x_2 = 5 + x_1 \Rightarrow x_1 + 5 + x_1 = 48$
9. $$\begin{array}{rcccccccccc}
A_2 & = & A_1 & & & & & & & & \\
A_3 & = & A_1 & + & 12 & & & & & & \\
\\
A_1 & + & A_2 & + & A_3 & & & = & 180 & & \\
A_1 & + & A_1 & + & A_1 & + & 12 & = & 180 & & \\
 & & & & & - & 12 & & -12 & & \\
 & & & & & & 3A_1 & = & 168 & & \\
\\
 & & & & & & A_1 & = & \dfrac{168}{3} & = & 56 \\
A_1 & = & 56^\circ & & & & & & & & \\
A_2 & = & 56^\circ & & & & & & & & \\
A_3 & = & 56^\circ & + & 12^\circ & = & 68^\circ & & & &
\end{array}$$
10. $$\begin{array}{rcccccccccc}
A_1 & = & A_2 & & & & & & & & \\
A_3 & = & A_1 & - & 12 & & & & & & \\
\\
A_1 & + & A_2 & + & A_3 & & & = & 180 & & \\
A_1 & + & A_1 & + & A_1 & - & 12 & = & 180 & & \\
 & & & & & + & 12 & & +12 & & \\
 & & & & & & 3A_1 & = & 192 & & \\
\\
 & & & & & & A_1 & = & \dfrac{192}{3} & = & 64 \\
A_1 & = & 64^\circ & & & & & & & & \\
A_2 & = & 64^\circ & & & & & & & & \\
A_3 & = & 64^\circ & - & 12^\circ & = & 52^\circ & & & &
\end{array}$$

11.
$$\begin{aligned} A_1 &= A_2 \\ A_3 &= 3A_1 \end{aligned}$$

$$\begin{array}{ccccccc} A_1 & + & A_2 & + & A_3 & = & 180 \\ A_1 & + & A_1 & + & 3A_1 & = & 180 \\ & & & & 5A_1 & = & 180 \end{array}$$

$$A_1 = \frac{180}{5} = 36$$

$$\begin{aligned} A_1 &= 36^\circ \\ A_2 &= 36^\circ \\ A_3 &= 3(36^\circ) = 108^\circ \end{aligned}$$

12.
$$\begin{aligned} A_2 &= 2A_1 \\ A_3 &= A_1 + 20 \end{aligned}$$

$$\begin{array}{ccccccccc} A_1 & + & A_2 & + & A_3 & & & = & 180 \\ A_1 & + & 2A_1 & + & A_1 & + & 20 & = & 180 \\ & & & & & - & 20 & = & -20 \\ & & & & & & 4A_1 & = & 160 \end{array}$$

$$A_1 = \frac{160}{4} = 40$$

$$\begin{aligned} A_1 &= 40^\circ \\ A_2 &= 2(40^\circ) = 80^\circ \\ A_3 &= 20^\circ + 40^\circ = 60^\circ \end{aligned}$$

13.
$$^1L = W + 15$$

$$\begin{array}{ccccccc} ^2P & = & 2L & + & 2W & & \\ 150 & = & 2(W & + & 15) & + & 2W \\ 150 & = & 2W & + & 30 & + & 2W \\ -30 & & & - & 30 & & \\ 120 & = & 4W & & & & \end{array}$$

$$W = \frac{120}{4} = 30 \text{ cm}$$

$$\begin{aligned} ^3L &= 30 + 15 \\ L &= 45 \text{ cm} \end{aligned}$$

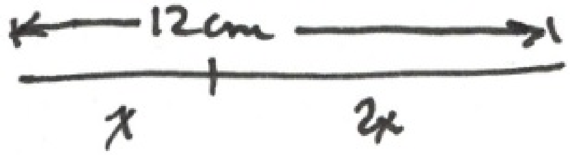
14.
$$\begin{array}{rcllllll}
{}^{1}L & = & W & + & 40 & & & \\
 & & & & & & & \\
{}^{2}P & = & 2L & + & 2W & & & \\
304 & = & 2(W & + & 40) & + & 2W & \\
304 & = & 2W & + & 80 & + & 2W & \\
-80 & & & - & 80 & & & \\
224 & = & 4W & & & & & \\
 & & & & & & & \\
W & = & \dfrac{224}{4} & = & 56 \text{ cm} & & & \\
 & & & & & & & \\
{}^{3}L & = & 56 & + & 40 & & & \\
L & = & 96 \text{ cm} & & & & &
\end{array}$$

15.
$$\begin{array}{rcllllll}
{}^{1}W & = & L & - & 22 & & & \\
 & & & & & & & \\
{}^{2}P & = & 2L & + & 2W & & & \\
152 & = & 2L & + & 2(L & - & 22) & \\
152 & = & 2L & + & 2L & - & 44 & \\
+44 & & & & & + & 44 & \\
196 & = & 4L & & & & & \\
 & & & & & & & \\
L & = & \dfrac{196}{4} & = & 49 \text{ m} & & & \\
 & & & & & & & \\
{}^{3}W & = & 49 & - & 22 & & & \\
L & = & 27 \text{ m} & & & & &
\end{array}$$

16.
$$\begin{array}{rcllllll}
{}^{1}W & = & L & - & 26 & & & \\
 & & & & & & & \\
{}^{2}P & = & 2L & + & 2W & & & \\
280 & = & 2L & + & 2(L & - & 26) & \\
280 & = & 2L & + & 2L & - & 52 & \\
+52 & & & & & + & 52 & \\
332 & = & 4L & & & & & \\
 & & & & & & & \\
L & = & \dfrac{332}{4} & = & 83 \text{ m} & & & \\
 & & & & & & & \\
{}^{3}W & = & 83 & - & 26 & & & \\
L & = & 57 \text{ m} & & & & &
\end{array}$$

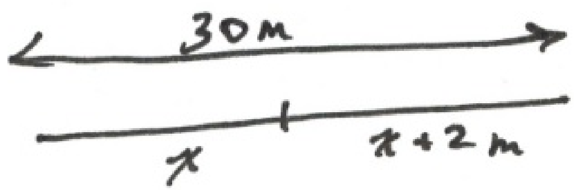
17.
$$\begin{aligned} x + 2x &= 12 \\ 3x &= 12 \\ x &= \frac{12}{3} = 4 \text{ cm} \\ \therefore\ 2x &= 2(4) = 8 \text{ cm} \end{aligned}$$

Pieces are 4 cm and 8 cm



18.
$$\begin{aligned} x + x + 2 &= 30 \\ -2 &\quad -2 \\ 2x &= 28 \\ x &= \frac{28}{2} = 14 \text{ m} \\ \therefore\ x + 2 &= 14 + 2 = 16 \text{ m} \end{aligned}$$

Pieces are 14 m and 16 m



Answer Key 4.6

Action taken:		**Each container holds:**
Step 1.	Fill B using A	$A = 11 \quad B = 13 \quad C = 0 \quad D = 0$
Step 2.	Fill D using B	$A = 11 \quad B = 8 \quad C = 0 \quad D = 5$
Step 3.	Fill C using B	$A = 11 \quad B = 0 \quad C = 8 \quad D = 5$
Step 4.	Pour D into A	$A = 16 \quad B = 0 \quad C = 8 \quad D = 0$
Step 5.	Fill B using A	$A = 3 \quad B = 13 \quad C = 8 \quad D = 0$
Step 6.	Fill D using B	$A = 3 \quad B = 8 \quad C = 8 \quad D = 5$
Step 7.	Pour D into A	$A = 8 \quad B = 8 \quad C = 8 \quad D = 0$

Mid Term 1: Review Questions Answer Key

1. True
2. Undefined
3. 15
4. 16
5. 12
6. 19
7. True
8. −18
9. 18
10. −16
11. 16
12. −16
13. $-(-6)-\sqrt{(-6)^2-4(8)(-2)}$
    $6-\sqrt{36+64}$
    $6-10$
    -4
14. $$\begin{array}{rrrrrrrrrrrr} & 3x & - & 12 & - & 27 & = & 7 & - & 5x & - & 30 \\ + & 5x & + & 12 & + & 27 & & & + & 5x & + & 12 \\ & & & & & & & & & & + & 7 \\ & & & & & & & & & & + & 27 \\ & & & & & 8x & = & 16 & & & & \\ & & & & & x & = & 2 & & & & \end{array}$$
15. $\left(\dfrac{1}{R}=\dfrac{1}{r_1}+\dfrac{1}{r_2}\right)(Rr_1r_2)$

    $r_1r_2=Rr_2+Rr_1$

    $r_1r_2=R(r_2+r_1)$

    $R=\dfrac{r_1r_2}{r_2+r_1}$
16. $$\left(\frac{x+3}{8}-\frac{3}{4}=\frac{x+6}{10}\right)(40)$$
    $$\begin{array}{rrrrrrrrrr} & 5(x & + & 3) & - & 3(10) & = & 4(x & + & 6) \\ & 5x & + & 15 & - & 30 & = & 4x & + & 24 \\ - & 4x & - & 15 & + & 30 & & -4x & - & 15 \\ & & & & & & & & + & 30 \\ & & & & x & & = & 39 & & \end{array}$$
17. Need graph drawn. $y=5$

18.

$$
\begin{aligned}
m &= \frac{\Delta y}{\Delta x} \\
\frac{2}{3} &= \frac{y-2}{x--1}
\end{aligned}
$$

$$
\begin{array}{rrrrrrrrr}
 & & 2(x & + & 1) & = & 3(y & - & 2) \\
 & & 2x & + & 2 & = & 3y & - & 6 \\
 & & -3y & + & 6 & & -3y & + & 6 \\
2x & - & 3y & + & 8 & = & 0 & & \\
 & & & & y & = & \frac{2}{3}x & + & \frac{8}{3}
\end{array}
$$

19.

1st slope

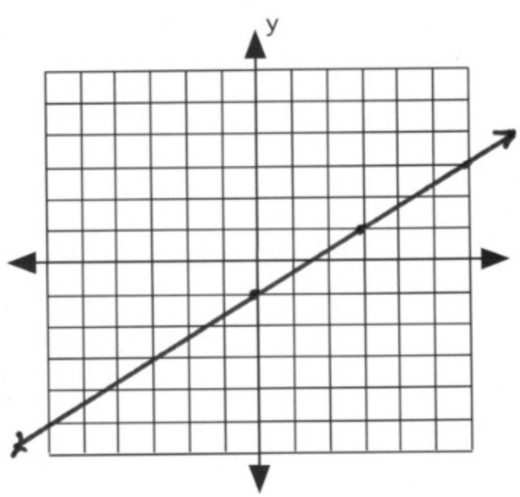
$$
\begin{aligned}
m &= \frac{\Delta y}{\Delta x} \\
m &= \frac{11--1}{2--2} \\
m &= \frac{12}{4} \\
m &= 3
\end{aligned}
$$

$$
\begin{array}{rrrrrrrrr}
 & & 3(x & + & 2) & = & y & + & 1 \\
 & & 3x & + & 6 & = & y & + & 1 \\
 & & -y & - & 1 & & -y & - & 1 \\
3x & - & y & + & 5 & = & 0 & & \\
 & & & & y & = & 3x & + & 5
\end{array}
$$

2nd slope

$$
\begin{aligned}
m &= \frac{\Delta y}{\Delta x} \\
3 &= \frac{y--1}{x--2}
\end{aligned}
$$

20.



Use slop intercept method.

21. $$\begin{aligned} d^2 &= \Delta x^2 + \Delta y^2 \\ d^2 &= (15-7)^2 + (3 - -3)^2 \\ d^2 &= 8^2 + 6^2 \\ d &= 10 \end{aligned}$$

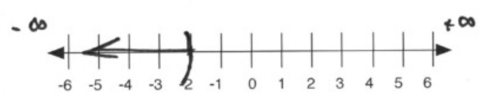
22.

$$x = \frac{x_1 + x_2}{2} = \frac{7+15}{2} = 11$$

$$y = \frac{y_1 + y_2}{2} = \frac{-3+3}{2} = 0$$

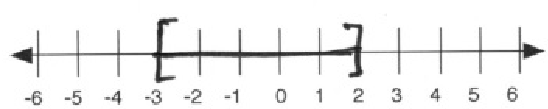
$(11, 0)$

23. True
24. True
25. $\{r, s, t\}$
26. True
27. $$\begin{array}{ccccrcr} 3x & - & 6 & - & 36x & > & 60 \\ & + & 6 & & & & +6 \\ & & & & -33x & > & 66 \\ & & & & x & < & -2 \end{array}$$



(−∞, −2)

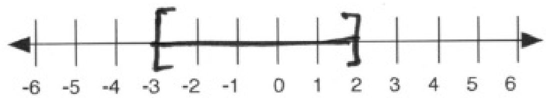
28. $$\begin{array}{ccccccc} -18 & \leq & 4x & - & 6 & \leq & 2 \\ +6 & & & + & 6 & & +6 \\ \frac{-12}{4} & \leq & & \frac{4x}{4} & & \leq & \frac{8}{4} \\ -3 & \leq & & x & & \leq & 2 \end{array}$$



[−3, 2]

29.

$$\begin{array}{rrrrr} 2x & - & 1 & < & -7 \\ & + & 1 & & +1 \\ & & \dfrac{2x}{2} & < & \dfrac{-6}{2} \\ & & x & < & -3 \end{array} \qquad \begin{array}{rrrrr} 7 & < & 2x & - & 1 \\ +1 & & & + & 1 \\ \dfrac{8}{2} & < & \dfrac{2x}{2} & & \\ 4 & < & x & & \end{array}$$



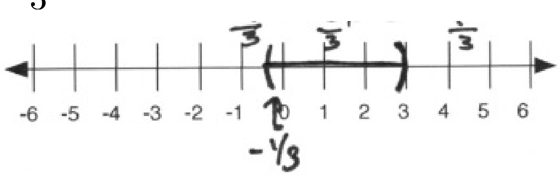
(−∞, −3) *or* (4, ∞) WRONG IMAGE

30.

$$\left|\frac{3x-4}{5}\right| < 1$$

$$\left(-1 < \frac{3x-4}{5} < 1\right)(5)$$

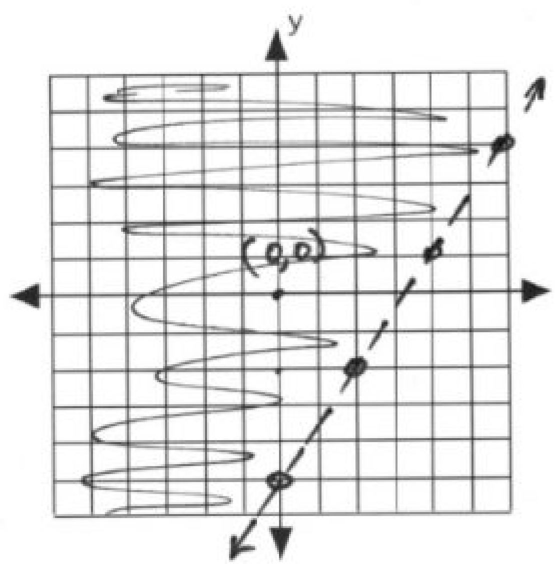
$$\begin{array}{rrrrrrr} -5 & < & 3x & - & 4 & < & 5 \\ +4 & & & + & 4 & & +4 \\ \dfrac{-1}{3} & < & & \dfrac{3x}{3} & & < & \dfrac{9}{3} \\ -\dfrac{1}{3} & < & & x & & < & 3 \end{array}$$



$$(-\frac{1}{3}, 3)$$

31.

$$\begin{array}{rrrrrrrr} & 3x & - & 2y & = & 10 & & \\ + & -10 & + & 2y & & -10 & + & 2y \\ & \dfrac{3x}{2} & - & \dfrac{10}{2} & = & \dfrac{2y}{2} & & \\ & & & y & = & \dfrac{3}{2}x & - & 5 \end{array}$$



Slope intercept method. Check (0, 0): 3(0) − 2(0) < 10. Shade the (0, 0) side.

32.

$$y = |x - 1| - 2$$

x	y
4	1
3	0
2	−1
1	−2
0	−1
−1	0
−2	1

33.

$$\begin{array}{rrrrcr} & 6L & + & 2S & = & 38 \\ + & 4L & - & 2S & = & 12 \\ & & & 10L & = & 50 \\ & & & L & = & 5 \\ \\ \therefore & 6(5) & + & 2S & = & 38 \\ & 30 & + & 2S & = & 38 \\ - & 30 & & & & -30 \\ & & & 2S & = & 8 \\ & & & S & = & 4 \end{array}$$

34. Insert diagram.

$$\begin{array}{rcl} 5x + x & = & 36 \\ 6x & = & 36 \\ x & = & 6 \\ \therefore 5x & = & 30 \end{array}$$

35.

$$\begin{array}{rrrcl} & (d & + & q & = & 14)(-10) \\ & 10d & + & 25q & = & 260 \\ \\ & -10d & - & 10q & = & -140 \\ + & 10d & + & 25q & = & 260 \\ & & & \dfrac{15q}{15} & = & \dfrac{120}{15} \\ \\ & & & q & = & 8 \\ \\ \therefore & d & + & 8 & = & 14 \\ & & - & 8 & & -8 \\ & & & d & = & 6 \end{array}$$

36.

$x, x+2$

$$\begin{array}{rrrrcrrr} x & + & x & + & 2 & = & x & - & 10 \\ & & 2x & + & 2 & = & x & - & 10 \\ & & -x & - & 2 & & -x & - & 2 \\ & & & & x & = & -12 & & \end{array}$$

numbers are −12, −10

37.

1st

$$y = \frac{kmn^2}{d}$$

2nd

$$\begin{array}{rcl} y & = & 12 \\ k & = & \text{find} \\ m & = & 3 \\ n & = & 4 \\ d & = & 8 \end{array}$$

$$12 = \frac{k(3)(4)^2}{8}$$

$$k = \frac{12(8)}{3 \cdot 16}$$

$$k = 2$$

3rd

$$\begin{array}{rcl} y & = & \text{find} \\ k & = & 2 \\ m & = & -3 \\ n & = & 3 \\ d & = & 6 \end{array}$$

$$y = \frac{(2)(-3)(3)^2}{6}$$

$$y = -9$$

1.

Midterm 1: Version A Answer Key

1. $-(-3)-\sqrt{(-3)^2-4(4)(-1)}$

$3-\sqrt{9+16}$

$3-\sqrt{25}$

$3-5$

-2

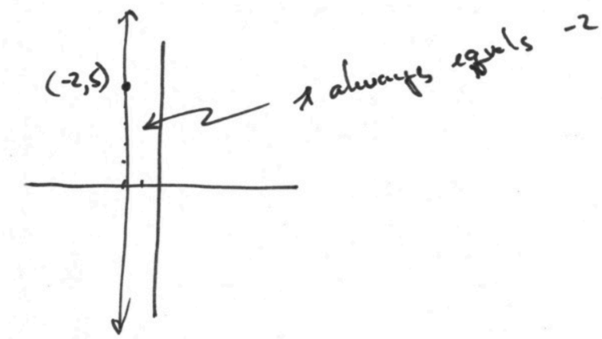
2. $$\begin{array}{rrrrrrrrrrrrrr}
& 2x & - & 10 & - & 85 & = & 3 & - & 9x & - & 54 \\
+ & 9x & + & 10 & + & 85 & & & + & 9x & + & 3 \\
& & & & & & & & & & + & 85 \\
& & & & & & & & & & + & 10 \\
& & & & & \dfrac{11x}{11} & = & \dfrac{44}{11} & & & & \\
& & & & & x & = & 4 & & & &
\end{array}$$

3. $$\begin{array}{rcl}
A(B-b) & = & h \\
B-b & = & \dfrac{h}{A} \\
-b & = & \dfrac{h}{A}-B
\end{array}$$

4. $$\left(\frac{x+1}{4}-\frac{5}{8}=\frac{x-1}{8}\right)(8)$$

$$\begin{array}{rrrrrrrrr}
2(x & + & 1) & - & 5 & = & x & - & 1 \\
2x & + & 2 & - & 5 & = & x & - & 1 \\
-x & - & 2 & + & 5 & & -x & + & 5 \\
& & & & & & & - & 2 \\
& & & & x & = & 2 & &
\end{array}$$

5.



$x = -2$

6.

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ \frac{2}{5} &= \frac{y - -2}{x - -1} \end{aligned}$$

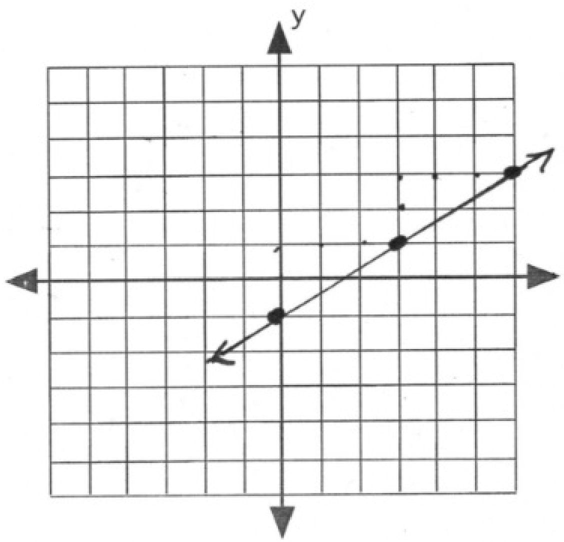
$$\begin{aligned} 2(x + 1) &= 5(y + 2) \\ 2x + 2 &= 5y + 10 \\ -5y - 10 & \quad -5y - 10 \\ 2x - 5y - 8 &= 0 \\ 0 &= 2x - 5y - 8 \\ y &= \frac{2}{5}x - \frac{8}{5} \end{aligned}$$

7. 1st slope

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ m &= \frac{4 - 0}{6 - -2} \\ m &= \frac{4}{8} \text{ or } \frac{1}{2} \end{aligned}$$

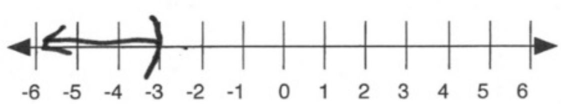
Now:

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ \frac{1}{2} &= \frac{y - 0}{x - -2} \\ 1(x + 2) &= 2(y - 0) \\ x + 2 &= 2y \\ \therefore y &= \frac{1}{2} + 1 \\ \text{or } x - 2y + 2 &= 0 \end{aligned}$$

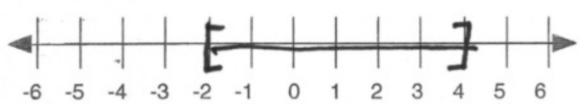


8.

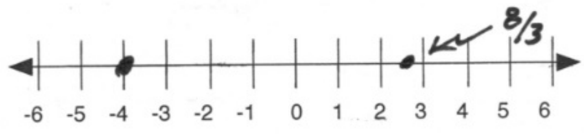
9.
$$\begin{array}{rcrcrcl} 6x & - & 5 & - & 30x & > & 67 \\ & + & 5 & & & & +5 \\ & & & & \dfrac{-24x}{-24} & > & \dfrac{72}{-24} \\ & & & & x & < & -3 \end{array}$$



10.
$$\begin{array}{rcccccc} -10 & \leq & 4x & - & 2 & \leq & 14 \\ +2 & & & + & 2 & & +2 \\ \dfrac{-8}{4} & \leq & & \dfrac{4x}{4} & & \leq & \dfrac{16}{4} \\ -2 & \leq & & x & & \leq & 4 \end{array}$$

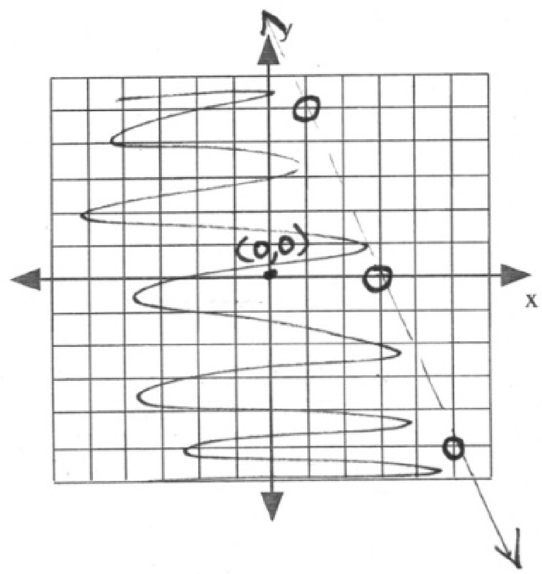


11.
$$\begin{array}{rcl} \dfrac{3x+2}{5} & = & 2 \\ 3x+2 & = & 10 \\ -2 & & -2 \\ \dfrac{3x}{3} & = & \dfrac{8}{3} \\ x & = & \dfrac{8}{3} \end{array} \qquad \begin{array}{rcl} \dfrac{3x+2}{5} & = & -2 \\ 3x+2 & = & -10 \\ -2 & & -2 \\ \dfrac{3x}{3} & = & \dfrac{-12}{3} \\ x & = & -4 \end{array}$$



12. $$5x + 2y < 15$$

x	y
0	7.5
3	0
1	5
5	−5



13.

$$\begin{array}{rrrrl} (5L & + & 3S & = & 47)(2) \\ (4L & - & 2S & = & 20)(3) \end{array}$$

$$\begin{array}{rrrrrl} & 10L & + & 6S & = & 94 \\ + & 12L & - & 6S & = & 60 \\ & & & \dfrac{22L}{22} & = & \dfrac{154}{22} \\ & & & L & = & 7 \end{array}$$

$$\begin{array}{rrrrrr} \therefore & 4L & - & 2S & = & 20 \\ & 4(7) & - & 2S & = & 20 \\ & 28 & - & 2S & = & 20 \\ - & 28 & & & & -28 \\ & & & \dfrac{-2S}{-2} & = & \dfrac{-8}{-2} \\ & & & S & = & 4 \end{array}$$

14.

$$\begin{array}{rcl} 36 \text{ cm} & = & 5x + x \\ 36 \text{ cm} & = & 6x \\ x & = & \dfrac{36 \text{ cm}}{6} \\ x & = & 6 \text{ cm} \\ 5x & = & 5(6) = 30 \text{ cm} \end{array}$$

15. <u>1st</u>

$$y = \frac{kmn}{d^2}$$

<u>2nd</u>

$$\begin{aligned} y &= 3 \\ k &= \text{find} \\ m &= 2 \\ n &= 8 \\ d &= 4 \end{aligned}$$

$$y = \frac{kmn}{d^2}$$

$$3 = \frac{k(2)(8)}{(4)^2}$$

$$k = \frac{3 \cdot (4)^2}{2 \cdot 8}$$

$$k = 3$$

<u>3rd</u>

$$\begin{aligned} y &= \text{find} \\ k &= 3 \\ m &= 15 \\ n &= 10 \\ d &= 5 \end{aligned}$$

$$y = \frac{kmn}{d^2}$$

$$y = \frac{(3)(15)(10)}{(5)^2}$$

$$y = 18$$

Midterm 1: Version B Answer Key

1. $-6 - \sqrt{6^2 - 4(5)(1)}$

   $-6 - \sqrt{36 - 20}$

   $-6 - \sqrt{16}$

   $-6 - 4$

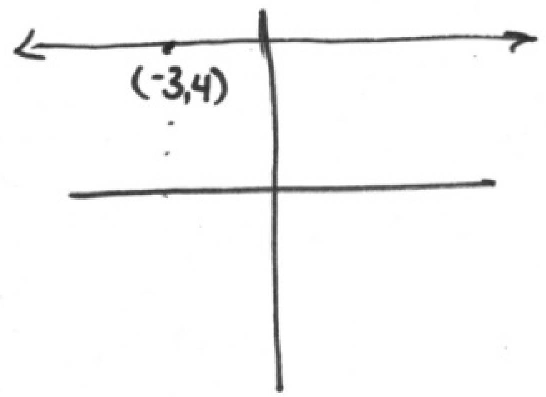
   -10
2. $$\begin{array}{rrrrcrrr} & 15x & - & 18 & = & 4[-6 & + & 3x] \\ & 15x & - & 18 & = & -24 & + & 12x \\ - & 12x & + & 18 & & +18 & - & 12x \\ & & & \dfrac{3x}{3} & = & \dfrac{-6}{3} & & \\ \\ & & & x & = & -2 & & \end{array}$$
3. $\left(A = \dfrac{h}{B \cdot b}\right)(b)$

   $\left(Ab = \dfrac{h}{B}\right) \div A$

   $b = \dfrac{h}{BA}$
4. $$\left(\frac{x+3}{5} - \frac{x}{2} = \frac{5-3x}{10}\right)(10)$$

   $$\begin{array}{rrrrrcrrr} 2(x & + & 3) & - & 5(x) & = & 5 & - & 3x \\ 2x & + & 6 & - & 5x & = & 5 & - & 3x \\ & - & 3x & + & 6 & = & 5 & - & 3x \\ & + & 3x & - & 6 & & -6 & + & 3x \\ & & & & 0 & = & -1 & & \end{array}$$

   No solution
5. $y = 4$



6.
$$\begin{aligned} \text{slope} &= \frac{\Delta y}{\Delta x} \\ \frac{1}{3} &= \frac{y-4}{x--1} \end{aligned}$$

$$\begin{array}{rcrcrcl} 1(x & + & 1) & = & 3(y & - & 4) \\ x & + & 1 & = & 3y & - & 12 \\ & - & 1 & & & - & 1 \\ & & x & = & 3y & - & 13 \end{array}$$

$$x - 3y + 13 = 0$$

$$y = \frac{1}{3}x + \frac{13}{3}$$

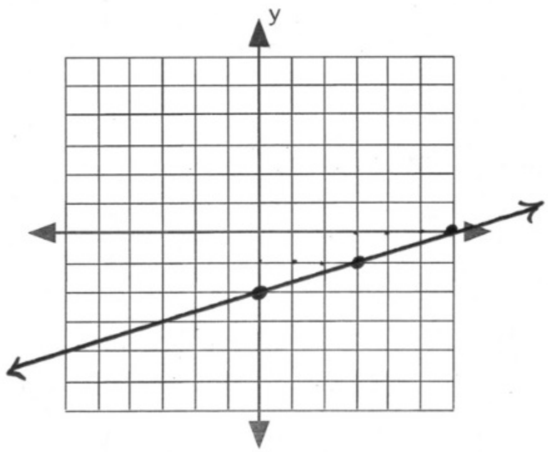
7.

$$\text{1st slope } \ldots\ m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{5-4}{-3-0} \Rightarrow -\frac{1}{3}$$

$$\text{now } \ldots\ m = \frac{\Delta y}{\Delta x} \Rightarrow -\frac{1}{3} \Rightarrow \frac{y-4}{x-0}$$

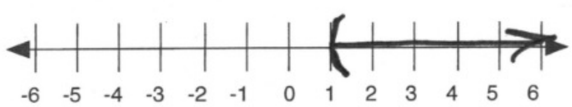
$$\begin{aligned} -1(x) &= 3(y-4) \\ -x &= 3y-12 \end{aligned}$$

$$\begin{aligned} x+3y-12 &= 0 \\ y &= -\frac{1}{3}x+4 \end{aligned}$$



8.

9.
$$\begin{array}{rrrrrrrrrr}
6x & - & 12 & + & 8x & > & 15 & - & 20x & + & 7 \\
 & + & 12 & + & 20x & & & + & 20x & + & 15 \\
 & & & + & 6x & & & & & + & 12 \\
 & & & & 34x & > & 34 & & & & \\
 & & & & x & > & 1 & & & &
\end{array}$$

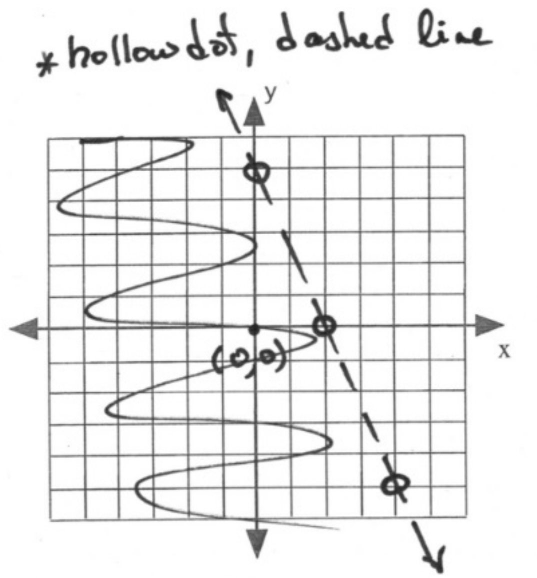


10.
$$\begin{array}{rrrrrrr}
-3 & \leq & 2x & + & 3 & < & 9 \\
-3 & & & - & 3 & & -3 \\
\frac{-6}{2} & \leq & & \frac{2x}{2} & & < & \frac{6}{2} \\
-3 & \leq & & x & & < & 3
\end{array}$$

11.
$$\left(-2 < \frac{3x+2}{5} < 2\right)(5)$$
$$\begin{array}{rrrrrrr}
-10 & < & 3x & + & 2 & < & 10 \\
-2 & & & - & 2 & & -2 \\
\frac{-12}{3} & < & & \frac{3x}{3} & & < & \frac{8}{3} \\
-4 & < & & x & & < & \frac{8}{3}
\end{array}$$

12.

$5x + 2y = 10$	
x	y
2	0
0	5
−2	10
4	−5



13.

$x, x+2$

$$\begin{array}{rrrrrrrr} x & + & (x & + & 2) & = & 5x & - & 16 \\ & & 2x & + & 2 & = & 5x & - & 16 \\ & + & -5x & - & 2 & & -5x & - & 2 \\ & & & & \dfrac{-3x}{-3} & = & \dfrac{-18}{-3} & & \\ & & & & x & = & 6 & & \end{array}$$

numbers are 6, 8

14.

$$\begin{aligned} 4x + x &= 40 \text{ cm} \\ 5x &= 40 \text{ cm} \\ x &= \frac{40 \text{ cm}}{5} \text{ or } 8 \text{ cm} \\ \therefore 4x &= 4(8) \text{ or } 32 \text{ cm} \end{aligned}$$

15.

$$\underline{\text{1st}}$$

$$P = \frac{kT}{V}$$

$$\underline{\text{2nd}}$$

$$\begin{aligned} &\textbf{1st data} \\ P &= 100 \\ k &= \text{find 1st} \\ T &= 200 \\ V &= 500 \end{aligned} \qquad \begin{aligned} &\textbf{2nd data} \\ P &= \text{find 2nd} \\ k &= 250 \\ T &= 100 \\ V &= 500 \end{aligned}$$

$$\begin{aligned} P &= \frac{kT}{V} \\ 100 &= \frac{k(200)}{500} \\ k &= \frac{100(500)}{2002} \\ k &= 250 \end{aligned} \qquad \begin{aligned} P &= \frac{kT}{V} \\ P &= \frac{(250)(100)}{500} \\ P &= 50 \end{aligned}$$

Midterm 1: Version C Answer Key

1. $-(4)-\sqrt{4^2-4(4)1}$

   $-4-\sqrt{16-16}$

   -4

2. $$\begin{array}{rrrrrrrrrrr} & 2x & - & 8 & + & 8 & = & 3 & - & 7x & - & 21 \\ + & 7x & & & & & & & + & 7x & & \\ & & & & & \dfrac{9x}{9} & = & \dfrac{-18}{9} & & & & \\ & & & & & x & = & -2 & & & & \end{array}$$

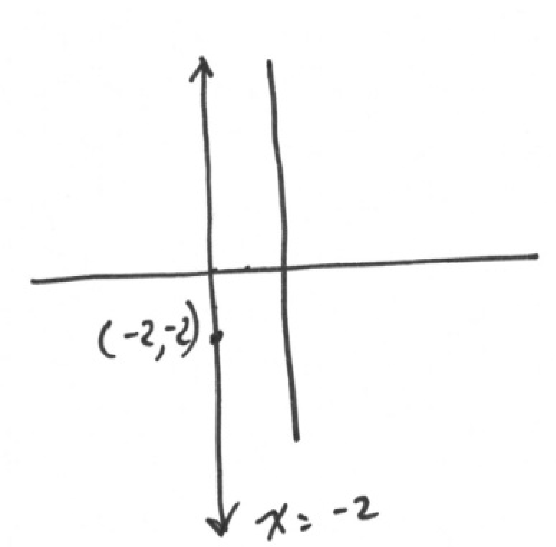
3. $$\left(A=\frac{h}{B+b}\right)(B+b)$$

   $$\begin{array}{rrrrrrr} \dfrac{A}{A}(B & + & b) & = & \dfrac{h}{A} & & \\ B & + & b & = & \dfrac{h}{A} & & \\ & - & b & & & - & b \\ & & B & = & \dfrac{h}{A} & - & b \end{array}$$

4. $$\left(\frac{x}{15}-\frac{x-3}{3}=\frac{1}{5}\right)(15)$$

   $$\begin{array}{rrrrrrr} x & - & 5(x & - & 3) & = & 3(1) \\ x & - & 5x & + & 15 & = & 3 \\ & & & - & 15 & & -15 \\ & & & & \dfrac{-4x}{-4} & = & \dfrac{-12}{-4} \\ & & & & x & = & 3 \end{array}$$

5. $x=-2$



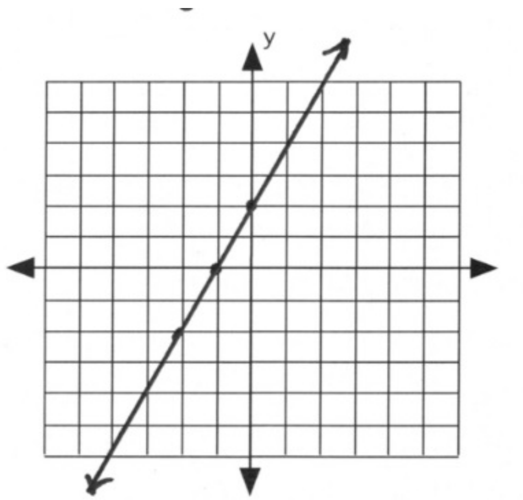
6.
$$
\begin{aligned}
y &= mx + 6 \\
\therefore y &= \frac{2}{3}x - 3 \\
\\
\text{or } 3y &= 2x - 9 \\
0 &= 2x - 3y - 9
\end{aligned}
$$

7.
$$
\begin{aligned}
& \textbf{1st slope} \\
m &= \frac{\Delta y}{\Delta x} \\
m &= \frac{4 - -6}{-1 - 14} \\
m &= \frac{10}{-15} \\
m &= -\frac{2}{3} \\
& \textbf{Now:} \\
m &= \frac{\Delta y}{\Delta x} \\
-\frac{2}{3} &= \frac{y - 4}{x - -1}
\end{aligned}
$$

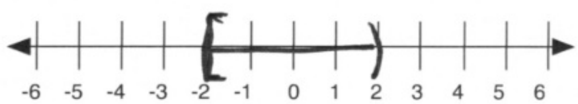
$$
\begin{array}{rrrrrrrrrr}
 & & & -2(x & + & 1) & = & 3(y & - & 4) \\
 & & & -2x & - & 2 & = & 3y & - & 12 \\
+ & & & -3y & + & 12 & & -3y & + & 12 \\
 & -2x & - & 3y & + & 10 & = & 0 & & \\
\text{or} & 2x & + & 3y & - & 10 & = & 0 & & \\
 & & & & & y & = & -\frac{2}{3}x & + & \frac{10}{3}
\end{array}
$$

8.

$2x - y = -2$	
x	y
0	2
−1	0
−2	−2

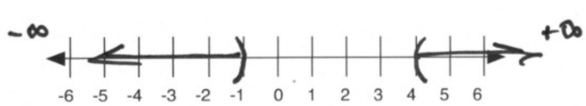


9.
$$\begin{array}{rcccccc} 0 & \leq & 2x & + & 4 & < & 8 \\ -4 & & & -4 & & & -4 \\ \dfrac{-4}{2} & \leq & & \dfrac{2x}{2} & & < & \dfrac{4}{2} \\ -2 & \leq & & x & & < & 2 \end{array}$$



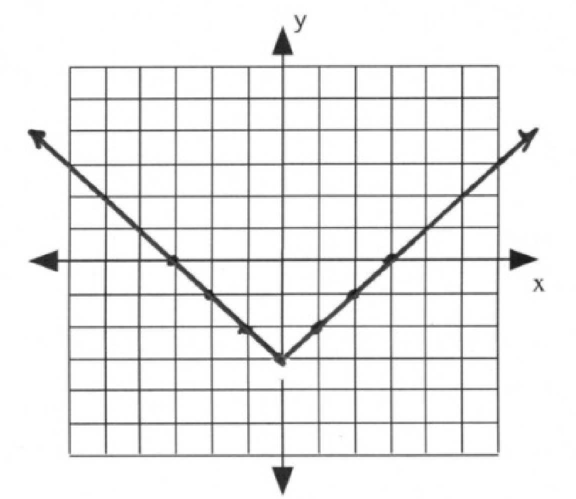
10.
$$\begin{array}{ccccccccccc} y & - & 1 & > & 3 & \text{or} & y & - & 1 & < & -3 \\ & + & 1 & & +1 & & & + & 1 & & +1 \\ & & y & > & 4 & \text{or} & & & y & < & -2 \end{array}$$

11.
$$\begin{array}{cccccccccc} 2x & - & 3 & < & -5 & \qquad & 2x & - & 3 & > & 5 \\ & + & 3 & & +3 & & & + & 3 & & +3 \\ & & 2x & < & -2 & & & & 2x & > & 8 \\ & & x & < & -1 & & & & x & > & 4 \end{array}$$



12. $y = |x| - 3$

x	y
3	0
2	−1
1	−2
0	−3
−1	−2
−2	−1
−3	0



13.
$$\begin{array}{rcrcl} 5L & + & 3S & = & 49 \\ 4L & - & 2S & = & 26 \end{array}$$

$$\frac{4L}{2} - \frac{2S}{2} = \frac{26}{2}$$

$$2L - S = 13$$

$$\text{or} \quad S = 2L - 13$$

$$\begin{array}{rcrcrcl} 5L & + & 3(2L & - & 13) & = & 49 \\ 5L & + & 6L & - & 39 & = & 49 \\ & & & + & 39 & & +39 \end{array}$$

$$\frac{11L}{11} = \frac{88}{11}$$

$$L = 8$$

$$\begin{aligned} \therefore S &= 2L - 13 \\ S &= 2(8) - 13 \\ S &= 16 - 13 \\ S &= 3 \end{aligned}$$

14.
$$\begin{aligned} 5x + x &= 42 \\ 6x &= 42 \\ x &= \frac{42}{6} \text{ or } 7 \\ \therefore 5x &= 5(7) \text{ or } 35 \end{aligned}$$

15. $y = \dfrac{km}{d^2}$

$$\begin{array}{rclcrcl} & & \textbf{1st} & & & & \textbf{2nd} \\ y &=& 3 & & y &=& \text{find} \\ k &=& \text{find 1st} & & k &=& 24 \\ m &=& 2 & & m &=& 25 \\ d &=& 4 & & d &=& 5 \end{array}$$

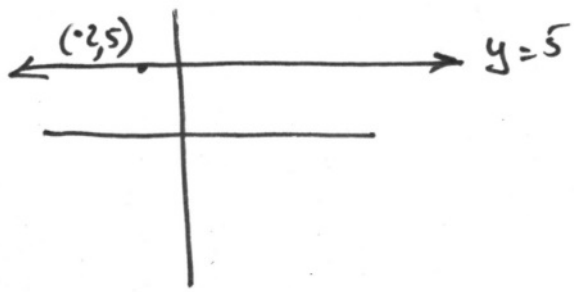
$$\begin{array}{rclcrcl} y &=& \dfrac{km}{d^2} & & y &=& \dfrac{km}{d^2} \\[2ex] 3 &=& \dfrac{k(2)}{(4)^2} & & y &=& \dfrac{(24)(25)}{(5)^2} \\[2ex] 3 &=& \dfrac{3(4)^2}{2} & & y &=& 24 \\[2ex] k &=& 24 & & & & \end{array}$$

Midterm 1: Version D Answer Key

1. $3(4)-\sqrt{4^2-4(4)(1)}$

   $12-\sqrt{16-16}$

   12
2. $$\begin{array}{rrrrrrrrr} 2x & - & 8 & + & 8 & = & -6 & + & 3x & + & 9 \\ &&&& 2x & = & 3x & + & 3 \\ &&&& -3x && -3x \\ &&&& -x & = & 3 \\ &&&& x & = & -3 \end{array}$$
3. $$\left(\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}\right)(Rr_1r_2)$$
   $$\begin{array}{rrrrrr} && r_1r_2 & = & Rr_2 & + & Rr_1 \\ && -Rr_2 && -Rr_2 \\ r_1r_2 & - & Rr_2 & = & Rr_1 \\ r_2(r_1 & - & R) & = & Rr_1 \\ \\ && r_2 & = & \dfrac{Rr_1}{r_1-R} \end{array}$$
4. $$\left(\frac{x}{15}-\frac{x-3}{3}=\frac{1}{3}\right)(15)$$
   $$\begin{array}{rrrrrrr} x & - & 5(x & - & 3) & = & 5 \\ x & - & 5x & + & 15 & = & 5 \\ &&& - & 15 && -15 \\ &&&& \dfrac{-4x}{-4} & = & \dfrac{-10}{-4} \\ \\ &&&& x & = & \dfrac{5}{2} \end{array}$$
5. $y=5$



6.
$$m = \frac{\Delta y}{\Delta x}$$

$$\frac{2}{3} = \frac{y-4}{x--2}$$

$$\begin{array}{rrrrrrrrr} & & 2(x & + & 2) & = & 3(y & - & 4) \\ & & 2x & + & 4 & = & 3y & - & 12 \\ & - & 3y & + & 12 & & -3y & + & 12 \\ 2x & - & 3y & + & 16 & = & 0 & & \end{array}$$

$$y = \frac{2}{3}x + \frac{16}{3}$$

7.

1st slope:

$$m = \frac{\Delta y}{\Delta x}$$

$$m = \frac{-9--7}{8-12}$$

$$m = \frac{-2}{-4}$$

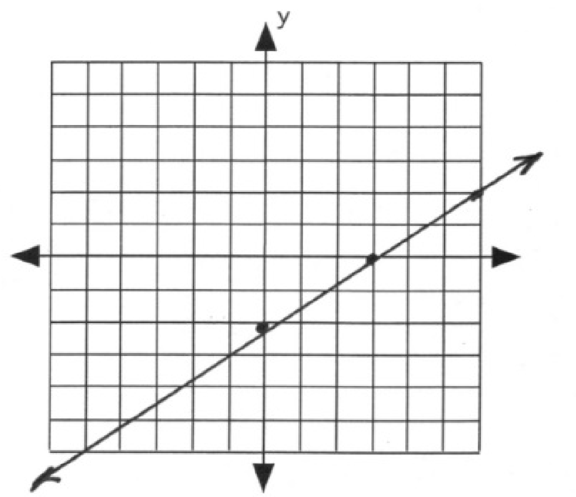
$$m = \frac{1}{2}$$

$$\begin{array}{rrrrrrrrr} & & x & - & 8 & = & 2(y & + & 9) \\ & & x & - & 8 & = & 2y & + & 18 \\ & - & 2y & - & 18 & & -2y & - & 18 \\ x & - & 2y & - & 26 & = & 0 & & \end{array}$$

Now:

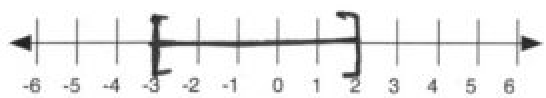
$$m = \frac{\Delta y}{\Delta x}$$

$$\frac{1}{2} = \frac{y--9}{x-8}$$



8.

9.
$$\begin{array}{rcrcrcr} -27 & \leq & 6x & - & 9 & \leq & 3 \\ +9 & & & + & 9 & & +9 \\ \dfrac{-18}{6} & \leq & & \dfrac{6x}{6} & & \leq & \dfrac{12}{6} \\ \\ -3 & \leq & & x & & \leq & 2 \end{array}$$



10.
$$\frac{2x+2}{6} = 2 \qquad\qquad \frac{2x+2}{6} = -2$$

$$\begin{array}{rcrcl} 2x & + & 2 & = & 2 \cdot 6 \\ 2x & + & 2 & = & 12 \\ & - & 2 & & -2 \\ & & \dfrac{2x}{2} & = & \dfrac{10}{2} \\ \\ & & x & = & 5 \end{array} \qquad \begin{array}{rcrcl} 2x & + & 2 & = & -2 \cdot 6 \\ 2x & + & 2 & = & -12 \\ & - & 2 & & -2 \\ & & \dfrac{2x}{2} & = & \dfrac{-14}{2} \\ \\ & & x & = & -7 \end{array}$$

11.
$$\begin{array}{rcrcr} 2x & - & 1 & < & -6 \\ & + & 1 & & +1 \\ & & \dfrac{2x}{2} & < & \dfrac{-5}{2} \\ \\ & & x & < & -\dfrac{5}{2} \end{array} \qquad \begin{array}{rcrcr} 6 & < & 2x & - & 1 \\ +1 & & & + & 1 \\ \dfrac{7}{2} & < & \dfrac{2x}{2} & & \\ \\ x & > & \dfrac{7}{2} & & \end{array}$$

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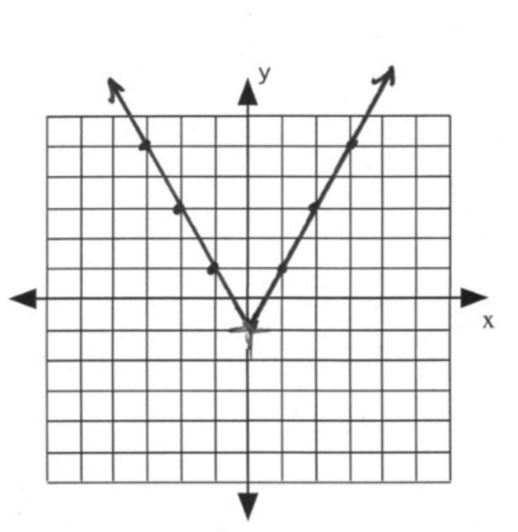
12. $y = |2x| - 1$

x	y
3	5
2	3
1	1
0	−1
−1	1
−2	3
−3	5



13.

$$\begin{aligned} A_1 &= A_2 \\ A_3 &= 2A_1 - 10^\circ \end{aligned}$$

$$\begin{aligned} A_1 + A_2 + A_3 &= 180^\circ \\ A_1 + A_1 + 2A_1 - 10^\circ &= 180^\circ \\ +10^\circ \quad & \quad +10^\circ \\ \frac{4A_1}{4} &= \frac{190^\circ}{4} \end{aligned}$$

$$\begin{aligned} A_1 &= 47.5^\circ \\ A_2 &= 47.5^\circ \\ A_3 &= 2A_1 - 10^\circ \\ A_3 &= 2(47.5^\circ) - 10^\circ \\ A_3 &= 95^\circ - 10^\circ \\ A_3 &= 85^\circ \end{aligned}$$

14.

$x, x + 2$

$$\begin{array}{ccccccccc} x & + & x & + & 2 & = & x & - & 20 \\ & & 2x & + & 2 & = & x & - & 20 \\ & & -x & - & 2 & & -x & - & 2 \\ & & & & x & = & -22 & & \end{array}$$

numbers are −22, −20

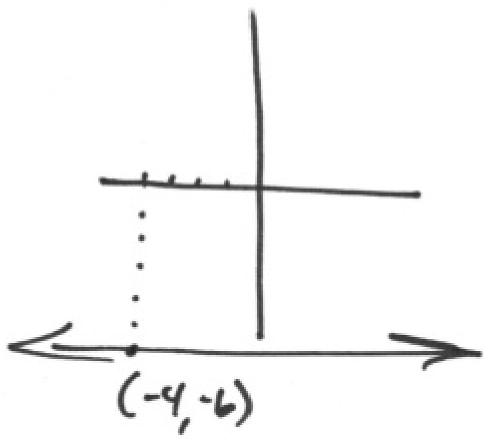
15. $$y = \frac{kmn^2}{d}$$

$$\begin{array}{rclcrcl} & & \textbf{1st data} & \quad & & & \textbf{2nd data} \\ y & = & 16 & & y & = & \text{find 2nd} \\ k & = & \text{find 1st} & & k & = & 2 \\ m & = & 3 & & m & = & -2 \\ n & = & 4 & & n & = & 4 \\ d & = & 6 & & d & = & 8 \end{array}$$

$$\begin{array}{rclcrcl} y & = & \dfrac{kmn^2}{d} & \quad & y & = & \dfrac{kmn^2}{d} \\[2ex] 16 & = & \dfrac{k(3)(4)^2}{6} & & y & = & \dfrac{(2)(-2)(4)^2}{8} \\[2ex] k & = & \dfrac{16 \cdot 6}{3 \cdot (4)^2} & & y & = & -8 \\[2ex] k & = & 2 & & & & \end{array}$$

Midterm 1: Version E Answer Key

1.
   a. −9
   b. 9
   c. −9
   d. 10
   e. −3
2. $$\begin{array}{rrrrrrrrrrr} 2x & - & 8 & + & 18 & = & -12 & + & 4x & + & 12 \\ -2x & & & & & & & - & 2x & & \\ & & & & \dfrac{10}{2} & = & \dfrac{2x}{2} & & & & \\ & & & & x & = & 5 & & & & \end{array}$$
3. $$\left(\frac{1}{R}-\frac{1}{r_1}=\frac{1}{r_2}\right)(Rr_1r_2)$$
$$\begin{array}{rrrrrrr} r_1r_2 & - & Rr_2 & = & Rr_1 & & \\ -Rr_1 & + & Rr_2 & & -Rr_1 & + & Rr_2 \\ r_1r_2 & - & Rr_1 & = & Rr_2 & & \\ r_1(r_2 & - & R) & = & Rr_2 & & \\ & & r_1 & = & \dfrac{Rr_2}{r_2-R} & & \end{array}$$
4. $$\left(\frac{x}{12}-\frac{x-4}{3}=\frac{2}{3}\right)(12)$$
$$\begin{array}{rrrrrrr} x & - & 4(x & - & 4) & = & 4(2) \\ x & - & 4x & + & 16 & = & 8 \\ & & & - & 16 & & -16 \\ & & & & \dfrac{-3x}{-3} & = & \dfrac{-8}{-3} \\ & & & & x & = & \dfrac{8}{3} \end{array}$$
5. $y=-6$



6.

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ \frac{2}{5} &= \frac{y-1}{x--1} \end{aligned}$$

$$\begin{array}{rrrrrrrrr} & & 2(x & + & 1) & = & 5(y & - & 1) \\ & & 2x & + & 2 & = & 5y & - & 5 \\ & - & 5y & + & 5 & & -5y & + & 5 \\ 2x & - & 5y & + & 7 & = & 0 & & \\ & & & & y & = & \frac{2}{5}x & + & \frac{7}{5} \end{array}$$

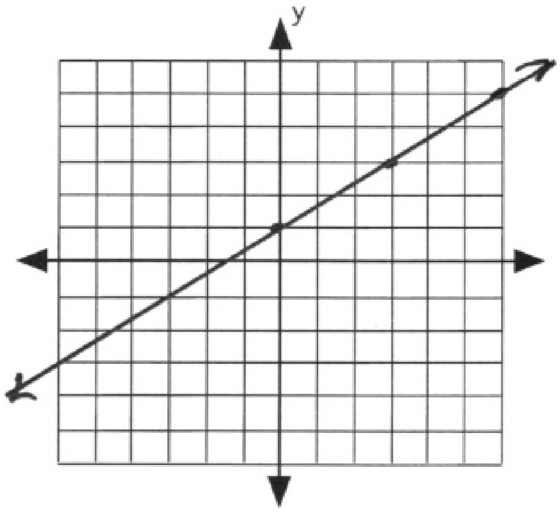
7.

1st slope:

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ m &= \frac{5--1}{2-0} \\ m &= \frac{6}{2} \\ m &= 3 \end{aligned}$$

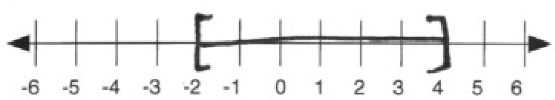
Now:

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ 3 &= \frac{y--1}{x-0} \\ 3x &= y+1 \\ 3x-y-1 &= 0 \\ \therefore 0 &= 3x-y-1 \\ y &= 3x-1 \end{aligned}$$

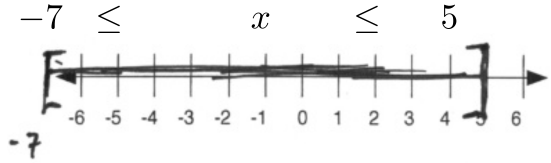


8.

9.
$$\begin{array}{rcccccc} -20 & \leq & 8x & - & 4 & \leq & 28 \\ +4 & & & + & 4 & & +4 \\ \dfrac{-16}{8} & \leq & & \dfrac{8x}{8} & & \leq & \dfrac{32}{8} \\ & & & & & & \\ -2 & \leq & & x & & \leq & 4 \end{array}$$

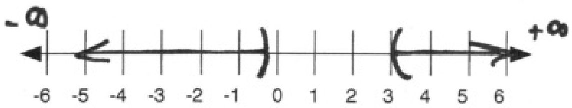


10.
$$\left(-2 \leq \frac{2x+2}{6} \leq 2\right)(6)$$
$$\begin{array}{rcccccc} -12 & \leq & 2x & + & 2 & \leq & 12 \\ -2 & & & - & 2 & & -2 \\ \dfrac{-14}{2} & \leq & & \dfrac{2x}{2} & & \leq & \dfrac{10}{2} \\ & & & & & & \\ -7 & \leq & & x & & \leq & 5 \end{array}$$



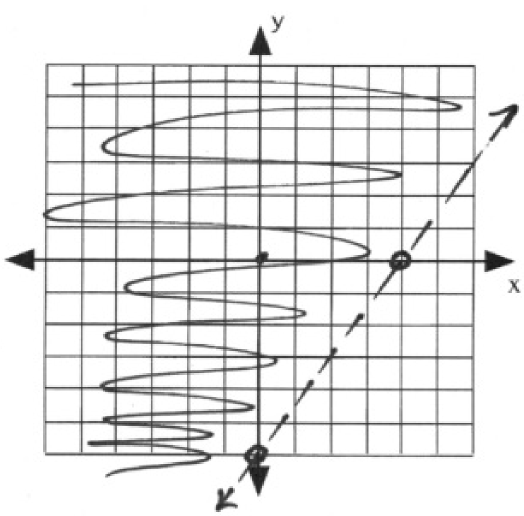
11. $\left(\dfrac{3x-4}{5}<-1\right)(5) \qquad \text{or} \qquad \left(1<\dfrac{3x-4}{5}\right)(5)$

$$\begin{array}{rrrrr} 3x & - & 4 & < & -5 \\ & + & 4 & & +4 \\ & & 3x & < & -1 \\ & & x & < & -\dfrac{1}{3} \end{array} \qquad \begin{array}{rrrrr} 5 & < & 3x & - & 4 \\ +4 & & & + & 4 \\ \dfrac{9}{3} & < & \dfrac{3x}{3} & & \\ x & > & 3 & & \end{array}$$



12. $3x-2y<12$

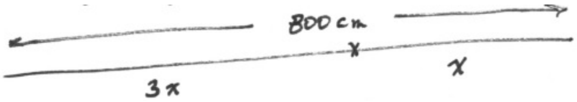
x	y
0	−6
4	0



13.
$x, x+2, x+4$

$$\begin{array}{rrrrrrrrrr} x & + & 2(x & + & 2) & + & 3(x & + & 4) & = & 94 \\ x & + & 2x & + & 4 & + & 3x & + & 12 & = & 94 \\ & & & & & & 6x & + & 16 & = & 94 \\ & & & & & & & - & 16 & & -16 \\ & & & & & & & & \dfrac{6x}{6} & = & \dfrac{78}{6} \\ & & & & & & & & x & = & 13 \end{array}$$

numbers are 13, 15, 17

14. 

$$\begin{aligned} 3x + x &= 800 \text{ cm} \\ 4x &= 800 \text{ cm} \\ x &= \frac{800 \text{ cm}}{4} \text{ or } 200 \text{ cm} \\ \\ 3x &= 600 \text{ cm} \end{aligned}$$

15. $$y = \frac{km}{n^2}$$

$$\begin{aligned} & \underline{\text{1st}} \\ y &= 12 \\ k &= \text{find} \\ m &= 3 \\ n &= 4 \end{aligned} \qquad \begin{aligned} & \underline{\text{2nd}} \\ y &= \text{find} \\ k &= 64 \\ m &= 3 \\ n &= -3 \end{aligned}$$

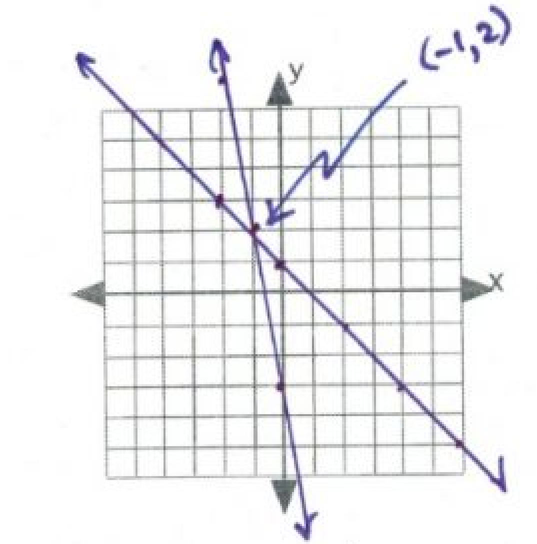
$$12 = \frac{k(3)}{(4)^2} \qquad y = \frac{(64)(3)}{(-3)^2}$$

$$k = \frac{12 \cdot (4)^2}{3} \qquad y = \frac{64}{3}$$

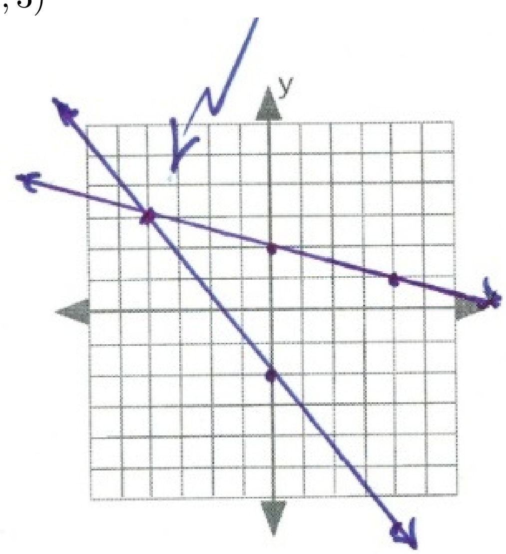
$$k = 64$$

Answer Key 5.1

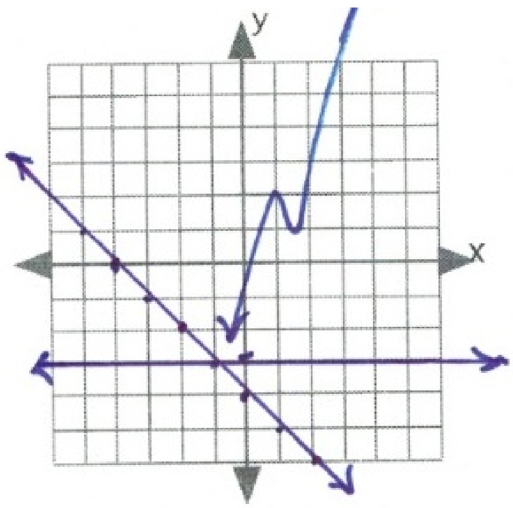
1. $(-1, 2)$



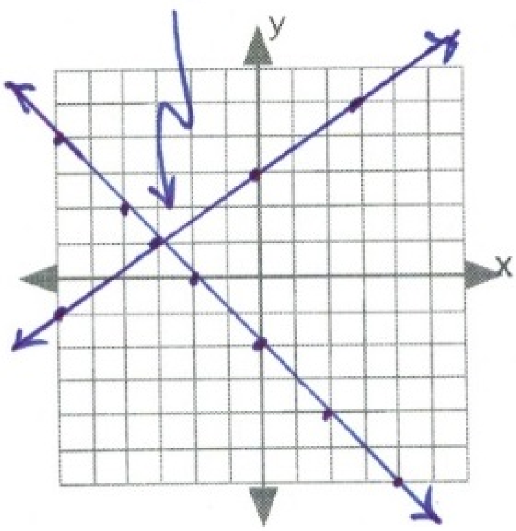
2. $(-4, 3)$



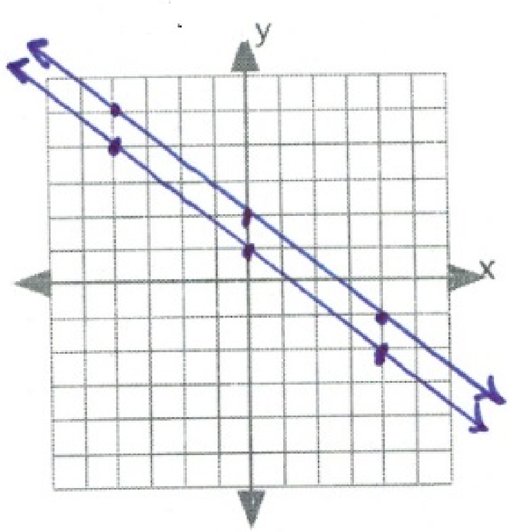
3. $(-1, -3)$



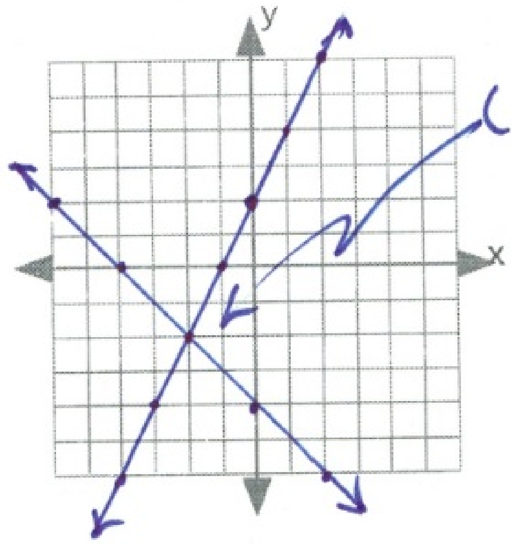
4. $(-3, 1)$



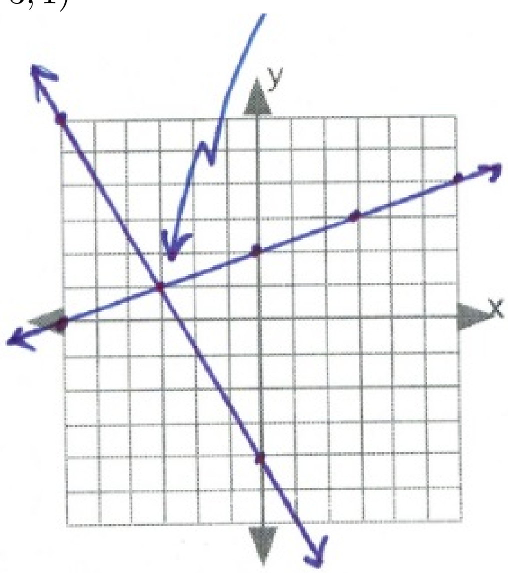
5. Parallel lines $\therefore$ no intersection



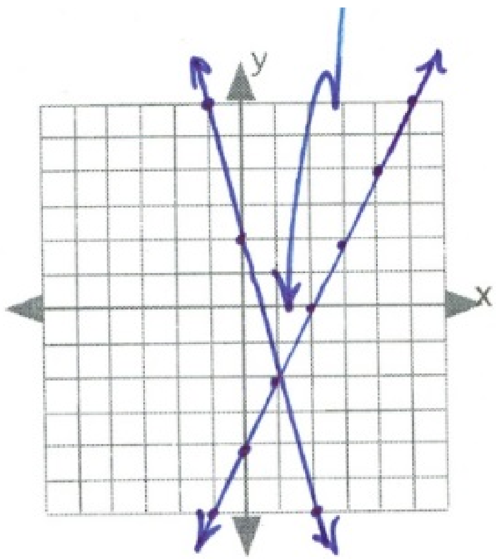
6. $(-2,-2)$



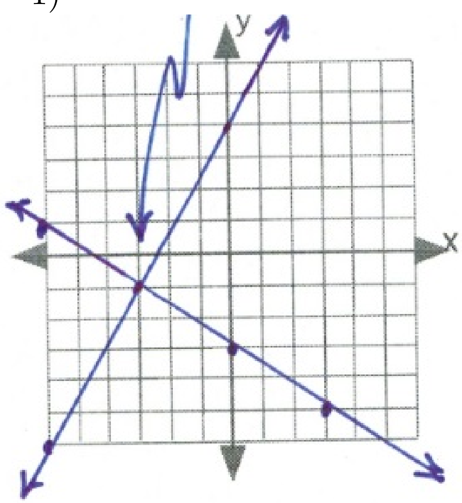
7. $(-3,1)$



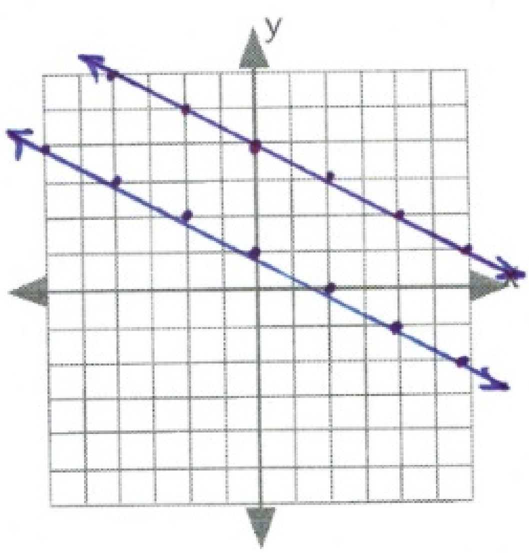
8. $(1,-2)$



9. $(-3, -1)$

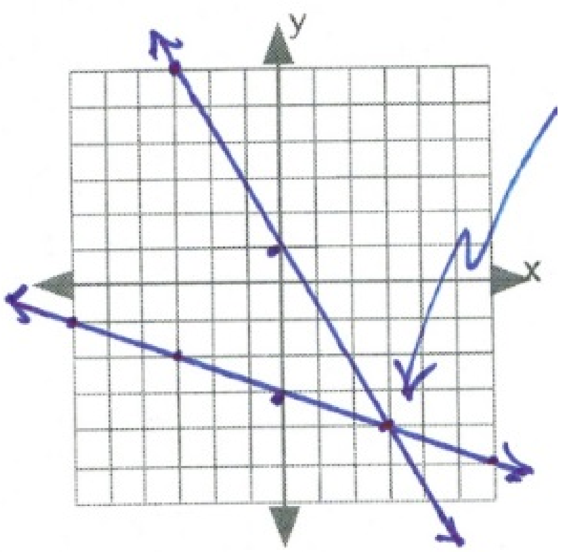


10. Parallel lines ∴ no intersection



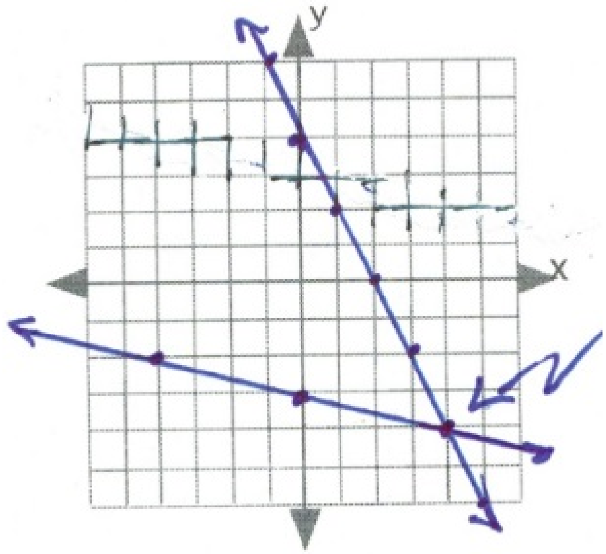
11.
$$\begin{array}{rcrcrcrcrcrcr}
x & + & 3y & = & -9 & & 5x & + & 3y & = & 3 \\
-x & & & & -x & & -5x & & & & -5x \\
\dfrac{3y}{3} & = & \dfrac{-x}{3} & - & \dfrac{9}{3} & & \dfrac{3y}{3} & = & \dfrac{-5x}{3} & + & \dfrac{3}{3} \\[2ex]
y & = & -\dfrac{1}{3}x & - & 3 & & y & = & -\dfrac{5}{3}x & + & 1
\end{array}$$

$(3, -4)$



12.
$$\begin{array}{rcrcrcrcrcrcr}
x & + & 4y & = & -12 & & 2x & + & y & = & 4 \\
-x & & & & -x & & -2x & & & & -2x \\
\dfrac{4y}{4} & = & \dfrac{-x}{4} & - & \dfrac{12}{4} & & y & = & -2x & + & 4 \\[2ex]
y & = & -\dfrac{1}{4}x & + & 3 & & y & = & -2x & + & 4
\end{array}$$

$(4, -4)$


y
x

Answer Key 5.2

1.
$$\begin{array}{rcrrr}
-3x & = & 6x & - & 9 \\
-6x & & -6x & & \\
\dfrac{-9x}{-9} & = & \dfrac{-9}{-9} & & \\
\\
x & = & 1 & & \\
\\
\therefore y & = & -3(1) & & \\
y & = & -3 & &
\end{array}$$

$(1, -3)$

2.
$$\begin{array}{rrrcrrr}
x & + & 5 & = & -2x & - & 4 \\
+2x & - & 5 & & +2x & - & 5 \\
& & \dfrac{3x}{3} & = & \dfrac{-9}{3} & & \\
\\
& & x & = & -3 & & \\
\\
& & \therefore y & = & -3 & + & 5 \\
& & y & = & 2 & &
\end{array}$$

$(-3, 2)$

3.
$$\begin{array}{rrrcrrr}
2x & - & 1 & = & -2x & - & 9 \\
+2x & + & 1 & & +2x & + & 1 \\
& & \dfrac{4x}{4} & = & \dfrac{-8}{4} & & \\
\\
& & x & = & -2 & & \\
\\
& & \therefore y & = & 2(-2) & - & 1 \\
& & y & = & -4 & - & 1 \\
& & y & = & -5 & &
\end{array}$$

$(-2, -5)$

4.
$$\begin{array}{rrrcrrr} 6x & + & 3 & = & -6x & + & 3 \\ +6x & - & 3 & & +6x & - & 3 \\ & & \dfrac{12x}{12} & = & \dfrac{0}{12} & & \\ & & x & = & 0 & & \\ & & \therefore y & = & 6(0) & + & 3 \\ & & y & = & 3 & & \end{array}$$

$(0, 3)$

5.
$$\begin{array}{rrrcrrr} 6x & + & 4 & = & -3x & - & 5 \\ +3x & - & 4 & & +3x & - & 4 \\ & & \dfrac{9x}{9} & = & \dfrac{-9}{9} & & \\ & & x & = & -1 & & \\ & & \therefore y & = & 6(-1) & + & 4 \\ & & y & = & -6 & + & 4 \\ & & y & = & -2 & & \end{array}$$

$(-1, -2)$

6.
$$\begin{array}{rrrcrrr} 3x & + & 13 & = & -2x & - & 22 \\ +2x & - & 13 & & +2x & - & 13 \\ & & \dfrac{5x}{5} & = & \dfrac{-35}{5} & & \\ & & x & = & -7 & & \\ & & \therefore y & = & 3(-7) & + & 13 \\ & & y & = & -21 & + & 13 \\ & & y & = & -8 & & \end{array}$$

$(-7, -8)$

7.
$$\begin{array}{rrrcrrr}
3x & + & 2 & = & -3x & + & 8 \\
+3x & - & 2 & & +3x & - & 2 \\
 & & \dfrac{6x}{6} & = & \dfrac{6}{6} & & \\
 & & & & & & \\
 & & x & = & 1 & & \\
 & & & & & & \\
 & & \therefore y & = & 3(1) & + & 2 \\
 & & y & = & 3 & + & 2 \\
 & & y & = & 5 & &
\end{array}$$

$(1, 5)$

8.
$$\begin{array}{rrrcrrr}
-2x & - & 9 & = & -5x & - & 21 \\
+5x & + & 9 & & +5x & + & 9 \\
 & & \dfrac{3x}{3} & = & \dfrac{-12}{3} & & \\
 & & & & & & \\
 & & x & = & -4 & & \\
 & & & & & & \\
 & & \therefore y & = & -2(-4) & - & 9 \\
 & & y & = & 8 & - & 9 \\
 & & y & = & -1 & &
\end{array}$$

$(-4, -1)$

9.
$$\begin{array}{rrrcrrr}
2x & - & 3 & = & -2x & + & 9 \\
+2x & + & 3 & & +2x & + & 3 \\
 & & \dfrac{4x}{4} & = & \dfrac{12}{4} & & \\
 & & & & & & \\
 & & x & = & 3 & & \\
 & & & & & & \\
 & & \therefore y & = & 2(3) & - & 3 \\
 & & y & = & 6 & - & 3 \\
 & & y & = & 3 & &
\end{array}$$

$(3, 3)$

10.
$$\begin{array}{rrrrrrr} 7x & - & 24 & = & -3x & + & 16 \\ +3x & + & 24 & & +3x & + & 24 \\ & & \dfrac{10x}{10} & = & \dfrac{40}{10} & & \\ & & x & = & 4 & & \\ & & \therefore y & = & 7(4) & - & 24 \\ & & y & = & 28 & - & 24 \\ & & y & = & 4 & & \end{array}$$

$(4, 4)$

11.
$$\begin{array}{rrrrrrr} 3x & - & 3(3x & - & 4) & = & -6 \\ 3x & - & 9x & + & 12 & = & -6 \\ & & & - & 12 & & -12 \\ & & & & \dfrac{-6x}{-6} & = & \dfrac{-18}{-6} \\ & & & & x & = & 3 \\ & & & & \therefore y & = & 3(3) - 4 \\ & & & & y & = & 9 - 4 \\ & & & & y & = & 5 \end{array}$$

$(3, 5)$

12.
$$\begin{array}{rrrrrrr} -x & + & 3(6x & + & 21) & = & 12 \\ -x & + & 18x & + & 63 & = & 12 \\ & & & - & 63 & & -63 \\ & & & & \dfrac{17x}{17} & = & \dfrac{-51}{17} \\ & & & & x & = & -3 \\ & & & & \therefore y & = & 6(-3) + 21 \\ & & & & y & = & -18 + 21 \\ & & & & y & = & 3 \end{array}$$

$(-3, 3)$

13.

$$\begin{array}{rrrcr} 3x & - & 6(-6) & = & 30 \\ 3x & + & 36 & = & 30 \\ & - & 36 & & -36 \\ & & \dfrac{3x}{3} & = & \dfrac{-6}{3} \\ \\ & & x & = & -2 \\ \\ & & y & = & -6 \end{array}$$

$(-2, -6)$

14.

$$\begin{array}{rrrrrcl} 6x & - & 4(-6x & + & 2) & = & -8 \\ 6x & + & 24x & - & 8 & = & -8 \\ & & & + & 8 & & +8 \\ & & & & \dfrac{30x}{30} & = & \dfrac{0}{30} \\ \\ & & & & \therefore x & = & 0 \\ \\ & & & & \therefore y & = & -6(0)+2 \\ & & & & y & = & 0+2 \\ & & & & y & = & 2 \end{array}$$

$(0, 2)$

15.

$$\begin{array}{rrrcr} 3x & + & 4(-5) & = & -17 \\ 3x & - & 20 & = & -17 \\ & + & 20 & & +20 \\ & & \dfrac{3x}{3} & = & \dfrac{3}{3} \\ \\ & & x & = & 1 \\ \\ & & y & = & -5 \end{array}$$

$(1, -5)$

16.

$$\begin{array}{rrrrrrl} 7x & + & 2(5x & + & 5) & = & -7 \\ 7x & + & 10x & + & 10 & = & -7 \\ & & & - & 10 & & -10 \\ & & & & \dfrac{17x}{17} & = & \dfrac{-17}{17} \\ \\ & & & & x & = & -1 \\ \\ & & & & \therefore y & = & 5(-1)+5 \\ & & & & y & = & -5+5 \\ & & & & y & = & 0 \end{array}$$

$(-1, 0)$

17.

$$\begin{array}{rrrrrrr} -6x & + & 6y & = & -12 & (\div & 6) \\ -x & + & y & = & -2 & & \\ +x & & & & +x & & \\ & & y & = & x & - & 2 \end{array}$$

$$\begin{array}{rrrrrrl} 8x & - & 3(x & - & 2) & = & 16 \\ 8x & - & 3x & + & 6 & = & 16 \\ & & & - & 6 & & -6 \\ & & & & \dfrac{5x}{5} & = & \dfrac{10}{5} \\ \\ & & & & x & = & 2 \\ \\ & & & & \therefore y & = & x-2 \\ & & & & y & = & 2-2=0 \end{array}$$

$(2, 0)$

18.
$$\begin{array}{rrrrrrr}
-8x & + & 2y & = & -6 & (\div & 2) \\
-4x & + & y & = & -3 & & \\
+4x & & & & +4x & & \\
 & & y & = & 4x & - & 3
\end{array}$$

$$\begin{array}{rrrrrrl}
-2x & + & 3(4x & - & 3) & = & 11 \\
-2x & + & 12x & - & 9 & = & 11 \\
 & & & + & 9 & & +9 \\
 & & & & \dfrac{10x}{10} & = & \dfrac{20}{10} \\
 & & & & x & = & 2 \\
 & & & & \therefore y & = & 4(2)-3 \\
 & & & & y & = & 8-3=5
\end{array}$$

$(2,5)$

19.
$$\begin{array}{rrrrrrr}
-7x & - & y & = & 20 & & \\
-20 & + & y & & -20 & + & y \\
-7x & - & 20 & = & y & &
\end{array}$$

$$\begin{array}{rrrrrrl}
2x & + & 3(-7x & - & 20) & = & 16 \\
2x & - & 21x & - & 60 & = & 16 \\
 & & & + & 60 & & +60 \\
 & & & & \dfrac{-19x}{-19} & = & \dfrac{76}{-19} \\
 & & & & x & = & -4 \\
 & & & & \therefore y & = & -7(-4)-20 \\
 & & & & y & = & 28-20 \\
 & & & & y & = & 8
\end{array}$$

$(-4,8)$

20.

$$\begin{array}{rrrrrrrr} & & (-x & - & 4y & = & -14) & (-1) \\ & & x & + & 4y & = & 14 & \\ & & & - & 4y & & -4y & \\ & & & & x & = & 14 & - & 4y \\ \\ & & (-6x & + & 8y & = & 12) & \div & 2 \\ & & -3x & + & 4y & = & 6 & \\ -3(14 & - & 4y) & + & 4y & = & 6 & \\ -42 & + & 12y & + & 4y & = & 6 & \\ +42 & & & & & & +42 & \\ & & & & \dfrac{16y}{16} & = & \dfrac{48}{16} & \\ \\ & & & & y & = & 3 & \\ \\ & & & & x & = & 14 & - & 4(3) \\ & & & & x & = & 14 & - & 12 \\ & & & & x & = & 2 & \end{array}$$

$(2, 3)$

Answer Key 5.3

1.
$$\begin{array}{rrcrcr} & 4x & + & 2y & = & 0 \\ + & -4x & - & 9y & = & -28 \\ & & & \dfrac{-7y}{-7} & = & \dfrac{-28}{-7} \\ \\ & & & y & = & 4 \\ \\ & 4x & + & 2(4) & = & 0 \\ & 4x & + & 8 & = & 0 \\ & & - & 8 & & -8 \\ & & & \dfrac{4x}{4} & = & \dfrac{-8}{4} \\ \\ & & & x & = & -2 \end{array}$$

$(-2, 4)$

2.
$$\begin{array}{rrcccr} & -7x & + & y & = & -10 \\ + & -9x & - & y & = & -22 \\ & & & \dfrac{-16x}{-16} & = & \dfrac{-32}{-16} \\ \\ & & & x & = & 2 \\ \\ & -7(2) & + & y & = & -10 \\ & -14 & + & y & = & -10 \\ & +14 & & & & +14 \\ & & & y & = & 4 \end{array}$$

$(2, 4)$

3.
$$\begin{array}{rrcrcr} & -9x & + & 5y & = & -22 \\ + & 9x & - & 5y & = & 13 \\ & & & 0 & = & -9 \end{array}$$
$\therefore$ Two parallel lines. No solution

4.
$$\begin{array}{rrcrcr} & -x & - & 2y & = & -7 \\ + & x & + & 2y & = & 7 \\ & & & 0 & = & 0 \end{array}$$
$\therefore$ Two identical lines. Infinite solutions

5.
$$\begin{array}{rrcrcr} & -6x & + & 9y & = & 3 \\ + & 6x & - & 9y & = & -9 \\ & & & 0 & = & -6 \end{array}$$
$\therefore$ Two parallel lines. No solution

6.
$$\begin{array}{rrrrrl} 5x & - & 5y & = & -15 & (\div 5) \therefore \text{Two identical lines. Infinite solutions} \\ (x & - & y & = & -3) & (-1) \end{array}$$

$$\begin{array}{rrrrrr} & x & - & y & = & -3 \\ + & -x & + & y & = & 3 \\ & & & 0 & = & 0 \end{array}$$

7.
$$\begin{array}{rrrrrr} & 4x & - & 6y & = & -10 \\ + & 4x & + & 6y & = & -14 \\ & & & \frac{8x}{8} & = & \frac{-24}{8} \\ & & & x & = & -3 \end{array}$$

$$\begin{array}{rrrrr} 4(-3) & - & 6y & = & -10 \\ -12 & - & 6y & = & -10 \\ +12 & & & & +12 \\ & & \frac{-6y}{-6} & = & \frac{2}{-6} \\ & & y & = & -\frac{1}{3} \end{array}$$

$(-3, -\frac{1}{3})$

8.
$$\begin{array}{rrrrrrr} -3x & + & 3y & = & -12 & \div & (-3) \\ -3x & + & 9y & = & -24 & \div & (3) \end{array}$$

$$\begin{array}{rrrrrr} & x & - & y & = & 4 \\ + & -x & + & 3y & = & -8 \\ & & & \frac{2y}{2} & = & \frac{-4}{2} \\ & & & y & = & -2 \end{array}$$

$$\begin{array}{rrrrr} \therefore x & - & y & = & 4 \\ x & - & -2 & = & 4 \\ x & + & 2 & = & 4 \\ & - & 2 & & -2 \\ & & x & = & 2 \end{array}$$

$(2, -2)$

9.

$$\begin{array}{rrrrr} & (-x & - & 5y & = & 28) & (-1) \\ \\ & x & + & 5y & = & -28 \\ + & -x & + & 4y & = & -17 \\ & & & \dfrac{9y}{9} & = & \dfrac{-45}{9} \\ \\ & & & y & = & -5 \\ \\ & x & + & 5(-5) & = & -28 \\ & x & - & 25 & = & -28 \\ & & + & 25 & & +25 \\ & & & x & = & -3 \end{array}$$

$(-3, -5)$

10.

$$\begin{array}{rrrrr} & (-10x & - & 5y & = & 0) & (-1) \\ \\ & 10x & + & 5y & = & 0 \\ + & -10x & - & 10y & = & -30 \\ & & & \dfrac{-5y}{-5} & = & \dfrac{-30}{-5} \\ \\ & & & y & = & 6 \\ \\ & 10x & + & 5(6) & = & 0 \\ & 10x & + & 30 & = & 0 \\ & & - & 30 & & -30 \\ & & & \dfrac{10x}{10} & = & \dfrac{-30}{10} \\ \\ & & & x & = & -3 \end{array}$$

$(-3, 6)$

11.

$$\begin{array}{rrrrrl} & (2x & - & y & = & 5) \quad (2) \\ \\ & 4x & - & 2y & = & 10 \\ + & 5x & + & 2y & = & -28 \\ & & & \dfrac{9x}{9} & = & \dfrac{-18}{9} \\ \\ & & & x & = & -2 \\ \\ & 2(x) & - & y & = & 5 \\ & 2(-2) & - & y & = & 5 \\ & -4 & - & y & = & 5 \\ & +4 & & & & +4 \\ & & & -y & = & 9 \\ \\ & & & y & = & -9 \end{array}$$

$(-2,-9)$

12.

$$\begin{array}{rrrrrl} & -5x & + & 6y & = & -17 \\ & (x & - & 2y & = & 5) \quad (3) \\ \\ & -5x & + & 6y & = & -17 \\ + & 3x & - & 6y & = & 15 \\ & & & \dfrac{-2x}{-2} & = & \dfrac{-2}{-2} \\ \\ & & & x & = & 1 \\ \\ & x & - & 2y & = & 5 \\ & 1 & - & 2y & = & 5 \\ & -1 & & & & -1 \\ & & & \dfrac{-2y}{-2} & = & \dfrac{4}{-2} \\ \\ & & & y & = & -2 \end{array}$$

$(1,-2)$

13.

$$\begin{array}{rrrrrl} & (10x & + & 6y & = & 24) \quad (\div 2) \\ & (-6x & + & y & = & 4) \quad (-3) \end{array}$$

$$\begin{array}{rrrrrr} & 5x & + & 3y & = & 12 \\ + & 18x & - & 3y & = & -12 \\ & & & 23x & = & 0 \\ & & & x & = & 0 \end{array}$$

$$\begin{array}{rrrrr} -6(x) & + & y & = & 4 \\ -6(0) & + & y & = & 4 \\ & & y & = & 4 \end{array}$$

$(0, 4)$

14.

$$\begin{array}{rrrrl} (10x & + & 6y & = & -10) \quad (\div -2) \end{array}$$

$$\begin{array}{rrrrrr} & x & + & 3y & = & -1 \\ + & -5x & - & 3y & = & 5 \\ & & & \dfrac{-4x}{-4} & = & \dfrac{4}{-4} \\ & & & x & = & -1 \end{array}$$

$$\begin{array}{rrrrr} -1 & + & 3y & = & -1 \\ +1 & & & & +1 \\ & & 3y & = & 0 \\ & & y & = & 0 \end{array}$$

$(-1, 0)$

15.

$$\begin{array}{rrrrrl} & (2x & + & 4y & = & 24) \quad (\div 2) \\ & (4x & - & 12y & = & 8) \quad (\div -4) \\ \\ & x & + & 2y & = & 12 \\ + & -x & + & 3y & = & -2 \\ & & & \dfrac{5y}{5} & = & \dfrac{10}{5} \\ \\ & & & y & = & 2 \\ \\ & x & + & 2(y) & = & 12 \\ & x & + & 2(2) & = & 12 \\ & x & + & 4 & = & 12 \\ & & - & 4 & & -4 \\ & & & x & = & 8 \end{array}$$

$(8, 2)$

16.

$$\begin{array}{rrrrrl} & (-6x & + & 4y & = & 12) \quad (\div 2) \\ & (12x & + & 6y & = & 18) \quad (\div -3) \\ \\ & -3x & + & 2y & = & 6 \\ + & -4x & - & 2y & = & -6 \\ & & & -7x & = & 0 \\ & & & x & = & 0 \\ \\ & \dfrac{-3(x)}{2} & + & \dfrac{2y}{2} & = & \dfrac{6}{2} \\ \\ & \dfrac{-3(0)}{2} & + & \dfrac{2y}{2} & = & \dfrac{6}{2} \\ \\ & & & y & = & 3 \end{array}$$

$(0, 3)$

17.

$$
\begin{array}{rrrrcrl}
 & (10x & - & 8y & = & -8) & (\div 2) \\
 \\
 & -7x & + & 4y & = & -4 & \\
+ & 5x & - & 4y & = & -4 & \\
 & & & \dfrac{-2x}{-2} & = & \dfrac{-8}{-2} & \\
 \\
 & & & x & = & 4 & \\
 \\
 & 5(4) & - & 4y & = & -4 & \\
 & 20 & - & 4y & = & -4 & \\
 & -20 & & & & -20 & \\
 & & & \dfrac{-4y}{-4} & = & \dfrac{-24}{-4} & \\
 \\
 & & & y & = & 6 &
\end{array}
$$

$(4, 6)$

18.

$$
\begin{array}{rrrrcrl}
 & (-6x & + & 4y & = & 4) & (\div 2) \\
 \\
 & -3x & + & 2y & = & 2 & \\
+ & 3x & - & y & = & 26 & \\
 & & & y & = & 28 & \\
 \\
 & 3x & - & 28 & = & 26 & \\
 & & + & 28 & & +28 & \\
 & & & \dfrac{3x}{3} & = & \dfrac{54}{3} & \\
 \\
 & & & x & = & 18 &
\end{array}
$$

$(18, 28)$

19.

$$\begin{array}{rrrrrl} & (-6x & - & 5y & = & -3) \quad (2) \\ \\ & 5x & + & 10y & = & 20 \\ + & -12x & - & 10y & = & -6 \\ & & & \dfrac{-7x}{-7} & = & \dfrac{14}{-7} \\ \\ & & & x & = & -2 \\ \\ & 5(-2) & + & 10y & = & 20 \\ & -10 & + & 10y & = & 20 \\ & +10 & & & & +10 \\ & & & \dfrac{10y}{10} & = & \dfrac{30}{10} \\ \\ & & & y & = & 3 \end{array}$$

$(-2, 3)$

20.

$$\begin{array}{rrrrrl} & (3x & - & 7y & = & -11) \quad (3) \\ \\ & -9x & - & 5y & = & -19 \\ + & 9x & - & 21y & = & -33 \\ & & & \dfrac{-26y}{-26} & = & \dfrac{-52}{-26} \\ \\ & & & y & = & 2 \\ \\ & 3x & - & 7(2) & = & -11 \\ & 3x & - & 14 & = & -11 \\ & & + & 14 & & +14 \\ & & & \dfrac{3x}{3} & = & \dfrac{3}{3} \\ \\ & & & x & = & 1 \end{array}$$

$(1, 2)$

21.

$$\begin{array}{rrrrrr}
(-7x & + & 5y & = & -8) & (3)\\
(-3x & - & 3y & = & 12) & (5)\\
\\
-21x & + & 15y & = & -24\\
+\quad -15x & - & 15y & = & 60\\
 & & \dfrac{-36x}{-36} & = & \dfrac{36}{-36}\\
\\
 & & x & = & -1\\
\\
-3(-1) & - & 3y & = & 12\\
3 & - & 3y & = & 12\\
-3 & & & & -3\\
 & & \dfrac{-3y}{-3} & = & \dfrac{9}{-3}\\
\\
 & & y & = & -3
\end{array}$$

$(-1,-3)$

22.

$$\begin{array}{rrrrrrrrr}
 & & (6x & + & 3y & = & -18) & \div & -3\\
\\
 & & -2x & - & y & = & 6\\
 & & +2x & & & & +2x\\
 & & & & -y & = & 2x & + & 6\\
 & & & & y & = & -2x & - & 6\\
\\
8x & + & 7(-2x & - & 6) & = & -24\\
8x & - & 14x & - & 42 & = & -24\\
 & & & + & 42 & & +42\\
 & & & & \dfrac{-6x}{-6} & = & \dfrac{18}{-6}\\
\\
 & & & & x & = & -3\\
\\
 & & & & y & = & -2(-3) & - & 6\\
 & & & & y & = & 6 & - & 6\\
 & & & & y & = & 0
\end{array}$$

$(-3,0)$

23.

$$
\begin{array}{rrrrrrrrr}
& & (-8x & - & 8y & = & -8) & \div & (-8) \\
\\
& & x & + & y & = & 1 & & \\
& & -x & & & & -x & & \\
& & & & y & = & 1 & - & x \\
\\
10x & + & 9(1 & - & x) & = & 1 & & \\
10x & + & 9 & - & 9x & = & 1 & & \\
& - & 9 & & & & -9 & & \\
& & & & x & = & -8 & & \\
\\
& & & & y & = & 1 & - & -8 \\
& & & & y & = & 9 & &
\end{array}
$$

$(-8, 9)$

24.

$$
\begin{array}{rrrrrrr}
& (-7x & + & 10y & = & 13) & (4) \\
& (4x & + & 9y & = & 22) & (7) \\
\\
& -28x & + & 40y & = & 52 & \\
+ & 28x & + & 63y & = & 154 & \\
& & & \dfrac{103y}{103} & = & \dfrac{206}{103} & \\
\\
& & & y & = & 2 & \\
\\
& 4x & + & 9(2) & = & 22 & \\
& 4x & + & 18 & = & 22 & \\
& & - & 18 & & -18 & \\
& & & \dfrac{4x}{4} & = & \dfrac{4}{4} & \\
\\
& & & x & = & 1 &
\end{array}
$$

$(1, 2)$

Answer Key 5.4

1.

$$\begin{array}{rrrrrrrrrr} & a & - & b & + & 2c & = & 2 \\ + & a & + & b & + & c & = & 3 \\ & & & 2a & + & 3c & = & 5 \end{array}$$

$$\begin{array}{rrrrrrrrrr} & a & - & b & + & 2c & = & 2 & \\ + & 2a & + & b & - & c & = & 2 & \\ & & & (3a & + & c & = & 4) & (-3) \\ & & & -9a & - & 3c & = & -12 & \end{array}$$

$$\begin{array}{rrrrrr} & -9a & - & 3c & = & -12 \\ + & 2a & + & 3c & = & 5 \\ & & & \dfrac{-7a}{-7} & = & \dfrac{-7}{-7} \\ & & & a & = & 1 \end{array}$$

$$\begin{array}{rrrrr} 3a & + & c & = & 4 \\ 3(1) & + & c & = & 4 \\ 3 & + & c & = & 4 \\ -3 & & & & -3 \\ & & c & = & 1 \end{array}$$

$$\begin{array}{rrrrrrr} a & + & b & + & c & = & 3 \\ (1) & + & b & + & (1) & = & 3 \\ & & b & + & 2 & = & 3 \\ & & & - & 2 & & -2 \\ & & & & b & = & 1 \end{array}$$

2.

$$\begin{array}{rrrrrrrr} & 2a & + & 3b & - & c & = & 12 \\ + & 3a & + & 4b & + & c & = & 19 \\ & & & 5a & + & 7b & = & 31 \end{array}$$

$$\begin{array}{rrrrrrrrr} & 2a & + & 3b & - & c & = & 12 & \\ + & a & - & 2b & + & c & = & -3 & \\ & & & (3a & + & b & = & 9) & (-7) \\ & & & -21a & - & 7b & = & -63 & \end{array}$$

$$\begin{array}{rrrrrr} & 5a & + & 7b & = & 31 \\ + & -21a & - & 7b & = & -63 \\ & & & \dfrac{-16a}{-16} & = & \dfrac{-32}{-16} \\ & & & a & = & 2 \end{array}$$

$$\begin{array}{rrrrr} 3a & + & b & = & 9 \\ 3(2) & + & b & = & 9 \\ 6 & + & b & = & 9 \\ -6 & & & & -6 \\ & & b & = & 3 \end{array}$$

$$\begin{array}{rrrrrrr} a & - & 2b & + & c & = & -3 \\ (2) & - & 2(3) & + & c & = & -3 \\ 2 & - & 6 & + & c & = & -3 \\ & & -4 & + & c & = & -3 \\ & & +4 & & & & +4 \\ & & & & c & = & 1 \end{array}$$

3.

$$\begin{array}{rrrrrrrrl}
 & (3x & + & y & - & z & = & 7) & (-1) \\
 & -3x & - & y & + & z & = & -7 & \\
+ & x & + & 3y & - & z & = & 5 & \\
 & & & (-2x & + & 2y & = & -2) & (\div 2) \\
 & & & (-x & + & y & = & -1) & (7) \\
 & & & -7x & + & 7y & = & -7 &
\end{array}$$

$$\begin{array}{rrrrrrrrl}
 & (3x & + & y & - & z & = & 7) & (2) \\
 & 6x & + & 2y & - & 2z & = & 14 & \\
+ & x & + & y & + & 2z & = & 3 & \\
 & & & 7x & + & 3y & = & 17 & \\
+ & & & -7x & + & 7y & = & -7 & \\
 & & & & & \dfrac{10y}{10} & = & \dfrac{10}{10} & \\
 & & & & & y & = & 1 &
\end{array}$$

$$\begin{array}{rrrrr}
-x & + & y & = & -1 \\
-x & + & (1) & = & -1 \\
-x & + & 1 & = & -1 \\
 & - & 1 & & -1 \\
 & & -x & = & -2 \\
 & & x & = & 2
\end{array}$$

$$\begin{array}{rrrrrrr}
x & + & y & + & 2z & = & 3 \\
(2) & + & (1) & + & 2z & = & 3 \\
 & & 2z & + & 3 & = & 3 \\
 & & & - & 3 & & -3 \\
 & & & & 2z & = & 0 \\
 & & & & z & = & 0
\end{array}$$

4.

$$\begin{array}{rrrrrrrrl}
 & x & + & y & + & z & = & 4 & (-1) \\
 & -x & - & y & - & z & = & -4 &
\end{array}$$

$$\begin{array}{rrrrrrrrl}
 & -x & - & y & - & z & = & -4 & \\
+ & x & + & 2y & + & 3z & = & 10 & \\
 & & & (y & + & 2z & = & 6) & (2) \\
 & & & 2y & + & 4z & = & 12 &
\end{array}$$

$$\begin{array}{rrrrrrrr}
 & -x & - & y & - & z & = & -4 \\
+ & x & - & y & + & 4z & = & 20 \\
 & & & -2y & + & 3z & = & 16 \\
+ & & & 2y & + & 4z & = & 12 \\
 & & & & & \dfrac{7z}{7} & = & \dfrac{28}{7} \\
 & & & & & z & = & 4
\end{array}$$

$$\begin{array}{rrrrr}
y & + & 2z & = & 6 \\
y & + & 2(4) & = & 6 \\
y & + & 8 & = & 6 \\
 & - & 8 & & -8 \\
 & & y & = & -2
\end{array}$$

$$\begin{array}{rrrrrrr}
x & + & y & + & z & = & 4 \\
x & + & (-2) & + & (4) & = & 4 \\
 & & x & + & 2 & = & 4 \\
 & & & - & 2 & & -2 \\
 & & & & x & = & 2
\end{array}$$

5.

$$\begin{array}{rrrrrrrr} & x & + & 2y & - & z & = & 0 \\ + & 3x & - & 2y & - & 4z & = & -5 \\ & & & 4x & - & 5z & = & -5 \end{array}$$

$$\begin{array}{rrrrrrrrr} & (2x & - & y & + & z & = & 15) & (2) \\ & 4x & - & 2y & + & 2z & = & 30 & \\ + & x & + & 2y & - & z & = & 0 & \\ & & & (5x & + & z & = & 30) & (5) \\ & & & 25x & + & 5z & = & 150 & \end{array}$$

$$\begin{array}{rrrrrr} & 4x & - & 5z & = & -5 \\ + & 25x & + & 5z & = & 150 \\ & & & \dfrac{29x}{29} & = & \dfrac{145}{29} \\ & & & x & = & 5 \end{array}$$

$$\begin{array}{rrrrr} 5x & + & z & = & 30 \\ 5(5) & + & z & = & 30 \\ 25 & + & z & = & 30 \\ -25 & & & & -25 \\ & & z & = & 5 \end{array}$$

$$\begin{array}{rrrrrrr} x & + & 2y & - & z & = & 0 \\ (5) & + & 2y & - & (5) & = & 0 \\ 5 & + & 2y & - & 5 & = & 0 \\ & & & & 2y & = & 0 \\ & & & & y & = & 0 \end{array}$$

6.

$$\begin{array}{rrrrrrrrr} & (x & - & y & + & 2z & = & -3) & (2) \\ & 2x & - & 2y & + & 4z & = & -6 & \\ + & x & + & 2y & + & 3z & = & 4 & \\ & & & (3x & + & 7z & = & -2) & (-1) \\ & & & -3x & - & 7z & = & 2 & \end{array}$$

$$\begin{array}{rrrrrrrr} & 2x & + & y & + & z & = & -3 \\ + & x & - & y & + & 2z & = & -3 \\ & & & 3x & + & 3z & = & -6 \\ + & & & -3x & - & 7z & = & 2 \\ & & & & & \dfrac{-4z}{-4} & = & \dfrac{-4}{-4} \\ & & & & & z & = & 1 \end{array}$$

$$\begin{array}{rrrrr} 3x & + & 3z & = & -6 \\ 3x & + & 3(1) & = & -6 \\ 3x & + & 3 & = & -6 \\ & - & 3 & & -3 \\ & & \dfrac{3x}{3} & = & \dfrac{-9}{3} \\ & & x & = & -3 \end{array}$$

$$\begin{array}{rrrrrrr} x & - & y & + & 2z & = & -3 \\ (-3) & - & y & + & 2(1) & = & -3 \\ -3 & - & y & + & 2 & = & -3 \\ & & -y & - & 1 & = & -3 \\ & & & + & 1 & & +1 \\ & & & & -y & = & -2 \\ & & & & y & = & 2 \end{array}$$

7.

$$\begin{array}{rrrrrrrrr}
 & x & + & y & + & z & = & 6 \\
+ & 2x & - & y & - & z & = & -3 \\
 & & & & & \dfrac{3x}{3} & = & \dfrac{3}{3} \\
 & & & & & x & = & 1
\end{array}$$

$$\begin{array}{rrrrrrrrr}
x & - & 2y & + & 3z & = & 6 & \\
(1) & - & 2y & + & 3z & = & 6 & \\
1 & - & 2y & + & 3z & = & 6 & \\
-1 & & & & & & -1 & \\
 & & -2y & + & 3z & = & 5 &
\end{array}$$

$$\begin{array}{rrrrrrrr}
x & + & y & + & z & = & 6 & \\
(1) & + & y & + & z & = & 6 & \\
1 & + & y & + & z & = & 6 & \\
-1 & & & & & & -1 & \\
 & & (y & + & z & = & 5) & (2) \\
 & & 2y & + & 2z & = & 10 &
\end{array}$$

$$\begin{array}{rrrrrrr}
 & -2y & + & 3z & = & 5 \\
+ & 2y & + & 2z & = & 10 \\
 & & & \dfrac{5z}{5} & = & \dfrac{15}{5} \\
 & & & z & = & 3
\end{array}$$

$$\begin{array}{rrrrrrr}
x & + & y & + & z & = & 6 \\
(1) & + & y & + & (3) & = & 6 \\
1 & + & y & + & 3 & = & 6 \\
 & & y & + & 4 & = & 6 \\
 & & & - & 4 & & -4 \\
 & & & & y & = & 2
\end{array}$$

8.

$$\begin{array}{rrrrrrrrr}
 & x & + & y & - & z & = & 0 \\
+ & 2x & + & y & + & z & = & 0 \\
 & & & 3x & + & 2y & = & 0
\end{array}$$

$$\begin{array}{rrrrrrrrr}
 & (x & + & y & - & z & = & 0) & (-4) \\
 & -4x & - & 4y & + & 4z & = & 0 & \\
+ & x & + & 2y & - & 4z & = & 0 & \\
 & & & -3x & - & 2y & = & 0 &
\end{array}$$

$$\begin{array}{rrrrrr}
 & -3x & - & 2y & = & 0 \\
+ & 3x & + & 2y & = & 0 \\
 & & & 0 & = & 0
\end{array}$$

$$\begin{array}{rrrr}
\therefore & x & = & 0 \\
 & y & = & 0 \\
 & z & = & 0
\end{array}$$

9.

$$\begin{array}{rrrrrrrl}
 & x & + & y & + & z & = & 2\\
+ & 2x & - & y & + & 3z & = & 9\\
 & & & 3x & + & 4z & = & 11
\end{array}$$

$$\begin{array}{rrrrrrrl}
 & 2x & - & y & + & 3z & = & 9\\
+ & & & y & - & z & = & -3\\
 & & & (2x & + & 2z & = & 6) \quad (-2)\\
 & & & -4x & - & 4z & = & -12
\end{array}$$

$$\begin{array}{rrrrrr}
 & 3x & + & 4z & = & 11\\
+ & -4x & - & 4z & = & -12\\
 & & & -x & = & -1\\
 & & & x & = & 1
\end{array}$$

$$\begin{array}{rrrrr}
2x & + & 2z & = & 6\\
2(1) & + & 2z & = & 6\\
2 & + & 2z & = & 6\\
-2 & & & & -2\\
 & & \dfrac{2z}{2} & = & \dfrac{4}{2}\\
 & & z & = & 2
\end{array}$$

$$\begin{array}{rrrrrrr}
x & + & y & + & z & = & 2\\
(1) & + & y & + & (2) & = & 2\\
 & & y & + & 3 & = & 2\\
 & & & - & 3 & & -3\\
 & & & & y & = & -1
\end{array}$$

10.

$$\begin{array}{rrrrrrrl}
 & (4x & & & + & z & = & 3) \quad (2)\\
 & 8x & & & + & 2z & = & 6\\
+ & 6x & - & y & - & 2z & = & -1\\
 & & & (14x & - & y & = & 5) \quad (3)\\
 & & & 42x & - & 3y & = & 15\\
+ & & & -2x & + & 3y & = & 5\\
 & & & & & \dfrac{40x}{40} & = & \dfrac{20}{40}\\
 & & & & & x & = & \dfrac{1}{2}
\end{array}$$

$$\begin{array}{rrrrr}
-2x & + & 3y & = & 5\\
-2\left(\dfrac{1}{2}\right) & + & 3y & = & 5\\
-1 & + & 3y & = & 5\\
+1 & & & & +1\\
 & & \dfrac{3y}{3} & = & \dfrac{6}{3}\\
 & & y & = & 2
\end{array}$$

$$\begin{array}{rrrrr}
4x & + & z & = & 3\\
4\left(\dfrac{1}{2}\right) & + & z & = & 3\\
2 & + & z & = & 3\\
-2 & & & & -2\\
 & & z & = & 1
\end{array}$$

11.

$$\begin{array}{rrrrrrrr} & & x & - & z & = & -2 \\ + & & y & + & z & = & 5 \\ & & x & + & y & = & 3 \end{array}$$

$$\begin{array}{rrrrrrrrr} & 2x & - & 3y & + & z & = & -1 & \\ + & x & & & - & z & = & -2 & \\ & & & (3x & - & 3y & = & -3) & (\div 3) \\ & & & x & - & y & = & -1 & \\ + & & & x & + & y & = & 3 & \\ & & & & & \frac{2x}{2} & = & \frac{2}{2} & \\ & & & & & x & = & 1 & \end{array}$$

$$\begin{array}{rrrrr} x & - & z & = & -2 \\ (1) & - & z & = & -2 \\ 1 & - & z & = & -2 \\ -1 & & & & -1 \\ & & -z & = & -3 \\ & & z & = & 3 \end{array}$$

$$\begin{array}{rrrrr} y & + & z & = & 5 \\ y & + & (3) & = & 5 \\ y & + & 3 & = & 5 \\ & - & 3 & & -3 \\ & & y & = & 2 \end{array}$$

12.

$$\begin{array}{rrrrrrrrr} & (3x & + & 4y & - & z & = & 11) & (2) \\ & 6x & + & 8y & - & 2z & = & 22 & \\ + & & & y & + & 2z & = & -4 & \\ & & & (6x & + & 9y & = & 18) & (\div 3) \\ & & & 2x & + & 3y & = & 6 & \\ + & & & -2x & + & y & = & -6 & \\ & & & & & 4y & = & 0 & \\ & & & & & y & = & 0 & \end{array}$$

$$\begin{array}{rrrrr} -2x & + & y & = & -6 \\ -2x & + & 0 & = & -6 \\ & & \frac{-2x}{-2} & = & \frac{-6}{-2} \\ & & x & = & 3 \end{array}$$

$$\begin{array}{rrrrr} y & + & 2z & = & -4 \\ 0 & + & 2z & = & -4 \\ & & \frac{2z}{2} & = & \frac{-4}{2} \\ & & z & = & -2 \end{array}$$

13.

$$\begin{array}{rrrrrrrrr} & & & (-2y & + & z & = & -6) & (3) \\ & & & -6y & + & 3z & = & -18 & \\ + & x & + & 6y & + & 3z & = & 30 & \\ & & & (x & + & 6z & = & 12) & (-2) \\ & & & -2x & - & 12z & = & -24 & \\ + & & & 2x & + & 2z & = & 4 & \\ & & & & & \frac{-10z}{-10} & = & \frac{-20}{-10} & \\ & & & & & z & = & 2 & \end{array}$$

$$\begin{array}{rrrrr} 2x & + & 2z & = & 4 \\ 2x & + & 2(2) & = & 4 \\ 2x & + & 4 & = & 4 \\ & - & 4 & & -4 \\ & & 2x & = & 0 \\ & & x & = & 0 \end{array}$$

$$\begin{array}{rrrrr} -2y & + & z & = & -6 \\ -2y & + & 2 & = & -6 \\ & - & 2 & & -2 \\ & & \frac{-2y}{-2} & = & \frac{-8}{-2} \\ & & y & = & 4 \end{array}$$

14.

$$\begin{array}{rrrrrrrl}
 & (x & - & y & + & 2z & = & 0)\quad(2) \\
 & 2x & - & 2y & + & 4z & = & 0 \\
+ & x & + & 2y & & & = & 1 \\
 & & & 3x & + & 4z & = & 1
\end{array}$$

$$\begin{array}{rrrrrl}
 & (2x & + & z & = & 4)\quad(-4) \\
 & -8x & - & 4z & = & -16 \\
+ & 3x & + & 4z & = & 1 \\
 & & & \dfrac{-5x}{-5} & = & \dfrac{-15}{-5} \\
 & & & x & = & 3
\end{array}$$

$$\begin{array}{rrrrr}
x & + & 2y & = & 1 \\
3 & + & 2y & = & 1 \\
-3 & & & & -3 \\
 & & \dfrac{2y}{2} & = & \dfrac{-2}{2} \\
 & & y & = & -1
\end{array}$$

$$\begin{array}{rrrrr}
2x & + & z & = & 4 \\
2(3) & + & z & = & 4 \\
6 & + & z & = & 4 \\
-6 & & & & -6 \\
 & & z & = & -2
\end{array}$$

15.

$$\begin{array}{rrrrrrrr}
 & x & + & y & + & z & = & 4 \\
+ & & - & y & - & z & = & -4 \\
 & & & & & x & = & 0
\end{array}$$

$$\begin{array}{rrrrr}
x & - & 2y & = & 0 \\
0 & - & 2y & = & 0 \\
 & & -2y & = & 0 \\
 & & y & = & 0
\end{array}$$

$$\begin{array}{rrrrr}
-y & - & z & = & -4 \\
0 & - & z & = & -4 \\
 & & -z & = & -4 \\
 & & z & = & 4
\end{array}$$

16.

$$\begin{array}{rrrrrrrl}
 & (x & + & y & - & z & = & 2)\quad(-2) \\
 & -2x & - & 2y & + & 2z & = & -4 \\
+ & 2x & & & + & z & = & 6 \\
 & & & -2y & + & 3z & = & 2 \\
+ & & & 2y & - & 4z & = & -4 \\
 & & & & & -z & = & -2 \\
 & & & & & z & = & 2
\end{array}$$

$$\begin{array}{rrrrr}
2x & + & z & = & 6 \\
2x & + & 2 & = & 6 \\
 & - & 2 & & -2 \\
 & & \dfrac{2x}{2} & = & \dfrac{4}{2} \\
 & & x & = & 2
\end{array}$$

$$\begin{array}{rrrrr}
2y & - & 4z & = & -4 \\
2y & - & 4(2) & = & -4 \\
2y & - & 8 & = & -4 \\
 & + & 8 & & +8 \\
 & & \dfrac{2y}{2} & = & \dfrac{4}{2} \\
 & & y & = & 2
\end{array}$$

Answer Key 5.5

1. $\begin{cases} d + q = 103 \\ 10d + 25q = 1525 \end{cases}$
2. $\begin{cases} n + f = 34 \\ 5n + 50f = 1340 \end{cases}$
3. $\begin{cases} A + C = 578 \\ 2A + 1.5C = 985 \end{cases}$
4. $\begin{cases} d + q = 21 \\ 10d + 25q = 390 \end{cases}$
5. $\begin{cases} 5n + 10d = 225 \\ d = 2n \end{cases}$
6. $\begin{cases} 25q + 50f = 375 \\ q = f + 3 \end{cases}$
7. $\begin{cases} I_{9.5} + I_{11} = 10000 \\ 0.095I_{9.5} + 0.11I_{11} = 1038.50 \end{cases}$
8. $7000(r) + 9000(r + 0.02) = 900$
9. $1600(r) + 2400(2r) = 256$
10. $3000(r) + 4500(r - 0.02) = 435$
11. $$\begin{array}{rrcrcl} & N & + & C & = & 203 \\ & 2N & + & 1.25C & = & 310 \\ \\ & (N & + & C & = & 203)(-2) \\ & -2N & - & 2.00C & = & -406 \\ + & 2N & + & 1.25C & = & 310 \end{array}$$

$$\frac{-0.75C}{-0.75} = \frac{-96}{-0.75}$$

$$C = 128$$

$$\begin{aligned} N &= 203 - C \\ N &= 203 - 128 \\ N &= 75 \end{aligned}$$

12.

$$\begin{array}{rcrcl} d & + & b & = & 131 \\ 2.50d & + & 2.75b & = & 342 \end{array}$$

$$\begin{array}{rrcrcl} & (d & + & b & = & 131)(-2.5) \\ & -2.50d & - & 2.50b & = & -327.5 \\ + & 2.50d & + & 2.75b & = & 342 \\ & & & \dfrac{0.25b}{0.25} & = & \dfrac{14.5}{0.25} \\ \\ & & & b & = & 58 \\ \\ & & & d & = & 131 - b \\ & & & d & = & 131 - 58 \\ & & & d & = & 73 \end{array}$$

13.

$$\begin{array}{rcrcl} d & + & q & = & 27 \\ 10d & + & 25q & = & 495 \end{array}$$

$$\begin{array}{rrcrcl} & (d & + & q & = & 27)(-10) \\ & -10d & - & 10q & = & -270 \\ + & 10d & + & 25q & = & 495 \\ & & & \dfrac{15q}{15} & = & \dfrac{225}{15} \\ \\ & & & q & = & 15 \\ \\ & & & d & = & 27 - q \\ & & & d & = & 27 - 15 \\ & & & d & = & 12 \end{array}$$

14.

$$\begin{array}{rcrcl} d & + & n & = & 18 \\ 10d & + & 5n & = & 115 \end{array}$$

$$\begin{array}{rrcrcl} & (d & + & n & = & 18)(-5) \\ & -5d & - & 5n & = & -90 \\ + & 10d & + & 5n & = & 115 \\ & & & \dfrac{5d}{5} & = & \dfrac{25}{5} \\ \\ & & & d & = & 5 \\ \\ & & & n & = & 18 - d \\ & & & n & = & 18 - 5 \\ & & & n & = & 13 \end{array}$$

15.

$$\begin{array}{rcrcl} S_{25} & + & S_{20} & = & 40 \\ 25S_{25} & + & 20S_{20} & = & 960 \end{array}$$

$$\begin{array}{rrcrcl} & (S_{25} & + & S_{20} & = & 40)(-20) \\ & -20S_{25} & - & 20S_{20} & = & -800 \\ + & 25S_{25} & + & 20S_{20} & = & 960 \\ & & & \frac{5S_{25}}{5} & = & \frac{160}{5} \\ & & & S_{25} & = & 32 \\ & & & S_{20} & = & 40 - S_{25} \\ & & & S_{20} & = & 40 - 32 \\ & & & S_{20} & = & 8 \end{array}$$

16.

$$\begin{array}{rcrcl} S_{15} & + & S_{25} & = & 15 \\ 15S_{15} & + & 25S_{25} & = & 315 \end{array}$$

$$\begin{array}{rrcrcl} & (S_{15} & + & S_{25} & = & 15)(-15) \\ & -15S_{15} & - & 15S_{25} & = & -225 \\ + & 15S_{15} & + & 25S_{25} & = & 315 \\ & & & \frac{10S_{25}}{10} & = & \frac{90}{10} \\ & & & S_{25} & = & 9 \\ & & & S_{15} & = & 15 - S_{25} \\ & & & S_{15} & = & 15 - 9 \\ & & & S_{15} & = & 6 \end{array}$$

17.

$$\begin{array}{rcrcl} I_{12} & + & I_{13} & = & 27000 \\ 0.12I_{12} & + & 0.13I_{13} & = & 3385 \end{array}$$

$$\begin{array}{rrcrcl} & (I_{12} & + & I_{13} & = & 27000)(-0.12) \\ & -0.12I_{12} & - & 0.12I_{13} & = & -3240 \\ + & 0.12I_{12} & + & 0.13I_{13} & = & 3385 \\ & & & \frac{0.01I_{13}}{0.01} & = & \frac{145}{0.01} \\ & & & I_{13} & = & 14500 \\ & & & I_{12} & = & 27000 - I_{13} \\ & & & I_{12} & = & 27000 - 14500 \\ & & & I_{12} & = & 12500 \end{array}$$

18.

$$\begin{array}{rcrcl} I_5 & + & I_{7.5} & = & 50000 \\ 0.05I_5 & + & 0.075I_{7.5} & = & 3250 \end{array}$$

$$\begin{array}{rrcrcl} & (I_5 & + & I_{7.5} & = & 50000)(-0.05) \\ & -0.05I_5 & - & 0.05I_{7.5} & = & -2500 \\ + & 0.05I_5 & + & 0.075I_{7.5} & = & 3250 \\ & & & \dfrac{0.025I_{7.5}}{0.025} & = & \dfrac{750}{0.025} \end{array}$$

$$I_{7.5} = 30000$$

$$\begin{array}{rcl} I_5 & = & 50000 - I_{7.5} \\ I_5 & = & 50000 - 30000 \\ I_5 & = & 20000 \end{array}$$

19.

$$\begin{array}{rcrcrcl} 10d & + & 25q & = & 605 & & \\ & & d & = & q & - & 6 \end{array}$$

$$\begin{array}{rcrcrcr} 10(q & - & 6) & + & 25q & = & 605 \\ 10q & - & 60 & + & 25q & = & 605 \\ & + & 60 & & & & +60 \\ & & & & \dfrac{35q}{35} & = & \dfrac{665}{35} \end{array}$$

$$q = 19$$

$$\begin{array}{rcrcl} d & = & q & - & 6 \\ d & = & 19 & - & 6 \\ d & = & 13 & & \end{array}$$

20.
$$\begin{array}{rcl} 5n + 10d &=& 275 \\ d &=& 2n - 10 \end{array}$$

$$\begin{array}{rcl} 5n + 10(2n - 10) &=& 275 \\ 5n + 20n - 100 &=& 275 \\ + 100 && +100 \\ \dfrac{25n}{25} &=& \dfrac{375}{25} \\ n &=& 15 \end{array}$$

$$\begin{array}{rcl} d &=& 2n - 10 \\ d &=& 2(15) - 10 \\ d &=& 30 - 10 \\ d &=& 20 \end{array}$$

21.
$$\begin{array}{rcl} 3000r + 24000(r - 0.04) &=& 2010 \\ 3000r + 24000r - 960 &=& 2010 \\ + 960 && +960 \\ \dfrac{27000r}{27000} &=& \dfrac{2970}{27000} \\ r &=& 0.11 = 11\% \\ 0.11 - 0.04 &=& 0.07 = 7\% \end{array}$$

22.
$$\begin{array}{rcl} 5000r + 11000\left(\dfrac{2}{3}r\right) &=& 1480 \\ 5000r + 7333.33r &=& 1480 \\ \dfrac{12333.33r}{12333.33} &=& \dfrac{1480}{12333.33} \\ r &=& 0.12 = 12\% \\ \dfrac{2}{3}r &=& 0.08 = 8\% \end{array}$$

23.

$$\begin{aligned}
5n + 10d + 25q &= 510\\
n &= 2d\\
q &= d - 3
\end{aligned}$$

$$\begin{aligned}
5(2d) + 10d + 25(d - 3) &= 510\\
10d + 10d + 25d - 75 &= 510\\
+75 &\quad +75\\
\frac{45d}{45} &= \frac{585}{45}\\
d &= 13\\
n &= 2(13)\\
n &= 26\\
q &= 13 - 3\\
q &= 10
\end{aligned}$$

24.

$$\begin{aligned}
5n + 10d + 25q &= 375\\
n + d + q &= 40\\
d &= 3q
\end{aligned}$$

$$\begin{aligned}
(n + d + q &= 40)(-5)\\
-5n - 5d - 5q &= -200\\
+ \quad 5n + 10d + 25q &= 375\\
\frac{5d + 20q}{5} &= \frac{175}{5}\\
d + 4q &= 35\\
3q + 4q &= 35\\
\frac{7q}{7} &= \frac{35}{7}\\
q &= 5
\end{aligned}$$

$$\begin{aligned}
d &= 3(5)\\
d &= 15\\
n + d + q &= 40\\
n + 15 + 5 &= 40\\
n + 20 &= 40\\
-20 &\quad -20\\
n &= 20
\end{aligned}$$

25.

$$\begin{array}{rrrrrrrrr}
 & & & & S_{22} & = & 3 & + & 4S_{40} \\
 & & 22S_{22} & + & 40S_{40} & = & 834 & & \\
 & & & & & & & & \\
22(3 & + & 4S_{40}) & + & 40S_{40} & = & 834 & & \\
66 & + & 88S_{40} & + & 40S_{40} & = & 834 & & \\
-66 & & & & & & -66 & & \\
 & & & & \frac{128S_{40}}{128} & = & \frac{768}{128} & & \\
 & & & & & & & & \\
 & & & & S_{40} & = & 6 & & \\
 & & & & & & & & \\
 & & & & S_{22} & = & 3 & + & 4(6) \\
 & & & & S_{22} & = & 3 & + & 24 \\
 & & & & S_{22} & = & 27 & &
\end{array}$$

Answer Key 5.6

1.

$$\begin{array}{rrrrrrrrrr}
 & t & + & x & + & y & + & z & = & 6\\
+ & -t & + & x & - & y & - & z & = & -2\\
 & & & & & & & \dfrac{2x}{2} & = & \dfrac{4}{2}\\
 & & & & & & & x & = & 2
\end{array}$$

$$\begin{array}{rrrrrrrrrr}
 & (-t & + & x & - & y & - & z & = & -2)(-1)\\
 & t & - & x & + & y & + & z & = & 2\\
+ & -t & + & 3x & + & y & - & z & = & 2\\
 & & & & & 2x & + & 2y & = & 4\\
 & & & & & 2(2) & + & 2y & = & 4\\
 & & & & & -4 & & & & -4\\
 & & & & & & & 2y & = & 0\\
 & & & & & & & y & = & 0
\end{array}$$

$$\begin{array}{rrrrrrrrrr}
 & t & + & 2x & + & 2y & + & 4z & = & 17\\
+ & -t & + & 3x & + & y & - & z & = & 2\\
 & & & 5x & + & 3y & + & 3z & = & 19\\
 & & & 5(2) & + & 3(0) & + & 3z & = & 19\\
 & & & & & 10 & + & 3z & = & 19\\
 & & & & & -10 & & & & -10\\
 & & & & & & & \dfrac{3z}{3} & = & \dfrac{9}{3}\\
 & & & & & & & z & = & 3
\end{array}$$

$$\begin{array}{rrrrrrrrr}
t & + & x & + & y & + & z & = & 6\\
t & + & 2 & + & 0 & + & 3 & = & 6\\
 & & & & t & + & 5 & = & 6\\
 & & & & & - & 5 & & -5\\
 & & & & & & t & = & 1
\end{array}$$

2.

$$\begin{array}{rrrrrrrrrr}
 & t & + & x & - & y & + & z & = & -1\\
+ & -t & + & 3x & + & y & - & z & = & 1\\
 & & & & & & & 4x & = & 0\\
 & & & & & & & x & = & 0
\end{array}$$

$$\begin{array}{rrrrrrrr}
t & - & y & + & z & = & -1\\
t & - & 2 & + & z & = & -1\\
 & + & 2 & & & & +2\\
 & & t & + & z & = & 1
\end{array}$$

$$\begin{array}{rrrrrrrrr}
\therefore & t & - & y & + & z & = & -1\\
 & -t & + & 2y & + & z & = & 3\\
 & -t & + & y & - & z & = & 1\\
 & -2t & + & y & - & 3z & = & 0
\end{array}$$

$$\begin{array}{rrrrrrrr}
 & -t & + & 2y & + & z & = & 3\\
+ & -t & + & y & - & z & = & 1\\
 & & & -2t & + & 3y & = & 4
\end{array}$$

$$\begin{array}{rrrrrrrr}
 & (-t & + & y & - & z & = & 1)(-3)\\
 & 3t & - & 3y & + & 3z & = & -3\\
+ & -2t & + & y & - & 3z & = & 0\\
 & & & (t & - & 2y & = & -3)(2)\\
 & & & 2t & - & 4y & = & -6\\
+ & & & -2t & + & 3y & = & 4\\
 & & & & & -y & = & -2\\
 & & & & & y & = & 2
\end{array}$$

$$\begin{array}{rrrrrrrr}
 & -t & + & 2y & + & z & = & 3\\
 & -t & + & 2(2) & + & z & = & 3\\
 & -t & + & 4 & + & z & = & 3\\
 & & - & 4 & & & & -4\\
 & & & -t & + & z & = & -1\\
+ & & & t & + & z & = & 1\\
 & & & & & 2z & = & 0\\
 & & & & & z & = & 0
\end{array}$$

$$\begin{array}{rrrrr}
t & + & z & = & 1\\
t & + & 0 & = & 1\\
 & & t & = & 1
\end{array}$$

Answer Key 5.7

1.

$$\begin{array}{rrrrr} & & 3H & = & 30 \\ & & H & = & 10 \\ \\ H & + & 4S & = & 18 \\ 10 & + & 4S & = & 18 \\ -10 & & & & -10 \\ & & \dfrac{4S}{4} & = & \dfrac{8}{4} \\ \\ & & S & = & 2 \end{array}$$

$$\begin{array}{rrrrr} 2S & - & 2B & = & 2 \\ 2(2) & - & 2B & = & 2 \\ 4 & - & 2B & = & 2 \\ -4 & & & & -4 \\ & & \dfrac{-2B}{-2} & = & \dfrac{-2}{-2} \\ \\ & & B & = & 1 \end{array}$$

$$\begin{array}{rrrrrrr} B & + & H & \times & S & = & ? \\ 1 & + & (10 & \times & 2) & = & ? \\ & & 1 & + & 20 & = & 21 \end{array}$$

2.

$$\begin{array}{rrrrr} & & 3B & = & 30 \\ & & B & = & 10 \\ \\ B & + & 2H & = & 20 \\ 10 & + & 2H & = & 20 \\ -10 & & & & -10 \\ & & \dfrac{2H}{2} & = & \dfrac{10}{2} \\ \\ & & H & = & 5 \end{array}$$

$$\begin{array}{rrrrr} H & + & 4M & = & 9 \\ 5 & + & 4M & = & 9 \\ -5 & & & & -5 \\ & & 4M & = & 4 \\ & & M & = & 1 \end{array}$$

$$\begin{array}{rrrrrrr} H & + & M & \times & B & = & ? \\ 10 & + & (5 & \times & 1) & = & ? \\ & & 10 & + & 5 & = & 15 \end{array}$$

3.

$$\begin{array}{rrcrcrcrcr}
\text{Row 2} & -a & - & 2b & - & 2c & + & 2d & = & -8 \\
\text{Row 3} & 2a & - & b & - & c & - & d & = & 5 \\
\text{Column 1} & a & - & b & + & c & + & 2d & = & -1 \\
\text{Column 3} & a & - & 2b & - & c & - & d & = & 3
\end{array}$$

$$\begin{array}{rrcrcrcrcl}
 & (-a & - & 2b & - & 2c & + & 2d & = & -8)(2) \\
 & -2a & - & 4b & - & 4c & + & 4d & = & -16 \\
+ & 2a & - & b & - & c & - & d & = & 5 \\
 & & & -5b & - & 5c & + & 3d & = & -11
\end{array}$$

$$\begin{array}{rrcrcrcrcl}
 & -a & - & 2b & - & 2c & + & 2d & = & -8 \\
+ & a & - & b & + & c & + & 2d & = & -1 \\
 & & & -3b & - & c & + & 4d & = & -9
\end{array}$$

$$\begin{array}{rrcrcrcrcl}
 & -a & - & 2b & - & 2c & + & 2d & = & -8 \\
+ & a & - & 2b & - & c & - & d & = & 3 \\
 & & & (-4b & - & 3c & + & d & = & -5)(-4) \\
 & & & 16b & + & 12c & - & 4d & = & 20 \\
 & & + & -3b & - & c & + & 4d & = & -9 \\
 & & & & & 13b & + & 11c & = & 11
\end{array}$$

$$\begin{array}{rrcrcrcrcl}
 & & & (-4b & - & 3c & + & d & = & -5)(-3) \\
 & & & 12b & + & 9c & - & 3d & = & 15 \\
 & & + & -5b & - & 5c & + & 3d & = & -11 \\
 & & & & & 7b & + & 4c & = & 4
\end{array}$$

$$\begin{array}{rrrrrrrrrl}
 & & & & (13b & + & 11c & = & 11)(-4) \\
 & & & & (\ 7b & + & 4c & = & 4)(11) \\
 & & & & & & & & & \\
 & & & & -52b & - & 44c & = & -44 \\
 & & & + & 77b & + & 44c & = & 44 \\
 & & & & & & 25b & = & 0 \\
 & & & & & & b & = & 0
\end{array}$$

$$\begin{array}{rrrrl}
7b & + & 4c & = & 4 \\
7(0) & + & 4c & = & 4 \\
 & & \dfrac{4c}{4} & = & \dfrac{4}{4} \\
 & & & & \\
 & & c & = & 1
\end{array}$$

$$\begin{array}{rrrrrrl}
-4b & - & 3c & + & d & = & -5 \\
-4(0) & - & 3(1) & + & d & = & -5 \\
 & & -3 & + & d & = & -5 \\
 & & +3 & & & & +3 \\
 & & & & d & = & -2
\end{array}$$

$$\begin{array}{rrrrrrrrr}
a & - & 2b & - & c & - & d & = & 3 \\
a & - & 2(0) & - & 1 & - & (-2) & = & 3 \\
 & & a & - & 1 & + & 2 & = & 3 \\
 & & & & a & + & 1 & = & 3 \\
 & & & & & - & 1 & & -1 \\
 & & & & & & a & = & 2
\end{array}$$

Answer Key 6.1

1. $4^{1+4+4} = 4^9$ or $262,144$
2. $4^{1+4+2} = 4^7$ or $16,384$
3. $2 \cdot 4 \cdot m^{4+2} n^{2+1} = 8m^6 n^3$
4. $x^{2+1} y^{4+2} = x^3 y^6$
5. $3^{3\cdot 4} = 3^{12}$ or $531,441$
6. $4^{3\cdot 4} = 4^{12}$ or $16,772,216$
7. $2^2 u^{3\cdot 2} v^{2\cdot 2} = 4u^6 v^4$
8. $x^3 y^3$
9. $4^{5-3} = 4^2$ or 16
10. $3^{7-3} = 3^4$ or 81
11. $3^{1-1} n^{1-1} m^2 = 3^0 n^0 m^2$
12. $4^{-1} x^{2-1} y^{4-1} = 4^{-1} xy^3$
13. $(2x^{2+3} y^{4+3})^2$
    $(2x^5 y^7)^2$
    $2^2 x^{5\cdot 2} y^{7\cdot 2}$
    $4x^{10} y^{14}$
14. $[2u^{2+4} v^2]^3$
    $[2u^6 v^2]^3$
    $2^3 u^{6\cdot 3} v^{2\cdot 3} \Rightarrow 8u^{18} v^6$
15. $[2^3 x^3 \div x^3]^2$
    $[2^3 x^{3-3}]^2$
    $2^{3\cdot 2} x^0$
    2^6 or 64
16. $2a^{2+7} b^2 \div b^2 a^{4\cdot 2}$
    $2a^9 b^2 \div b^2 a^8$
    $2a^{9-8} b^{2-2}$
    $2a$
17. $[2y^{17} \div 2^4 x^{2\cdot 4} y^{4\cdot 4}]^3$
    $[2y^{17} \div 2^4 x^8 y^{16}]^3$
    $[2^{1-4} y^{17-16} x^{-8}]^3$
    $[2^{-3} y x^{-8}]^3$
    $2^{-9} y^3 x^{-24}$
18. $[xy^2 \cdot y^8] \div 2y^4$
    $xy^{10} \div 2y^4$
    $2^{-1} xy^{10-4}$
    $2^{-1} xy^6$
19. $4x^3 y^8 \div 2xy^7$
    $2x^2 y$
20. $2y^3 x^2 \div x^4 y^4$
    $2x^{-2} y^{-1}$
21. $[q^3 r^2 \cdot 4p^4 q^4 r^6] \div 2p^3$
    $4p^4 q^7 r^8 \div 2p^3$
    $2pq^7 r^8$

22. $4x^6y^{12}z^{10} \div x^4y^8z^8$
$4x^2y^4z^2$

Answer Key 6.2

1. $2x^4y^{-2} \cdot 2^4x^4y^{12}$
   $2^5x^8y^{10} \Rightarrow 32x^8y^{10}$
2. $2a^{-2}b^{-3} \cdot 2^4b^{16}$
   $32a^{-2}b^{13} \Rightarrow \dfrac{32b^{13}}{a^2}$
3. $2^4x^8y^8 \cdot x^{-4}$
   $2^4x^4y^8 \Rightarrow 16x^4y^8$
4. 1
5. $2x^{-3--3}y^{2-3}3^{-2}$
   $2x^0y^{-1}3^{-2} \Rightarrow \dfrac{2}{9y}$
6. $3y^3 \div [6x^7y^{-2}]$
   $3x^{-7}y^{3--2} \div 6$
   $3x^{-7}y^5 \div 6 \Rightarrow \dfrac{y^5}{2x^7}$
7. $2y \div y^8$
   $2y^{1-8}$
   $2y^{-7} \Rightarrow \dfrac{2}{y^7}$
8. $a^{16} \div 2b \Rightarrow \dfrac{a^{16}}{2b}$
9. $2^4a^8b^{12} \div a^{-1}$
   $16a^{8+1}b^{12}$
   $16a^9b^{12}$
10. $2^{-2}y^8 \div x^2 \Rightarrow \dfrac{y8}{2^2x^2} \Rightarrow \dfrac{y^8}{4x^2}$
11. $2^4m^4n^{4--2}$
    $16m^4n^6$
12. $2x^{-3} \div x^{-4}y^3$
    $2x^{-3--4}y^{-3} \Rightarrow 2xy^{-3} \Rightarrow \dfrac{2x}{y^3}$
13. $[2u^{-2}v^3 \cdot 2^{-1}u^{-1}v^{-4}] \div 2u^{-4}$
    $[u^{-3}v^{-1}] \div 2u^{-4}$
    $2^{-1}u^{-3--4}v^{-1}$
    $2^{-1}uv^{-1} \Rightarrow \dfrac{u}{2v}$
14. $2y \div 2^{-1}y^{-4}$
    $2y \cdot 2y^4 \Rightarrow 4y^5$
15. $b^{-1} \div 2a^{-3}b^2$
    $b^{-1} \cdot 2^{-1}a^3b^{-2} \Rightarrow \dfrac{a^3}{2b^3}$
16. $2x^2yz \div [2x^4y^4z^{-2}z^4y^8]$
    $2x^2yz \div [2x^4y^{12}z^2]$

$2x^2yz \cdot z^{-1}x^{-4}y^{-12}z^{-2}$
$x^{-2}y^{-11}z^{-1} \Rightarrow \dfrac{1}{x^2y^{11}z}$

17. $2a^{-3}b^2c^2b^6 \div a^9b^{-6}c^9$
$2a^{-3}b^8c^2 \cdot a^{-9}b^6c^{-9}$
$2a^{-12}b^{14}c^{-7} \Rightarrow \dfrac{2b^{14}}{a^{12}c^7}$

18. $2m^2p^2q^5 \div [2^3m^3 \cdot 4^3p^6]$
$2m^2p^2q^5 \div [2^3m^3 \cdot 2^6p^6]$
$2m^2p^2q^5 \div 2^9m^3p^6$
$2m^2p^2q^5 \cdot 2^{-9}m^{-3}p^{-6}$
$2^{-8}m^{-1}p^{-4}q^5 \Rightarrow \dfrac{q^5}{2^8mp^4} \Rightarrow \dfrac{q^5}{256mp^4}$

19. $y^{-1}x^4z^{-2} \div x^2y^3z^2$
$y^{-1}x^4z^{-2} \cdot x^{-2}y^{-3}z^{-2}$
$x^2y^{-4}z^{-4} \Rightarrow \dfrac{x^2}{y^4z^4}$

20. $2mpn^{-3} \div 2n^2n^{-12}p^6$
$2mpn^{-3} \div 2n^{-10}p^6$
$2mpn^{-3} \cdot 2^{-1}n^{10}p^{-6}$
$mp^{-5}n^7 \Rightarrow \dfrac{mn^7}{p^5}$

Answer Key 6.3

1. 8.853×10^{2}
2. 7.44×10^{-4}
3. 8.1×10^{-2}
4. 1.09×10^{0}
5. 3.9×10^{-2}
6. 1.5×10^{4}
7. 870,000
8. 256
9. 0.0009
10. 50,000
11. 2
12. 0.00006
13. $14 \times 10^{-1 + -3}$
    14×10^{-4} or 1.4×10^{-3}
14. $17.6 \times 10^{-6 + -5}$
    17.6×10^{-11} or 1.76×10^{-10}
15. 1.66×10^{-6}
16. 5.02×10^{6}
17. 1.18×10^{-2}
18. 2.39×10^{1}
19. 1.695×10^{2}
20. 1.33×10^{13}

Answer Key 6.4

1. $-(-4)^3-(-4)^2+6(-4)-21$
   $64-16-24-21$
   3
2. $(-6)^2+3(-6)-11$
   $36-18-11$
   7
3. $-5(-1)^4-11(-1)^3-9(-1)^2-(-1)-5$
   $-5+11-9+1-5$
   -7
4. $(5)^4-5(5)^3-(5)+13$
   $625-625-5+13$
   8
5. $(-3)^2+9(-3)+23$
   $9-27+23$
   5
6. $-6(6)^3+41(6)^2-32(6)+11$
   $-1296+1476-192+11$
   -1
7. $(6)^4-6(6)^3+(6)^2-24$
   $1296-1296+36-24$
   12
8. $(-6)^4+8(-6)^3+14(-6)^2+13(-6)+5$
   $1296-1728+504-78+5$
   -1
9. $\begin{array}{rcr} -5p^4 & + & 5p \\ +8p^4 & - & 8p \\ 3p^4 & - & 3p \end{array}$
10. $\begin{array}{rcr} 5m^3 & + & 7m^2 \\ -6m^3 & + & 5m^2 \\ -m^3 & + & 12m^2 \end{array}$
11. $\begin{array}{rcr} 5p^3 & + & 1 \\ 8p^3 & - & 1 \\ 13p^3 & & \end{array}$
12. $\begin{array}{rcr} 6x^3 & + & 5x \\ -6x^3 & - & 8x \\ & & -3x \end{array}$
13. $\begin{array}{rcrcr} 5n^4 & + & 6n^3 & & \\ -5n^4 & - & 3n^3 & + & 8 \\ & & 3n^3 & + & 8 \end{array}$
14. $\begin{array}{rcrcr} & & 8x^2 & + & 1 \\ x^4 & + & x^2 & - & 6 \\ x^4 & + & 9x^2 & - & 5 \end{array}$

15.
$$\begin{array}{rcrcr}
2a^4 & & & + & 2a \\
5a^4 & - & 3a^2 & - & 4a \\
7a^4 & - & 3a^2 & - & 2a
\end{array}$$

16.
$$\begin{array}{rcrcr}
8v^3 & + & 6v & & \\
4v^3 & - & 3v & + & 3 \\
12v^3 & + & 3v & + & 3
\end{array}$$

17.
$$\begin{array}{rcrcr}
4p^2 & - & 2p & - & 3 \\
-3p^2 & + & 6p & - & 3 \\
p^2 & + & 4p & - & 6
\end{array}$$

18.
$$\begin{array}{rcrcr}
8m^4 & + & 4m & + & 7 \\
-5m^4 & - & 6m & - & 1 \\
3m^4 & - & 2m & + & 6
\end{array}$$

19.
$$\begin{array}{rcrcrcr}
4n^4 & & & + & 2n^2 & + & 3 \\
-4n^4 & + & n^3 & - & 7n^2 & & \\
 & & n^3 & - & 5n^2 & + & 3
\end{array}$$

20.
$$\begin{array}{rcrcr}
2x^4 & + & 7x^3 & + & 7x^2 \\
-8x^4 & + & 6x^3 & - & 7x^2 \\
-6x^4 & + & 13x^3 & &
\end{array}$$

21.
$$\begin{array}{rcrcrcr}
8r^4 & - & 5r^3 & + & 5r^2 & & \\
-7r^4 & + & 2r^3 & + & 2r^2 & + & 1 \\
r^4 & - & 3r^3 & + & 7r^2 & + & 1
\end{array}$$

22.
$$\begin{array}{rcrcrcr}
4x^3 & - & 7x^2 & + & x & & \\
6x^3 & + & x^2 & + & 2x & - & 8 \\
10x^3 & - & 6x^2 & + & 3x & - & 8
\end{array}$$

23.
$$\begin{array}{rcrcrcr}
7n^4 & & & + & 2n^2 & - & 2 \\
2n^4 & + & 2n^3 & + & 4n^2 & + & 2 \\
9n^4 & + & 2n^3 & + & 6n^2 & &
\end{array}$$

24.
$$\begin{array}{rcrcrcr}
4b^4 & + & 7b^3 & & & - & 4b \\
-2b^4 & - & 8b^3 & + & 4b^2 & + & 8b \\
2b^4 & - & b^3 & + & 4b^2 & + & 4b
\end{array}$$

25.
$$\begin{array}{rcrcrcrcr}
 & & 7b^3 & & & - & b & + & 8 \\
-3b^4 & & & - & 7b^2 & - & 7b & + & 8 \\
 & & 6b^3 & & & - & 3b & + & 3 \\
-3b^4 & + & 13b^3 & - & 7b^2 & - & 11b & + & 19
\end{array}$$

26.
$$\begin{array}{rcrcrcr}
-3n^4 & - & 8n^3 & & & + & 1 \\
7n^4 & + & 3n^3 & - & 6n^2 & + & 2 \\
8n^4 & + & 4n^3 & & & + & 7 \\
12n^4 & - & n^3 & - & 6n^2 & + & 10
\end{array}$$

27.
$$\begin{array}{rcrcrcr}
8x^4 & + & 2x^3 & + & 2x & & \\
-x^4 & - & 2x^3 & + & 2x & + & 2 \\
-5x^4 & - & x^3 & - & 8x & & \\
2x^4 & - & x^3 & - & 4x & + & 2
\end{array}$$

28. $$\begin{array}{rcrcrcr} -5x^4 & - & 4x^2 & + & 6x & & \\ 4x^4 & + & 7x^2 & - & 2x & + & 8 \\ 4x^4 & + & 6x^2 & & & - & 8 \\ 3x^4 & + & 9x^2 & + & 4x & & \end{array}$$

Answer Key 6.5

1. $6p - 42$
2. $32k^2 + 16k$
3. $12x + 6$
4. $18n^3 + 21n^2$
5. $$\begin{array}{rrrrrrr} & 8n & + & 8 & & \\ \times & 4n & + & 6 & & \\ & 32n^2 & + & 32n & & \\ & & & 48n & + & 48 \\ & 32n^2 & + & 80n & + & 48 \end{array}$$
6. $$\begin{array}{rrrrrr} & x & - & 4 & & \\ \times & 2x & + & 1 & & \\ & 2x^2 & - & 8x & & \\ & & & x & - & 4 \\ & 2x^2 & - & 7x & - & 4 \end{array}$$
7. $$\begin{array}{rrrrrr} & 7b & - & 5 & & \\ \times & 8b & + & 3 & & \\ & 56b^2 & - & 40b & & \\ & & + & 21b & - & 15 \\ & 56b^2 & - & 19b & - & 15 \end{array}$$
8. $$\begin{array}{rrrrrr} & 4r & + & 8 & & \\ \times & r & + & 8 & & \\ & 4r^2 & + & 8r & & \\ & & + & 32r & + & 64 \\ & 4r^2 & + & 40r & + & 64 \end{array}$$
9. $$\begin{array}{rrrrrr} & 5v & - & 2 & & \\ \times & 3v & - & 4 & & \\ & 15v^2 & - & 6v & & \\ & & - & 20v & + & 8 \\ & 15v^2 & - & 26v & + & 8 \end{array}$$
10. $$\begin{array}{rrrrrr} & a & - & 8 & & \\ \times & 6a & + & 4 & & \\ & 6a^2 & - & 48a & & \\ & & + & 4a & - & 32 \\ & 6a^2 & - & 44a & - & 32 \end{array}$$

11.

$$\begin{array}{rrrrrrr} & 6x & - & 4y & & \\ \times & 5x & + & y & & \\ & 30x^2 & - & 20xy & & \\ & & + & 6xy & - & 4y^2 \\ & 30x^2 & - & 14xy & - & 4y^2 \end{array}$$

12.

$$\begin{array}{rrrrrrr} & 8u & - & 7v & & \\ \times & 2u & + & 3v & & \\ & 16u^2 & - & 14uv & & \\ & & + & 24uv & - & 21v^2 \\ & 16u^2 & + & 10uv & - & 21v^2 \end{array}$$

13.

$$\begin{array}{rrrrrrr} & 8x & + & 3y & & \\ \times & 7x & + & 5y & & \\ & 56x^2 & + & 21xy & & \\ & & + & 40xy & + & 15y^2 \\ & 56x^2 & + & 61xy & + & 15y^2 \end{array}$$

14.

$$\begin{array}{rrrrrrr} & a & - & 3b & & \\ \times & 5a & + & 8b & & \\ & 5a^2 & - & 15ab & & \\ & & + & 8ab & - & 24b^2 \\ & 5a^2 & - & 7ab & - & 24b^2 \end{array}$$

15.

$$\begin{array}{rrrrrrrrr} & 6r^2 & - & r & + & 5 & & \\ \times & & & r & - & 7 & & \\ & 6r^3 & - & r^2 & + & 5r & & \\ & & - & 42r^2 & + & 7r & - & 35 \\ & 6r^3 & - & 43r^2 & + & 12r & - & 35 \end{array}$$

16.

$$\begin{array}{rrrrrrrrr} & 4x^2 & + & 3x & + & 5 & & \\ \times & & & 4x & + & 8 & & \\ & 16x^3 & + & 12x^2 & + & 20x & & \\ & & + & 32x^2 & + & 24x & + & 40 \\ & 16x^3 & + & 44x^2 & + & 44x & + & 40 \end{array}$$

17.

$$\begin{array}{rrrrrrrrr} & 2n^2 & - & 2n & + & 5 & & \\ \times & & & 6n & - & 4 & & \\ & 12n^3 & - & 12n^2 & + & 30n & & \\ & & - & 8n^2 & + & 8n & - & 20 \\ & 12n^3 & - & 20n^2 & + & 38n & - & 20 \end{array}$$

18.

$$\begin{array}{rrrrrrr} & 4b^2 & + & 4b & + & 4 & \\ \times & & & 2b & - & 3 & \\ & 8b^3 & + & 8b^2 & + & 8b & \\ & & - & 12b^2 & - & 12b & - \ 12 \\ & 8b^3 & - & 4b^2 & - & 4b & - \ 12 \end{array}$$

19.

$$\begin{array}{rrrrrrrrr} & 6x^2 & - & 7xy & + & 4y^2 & & \\ \times & & & 6x & + & 3y & & \\ & 36x^3 & - & 42x^2y & + & 24xy^2 & & \\ & & + & 18x^2y & - & 21xy^2 & + & 12y^3 \\ & 36x^3 & - & 24x^2y & + & 3xy^2 & + & 12y^3 \end{array}$$

20.

$$\begin{array}{rrrrrrrr} & 7m^2 & + & 6mn & + & 4n^2 & & \\ \times & & & 3m & - & 2n & & \\ & 21m^3 & + & 18m^2n & + & 12mn^2 & & \\ & & - & 14m^2n & - & 12mn^2 & - & 8n^3 \\ & 21m^3 & + & 4m^2n & - & 8n^3 & & \end{array}$$

21.

$$\begin{array}{rrrrrrrrrr} & 6n^2 & - & 5n & + & 6 & & & & \\ \times & 8n^2 & + & 4n & + & 6 & & & & \\ & 48n^4 & - & 40n^3 & + & 48n^2 & & & & \\ & & + & 24n^3 & - & 20n^2 & + & 24n & & \\ & & & & + & 36n^2 & - & 30n & + & 36 \\ & 48n^4 & - & 16n^3 & + & 64n^2 & - & 6n & + & 36 \end{array}$$

22.

$$\begin{array}{rrrrrrrrrr} & 7a^2 & - & 6a & + & 1 & & & & \\ \times & 2a^2 & + & 6a & + & 3 & & & & \\ & 14a^4 & - & 12a^3 & + & 2a^2 & & & & \\ & & + & 42a^3 & - & 36a^2 & + & 6a & & \\ & & & & + & 21a^2 & - & 18a & + & 3 \\ & 14a^4 & + & 30a^3 & - & 13a^2 & - & 12a & + & 3 \end{array}$$

23.

$$\begin{array}{rrrrrrrrrr} & 3k^2 & + & 3k & + & 6 & & & & \\ \times & 5k^2 & + & 3k & + & 3 & & & & \\ & 15k^4 & + & 15k^3 & + & 30k^2 & & & & \\ & & + & 9k^3 & + & 9k^2 & + & 18k & & \\ & & & & + & 9k^2 & + & 9k & + & 18 \\ & 15k^4 & + & 24k^3 & + & 48k^2 & + & 27k & + & 18 \end{array}$$

24.

$$\begin{array}{rrrrrrrrrr} & 6u^2 & + & 4uv & + & 3v^2 & & & & \\ \times & 7u^2 & + & 8uv & - & 6v^2 & & & & \\ & 42u^4 & + & 28u^3v & + & 21u^2v^2 & & & & \\ & & + & 48u^3v & + & 32u^2v^2 & + & 24uv^3 & & \\ & & & & - & 36u^2v^2 & - & 24uv^3 & - & 18v^4 \\ & 42u^4 & + & 76u^3v & + & 17u^2v^2 & - & 18v^4 & & \end{array}$$

25.

$$\begin{array}{rrrrrrrrrrrrrr}
 & 2n^3 & - & 8n^2 & + & 3n & + & 6 \\
\times & n^3 & - & 6n^2 & - & 2n & + & 3 \\
 & 2n^6 & - & 8n^5 & + & 3n^4 & + & 6n^3 \\
 & & - & 12n^5 & + & 48n^4 & - & 18n^3 & - & 36n^2 \\
 & & & & - & 4n^4 & + & 16n^3 & - & 6n^2 & - & 12n \\
 & & & & & & + & 6n^3 & - & 24n^2 & + & 9n & + & 18 \\
 & 2n^6 & - & 20n^5 & + & 47n^4 & + & 10n^3 & - & 66n^2 & - & 3n & + & 18
\end{array}$$

26.

$$\begin{array}{rrrrrrrrrrrrrr}
 & a^3 & + & 2a^2 & + & 3a & + & 3 \\
\times & a^3 & + & 2a^2 & - & 4a & + & 1 \\
 & a^6 & + & 2a^5 & + & 3a^4 & + & 3a^3 \\
 & & + & 2a^5 & + & 4a^4 & + & 6a^3 & + & 6a^2 \\
 & & & & - & 4a^4 & - & 8a^3 & - & 12a^2 & - & 12a \\
 & & & & & & + & a^3 & + & 2a^2 & + & 3a & + & 3 \\
 & a^6 & + & 4a^5 & + & 3a^4 & + & 2a^3 & - & 4a^2 & - & 9a & + & 3
\end{array}$$

27.

$$\begin{array}{rrrrrr}
 & 2x & + & 1 \\
\times & 3x & - & 4 \\
 & 6x^2 & + & 3x \\
 & & - & 8x & - & 4 \\
 & 3(6x^2 & - & 5x & - & 4) \\
 & 18x^2 & - & 15x & - & 12
\end{array}$$

28.

$$\begin{array}{rrrrrr}
 & 2x & - & 3 \\
\times & x & - & 4 \\
 & 2x^2 & - & 3x \\
 & & - & 8x & + & 12 \\
 & 5(2x^2 & - & 11x & + & 12) \\
 & 10x^2 & - & 55x & + & 60
\end{array}$$

29.

$$\begin{array}{rrrrrr}
 & 4x & - & 5 \\
\times & 2x & + & 1 \\
 & 8x^2 & - & 10x \\
 & & + & 4x & - & 5 \\
 & 3(8x^2 & - & 6x & - & 5) \\
 & 24x^2 & - & 18x & - & 15
\end{array}$$

30.

$$\begin{array}{rrrrrr}
 & 2x & - & 6 \\
\times & 4x & + & 1 \\
 & 8x^2 & - & 24x \\
 & & + & 2x & - & 6 \\
 & 2(8x^2 & - & 22x & - & 6) \\
 & 16x^2 & - & 44x & - & 12
\end{array}$$

Answer Key 6.6

1. $\begin{array}{ccc}(x)^2 & - & (8)^2 \\ x^2 & - & 64\end{array}$
2. $\begin{array}{ccc}(a)^2 & - & (4)^2 \\ a^2 & - & 16\end{array}$
3. $\begin{array}{ccc}(1)^2 & - & (3p)^2 \\ 1 & - & 9p^2\end{array}$
4. $\begin{array}{ccc}(x)^2 & - & (3)^2 \\ x^2 & - & 9\end{array}$
5. $\begin{array}{ccc}(1)^2 & - & (7n)^2 \\ 1 & - & 49n^2\end{array}$
6. $\begin{array}{ccc}(8m)^2 & - & (5)^2 \\ 64m^2 & - & 25\end{array}$
7. $\begin{array}{ccc}(4y)^2 & - & (x)^2 \\ 16y^2 & - & x^2\end{array}$
8. $\begin{array}{ccc}(7a)^2 & - & (7b)^2 \\ 49a^2 & - & 49b^2\end{array}$
9. $\begin{array}{ccc}(4m)^2 & - & (8n)^2 \\ 16m^2 & - & 64n^2\end{array}$
10. $\begin{array}{ccc}(3y)^2 & - & (3x)^2 \\ 9y^2 & - & 9x^2\end{array}$
11. $\begin{array}{ccc}(6x)^2 & - & (2y)^2 \\ 36x^2 & - & 4y^2\end{array}$
12. $\begin{array}{ccccc}(1)^2 & + & 2(1)(5n) & + & (5n)^2 \\ 1 & + & 10n & + & 25n^2\end{array}$
13. $a^2 + 10a + 25$
14. $x^2 - 16x + 64$
15. $1 - 12n + 36n^2$
16. $16x^2 - 40x + 25$
17. $25m^2 - 80m + 64$
18. $9a^2 + 18ab + 9b^2$
19. $25x^2 + 70xy + 49y^2$
20. $16m^2 - 8mn + n^2$
21. $25 + 20r + 4r^2$
22. $m^2 - 14m + 49$
23. $16v^2 - 49$
24. $b^2 - 16$

Answer Key 6.7

1. $\dfrac{20x^4}{4x^3} + \dfrac{x^3}{4x^3} + \dfrac{2x^2}{4x^3}$

   $5x + \dfrac{1}{4} + \dfrac{1}{2x}$

2. $\dfrac{5x^4}{9x} + \dfrac{45x^3}{9x} + \dfrac{4x^2}{9x}$

   $\dfrac{5}{9}x^3 + 5x^2 + \dfrac{4}{9}x$

3. $\dfrac{20n^4}{10n} + \dfrac{n^3}{10n} + \dfrac{40n^2}{10n}$

   $2n^3 + \dfrac{n^2}{10} + 4n$

4. $\dfrac{3k^3}{8k} + \dfrac{4k^2}{8k} + \dfrac{2k}{8k}$

   $\dfrac{3}{8}k^2 + \dfrac{k}{2} + \dfrac{1}{4}$

5. $\dfrac{12x^4}{6x} + \dfrac{24x^3}{6x} + \dfrac{3x^2}{6x}$

   $2x^3 + 4x^2 + \dfrac{x}{2}$

6. $\dfrac{5p^4}{4p} + \dfrac{16p^3}{4p} + \dfrac{16p^2}{4p}$

   $\dfrac{5}{4}p^3 + 4p^2 + 4p$

7. $\dfrac{10n^4}{10n^2} + \dfrac{50n^3}{10n^2} + \dfrac{2n^2}{10n^2}$

   $n^2 + 5n + \dfrac{1}{5}$

8. $\dfrac{3m^4}{9m^2} + \dfrac{18m^3}{9m^2} + \dfrac{27m^2}{9m^2}$

   $\dfrac{m^2}{3} + 2m + 3$

9. $45x^2 + 56x + 169x + 4$

10. $6x^2 + 16x + 166x - 2$ or $x + 3 + \dfrac{22}{6x - 2}$
11. $10x^2 - 32x + 610x - 2$
12. $x^2 + 7x + 12x + 4$
13. $4x^2 - 33x + 354x - 5$
14. $4x^2 - 23x - 354x + 5$
15. $x^3 + 15x^2 + 49x - 49x + 7$
16. $6x^3 - 12x^2 - 43x - 20x - 4$
17. $x^3 - 6x - 40x + 4$ or $x^2 - 4x + 10 - \dfrac{80}{x + 4}$
18. $x^3 - 16x^2 + 512x - 8$
19. $x^3 - x^2 - 8x - 16x - 4$
20. $2x^3 + 6x^2 + 4x + 122x + 6$
21. $12x^3 + 12x^2 - 15x - 92x + 3$
22. $6x + 18 - 21x^2 + 4x^3 4x + 3$

Answer Key 6.8

1.

Name	Amount	Value	Equation
S_{40}	8000	0.40	0.40 (8000)
W	x	0	0
S_{30}	$x + 8000$	0.30	$0.30(x + 8000)$

$$0.40(8000) = 0.30(x + 8000)$$

2.

Name	Amount	Value	Equation
A_{100}	x	1.00	$1.00x$
A_{30}	5	0.30	0.30 (5)
A_{50}	$x + 5$	0.50	$0.50(x + 5)$

$$1.00x + 0.30(5) = 0.50(x + 5)$$

3.

Name	Amount	Value	Equation
S_{10}	12	0.10	0.10 (12)
S_3	x	0.03	$0.03(x)$
S_5	$x + 12$	0.05	$0.05(x + 12)$

$$0.10(12) + 0.03(x) = 0.05(x + 12)$$

4.

Name	Amount	Value	Equation
A_{100}	x	1.00	$1.00x$
A_{14}	24	0.14	0.14 (24)
A_{20}	$x + 24$	0.20	$0.20(x + 24)$

$$1.00x + 0.14(24) = 0.20(x + 24)$$

5.

Name	Amount	Value	Equation
B	x	1.60	$1.60x$
Mag	18	2.50	2.50 (18)
Mix	$x + 18$	1.90	$1.90(x + 18)$

$$1.60x + 2.50(18) = 1.90(x + 18)$$

6.

Name	Amount	Value	Equation
A_{100}	x	1.00	$1.00x$
A_{20}	40	0.20	0.20 (40)
A_{36}	$x + 40$	0.36	$0.36(x + 40)$

$$x + 0.20(40) = 0.36(x + 40)$$

7.

Name	Amount	Value	Equation
O_{40}	100	0.40	100 (0.40)
O_{100}	x	1.00	$x(1.00)$
O_{50}	$100 + x$	0.50	$(100 + x)(0.50)$

$$100(0.40) + x = (100 + x)(0.50)$$

8.

Name	Amount	Value	Equation
P_{220}	20	220	220 (20)
P_{400}	x	400	$400(x)$
P_{300}	$x + 20$	300	$300(x + 20)$

$$220(20) + 400x = 300(x + 20)$$

9.

Name	Amount	Value	Equation
$T_{4.20}$	x	4.20	$4.20(x)$
$T_{2.25}$	12	2.25	2.25 (12)
$T_{3.40}$	$x + 12$	3.40	$3.40(x + 12)$

$$4.20x + 2.25(12) = 3.40(x + 12)$$

10.

Name	Amount	Value	Equation
S_{80}	x	80	$80(x)$
S_{25}	6	25	25 (6)
S_{36}	$x + 6$	36	$36(x + 6)$

$$\begin{array}{rcrcrcr}
80x & + & 25(6) & = & 36(x & + & 6) \\
80x & + & 150 & = & 36x & + & 216 \\
-36x & - & 150 & & -36x & - & 150 \\
 & & \dfrac{44x}{44} & = & \dfrac{66}{44} & & \\
 & & x & = & \dfrac{3}{2} & \text{or} & 1.5 \text{ L}
\end{array}$$

11.

Name	Amount	Value	Equation
C	x	7.50	$7.50(x)$
J	24	3.25	3.25 (24)
M	$x + 24$	4.50	$4.50(x + 24)$

$$\begin{array}{rcrcrcr}
7.50x & + & 3.25(24) & = & 4.50(x & + & 24) \\
7.50x & + & 78 & = & 4.50x & + & 108 \\
-4.50x & - & 78 & & -4.50x & - & 78 \\
& & \dfrac{3.00x}{3.00} & = & \dfrac{30}{3.00} & & \\
& & x & = & 10 \text{ kg} & &
\end{array}$$

12.

Name	Amount	Value	Equation
S	x	7.00	$7.00(x)$
A	20	3.50	3.50 (20)
F	$x+20$	4.50	$4.50(x+20)$

$$\begin{array}{rcrcrcr}
7.00(x) & + & 3.50(20) & = & 4.50(x & + & 20) \\
7.00x & + & 70 & = & 4.50x & + & 90 \\
-4.50x & - & 70 & & -4.50x & - & 70 \\
& & \dfrac{2.5x}{2.5} & = & \dfrac{20}{2.5} & & \\
& & x & = & 8 & &
\end{array}$$

13.

Name	Amount	Value	Equation
$C_{1.8}$	$C_{1.8}$	1.80	1.80 ($C_{1.8}$)
$C_{3.0}$	15 – $C_{1.8}$	3.00	3.00 (15 – $C_{1.8}$)
$C_{2.2}$	15	2.20	2.20 (15)

$$\begin{array}{rcrcrcl}
1.80C_{1.8} & + & 3.00(15 & - & C_{1.8}) & = & 2.20(15) \\
1.80C_{1.8} & + & 45 & - & 3C_{1.8} & = & 33 \\
& - & 45 & & & & -45 \\
& & & & \dfrac{-1.2C_{1.8}}{-1.2} & = & \dfrac{-12}{-1.2} \\
& & & & C_{1.8} & = & 10 \\
& & & & C_{3.0} & = & 15-10 \\
& & & & C_{3.0} & = & 5
\end{array}$$

14.

Name	Amount	Value	Equation
M_{10}	100 – C	0.10	0.10 (100 – C)
C	C	0.60	0.60 (C)
M_{45}	100	0.45	0.45 (100)

$$\begin{aligned}
0.10(100 - C) + 0.60(C) &= 0.45(100)\\
10 - 0.10C + 0.60C &= 45\\
-10 \qquad\qquad &\quad -10\\
\frac{0.50C}{0.50} &= \frac{35}{0.50}\\
C &= 70\\
M_{10} &= 100 - 70\\
M_{10} &= 30
\end{aligned}$$

15.

Name	**Amount**	**Value**	**Equation**
A	A	0.50	0.50A
B	100 cc − A	0.80	0.80 (100 − A)
Mix	100 cc	0.68	0.68 (100)

$$\begin{aligned}
0.50A + 0.80(100 - A) &= 0.68(100)\\
0.50A + 80 - 0.80A &= 68\\
- 80 \qquad &= -80\\
\frac{-0.30A}{-0.30} &= \frac{-12}{-0.30}\\
A &= 40 \text{ cc}\\
B &= 100 - 40\\
B &= 60 \text{ cc}
\end{aligned}$$

16.

Name	**Amount**	**Value**	**Equation**
G_{21}	G_{21}	0.21	0.21 (G_{21})
G_{15}	600 − G_{21}	0.15	0.15 (600 − G_{21})
G_{19}	600	0.19	0.19 (600)

$$\begin{aligned}
0.21G_{21} + 0.15(600 - G_{21}) &= 0.19(600)\\
0.21G_{21} + 90 - 0.15G_{21} &= 114\\
- 90 \qquad &= -90\\
\frac{0.06G_{21}}{0.06} &= \frac{24}{0.06}\\
G_{21} &= 400\\
G_{15} &= 600 - 400\\
G_{15} &= 200
\end{aligned}$$

17.

Name	**Amount**	**Value**	**Equation**
C_{40}	C_{40}	0.40	0.40C_{40}
C_{30}	80 − C_{40}	0.30	0.30 (80 − C_{40})
C_{32}	80	0.32	0.32 (80)

$$
\begin{aligned}
0.40C_{40} + 0.30(80 - C_{40}) &= 0.32(80) \\
0.40C_{40} + 24 - 0.30C_{40} &= 25.6 \\
-24 &= -24 \\
\frac{0.10C_{40}}{0.10} &= \frac{1.6}{1.0} \\
C_{40} &= 16 \\
C_{30} &= 80 - 16 \\
C_{30} &= 64
\end{aligned}
$$

18.

Name	Amount	Value	Equation
A	3.75	0.40	0.40 (3.75)
F	x	0	0
P	$x + 3.75$	0.06	$0.06(x + 3.75)$

$$
\begin{aligned}
0.40(3.75) &= 0.06(x + 3.75) \\
1.5 &= 0.06x + 0.225 \\
-0.225 & \qquad\qquad -0.225 \\
\frac{1.275}{0.06} &= \frac{0.06x}{0.06} \\
x &= 21.25
\end{aligned}
$$

19.

Name	Amount	Value	Equation
A_{70}	$20 - x$	0.70	$0.70(20 - x)$
W	x	0	0
A_{18}	20	0.18	0.18 (20)

$$
\begin{aligned}
0.70(20 - x) &= 0.18(20) \\
14 - 0.7x &= 3.6 \\
-14 \qquad & \quad -14.0 \\
\frac{-0.7x}{-0.7} &= \frac{-10.4}{-0.7} \\
x &= 14.86 \text{ L of water} \\
0.70(20 - 14.86) &= 5.14 \text{ L of } 70\%
\end{aligned}
$$

20.

Name	Amount	Value	Equation
W	x	0	0
A_{100}	50	1.00	1.00 (50)
A_{40}	$x + 50$	0.40	$0.40(x + 50)$

$$\begin{aligned}
1.00(50) &= 0.40(x + 50)\\
50 &= 0.40x + 20\\
-20 &= \quad\quad\; - 20\\
\frac{30}{0.40} &= \frac{0.40x}{0.40}\\
x &= 75 \text{ mL}
\end{aligned}$$

21.

Name	Amount	Value	Equation
W	x	0	0
S_{12}	50	0.12	0.12 (50)
S_{15}	$50 - x$	0.15	$0.15(50 - x)$

$$\begin{aligned}
0.12(50) &= 0.15(50 - x)\\
6.0 &= 7.5 - 0.15x\\
-7.5 & \quad -7.5\\
\frac{-1.5}{-0.15} &= \frac{-0.15x}{-0.15}\\
x &= 10 \text{ L}
\end{aligned}$$

Answer Key 6.9

Problem: Use Pascal's triangle to expand the binomial $(a+b)^{12}$.

Row 10	$1+10+44+120+210+252+210+120+44+10+1$
Row 11	$1+11+54+164+330+462+462+330+164+54+11+1$
Row 12	$1+12+65+218+494+792+924+792+494+218+65+12+1$

Equation:

$$a^{12}+12a^{11}b+65a^{10}b^2+218a^9b^3+494a^8b^4+792a^7b^5+924a^6b^6+792a^5b^7+494a^4b^8+218a^3b^9+65a^2b^{10}+12ab^{11}+b^{12}$$

Answer Key 7.1

1. $9 + 8b^2$
2. $x - 5$
3. $5(9x^2 - 5)$
4. $1 + 2n^2$
5. $7(8 - 5p)$
6. $10(5x - 8y)$
7. $7ab(1 - 5a)$
8. $9x^2y^2(3y^3 - 8x)$
9. $3a^2b(-1 + 2ab)$
10. $4x^3(2y^2 + 1)$
11. $5x^2(-1 - x - 3x^2)$ or $-5x^2(1 + x + 3x^2)$
12. $8n^5(-4n^4 + 4n + 5)$
13. $4(7m^4 + 10m^3 + 2)$
14. $2x(-5x^3 + 10x + 6)$
15. $5(6b^9 + ab - 3a^2)$
16. $3y^2(9y^5 + 4x + 3)$
17. $-8a^2b(6b + 7a + 7a^3)$
18. $5(6m^6 + 3mn^2 - 5)$
19. $5x^3y^2z(4x^5z + 3x^2 + 7y)$
20. $3(p + 4q - 5q^2r^2)$
21. $3(-6n^5 + n^3 - 7n + 1)$
22. $3a^2(10a^6 + 2a^3 + 9a + 7)$
23. $-10x^{11}(4 + 2x - 5x^2 + 5x^3)$
24. $4x^2(-6x^4 - x^2 + 3x + 1)$
25. $4mn(-8n^7 + m^5 + 3n^3 + 4)$
26. $2y^7(-5 + 3y^3 - 2y^3x - 4yx)$

Answer Key 7.2

1. $8r^2(5r-1)-5(5r-1)$
$(5r-1)(8r^2-5)$
2. $5x^2(7x-2)-8(7x-2)$
$(7x-2)(5x^2-8)$
3. $n^2(3n-2)-3(3n-2)$
$(3n-2)(n^2-3)$
4. $2v^2(7r+5)-1(7r+5)$
$(7v+5)(2v^2-1)$
5. $3b^2(5b+7)-7(5b+7)$
$(5b+7)(3b^2-7)$
6. $6x^2(x-8)+5(x-8)$
$(x-8)(6x^2+5)$
7. $7x^2(5x-4)-4(5x-4)$
$(5x-4)(7x^2-4)$
8. $7n^2(n+3)-5(n+3)$
$(n+3)(7n^2-5)$
9. $7x(y-7)+5(y-7)$
$(y-7)(7x+5)$
10. $7r^2(6r-7)+3(6r-7)$
$(6r-7)(7r^2+3)$
11. $8x(2y-7)+1(2y-7)$
$(2y-7)(8x+1)$
12. $m(3n-8)+5(3n-8)$
$(3n-8)(m+5)$
13. $2x(y-4x)+7y^2(y-4x)$
$(y-4x)(2x+7y^2)$
14. $m(5n+2)-5(5n+2)$
$(5n+2)(m-5)$
15. $5x(8y+7)-y(8y+7)$
$(8y+7)(5x-y)$
16. $8x(y+7)-1(y+7)$
$(y+7)(8x-1)$
17. $12y+10xy+30+25x$
$2y(6+5x)+5(6+5x)$
$(6+5x)(2y+5)$
18. $24xy-20x+25y^2-30y^3$
$4x(6y-5)-5y^2(6y-5)$
$(6y-5)(4x-5y^2)$
19. $-6u^2+3uv+14u-7v$
$-3u(2u-v)+7(2u-v)$
$(2u-v)(-3u+7)$
20. $56ab-16b-49a+14$
$8b(7a-2)-7(7a-2)$

$$(7a-2)(8b-7)$$

Answer Key 7.3

1. $9 \times 8 = 72$
   $9 + 8 = 17$
   $p^2 + 9p + 8p + 72$
   $p(p+9) + 8(p+9)$
   $(p+9)(p+8)$
2. $9 \times -8 = -72$
   $9 + -8 = 1$
   $x^2 + 9x - 8x - 72$
   $x(x+9) - 8(x+9)$
   $(x+9)(x-8)$
3. $-8 \times -1 = 8$
   $-8 + -1 = -9$
   $n^2 - n - 8n + 8$
   $n(n-1) - 8(n-1)$
   $(n-1)(n-8)$
4. $6 \times -5 = -30$
   $6 + -5 = 1$
   $x^2 + 6x - 5x - 30$
   $x(x+6) - 5(x+6)$
   $(x+6)(x-5)$
5. $-10 \times 1 = -10$
   $-10 + 1 = -9$
   $x^2 - 10x + x - 10$
   $x(x-10) + 1(x-10)$
   $(x-10)(x+1)$
6. $8 \times 5 = 40$
   $8 + 5 = 13$
   $x^2 + 8x + 5x + 40$
   $x(x+8) + 5(x+8)$
   $(x+8)(x+5)$
7. $4 \times 8 = 32$
   $4 + 8 = 12$
   $b^2 + 4b + 8b + 32$
   $b(b+4) + 8(b+4)$
   $(b+4)(b+8)$
8. $-7 \times -10 = 70$
   $-7 + -10 = -17$
   $b^2 - 7b - 10b + 70$
   $b(b-7) - 10(b-7)$
   $(b-7)(b-10)$
9. $-3 \times -5 = 15$
   $-3 + -5 = -8$
   $u^2 - 3uv - 5uv + 15v^2$
   $u(u-3v) - 5v(u-3v)$

$(u-3v)(u-5v)$

10. $-8\times 5=-40$
$-8+5=-3$
$m^2-8mn+5mn-40n^2$
$m(m-8n)+5n(m-8n)$
$(m-8n)(m+5n)$

11. $4\times -2=-8$
$4+-2=2$
$m^2+4mn-2mn-8n^2$
$m(m+4n)-2n(m+4n)$
$(m+4n)(m-2n)$

12. $8\times 2=16$
$8+2=10$
$x^2+8xy+2xy+16y^2$
$x(x+8y)+2y(x+8y)$
$(x+8y)(x+2y)$

13. $-9\times -2=18$
$-9+-2=-11$
$x^2-9xy-2xy+18y^2$
$x(x-9y)-2y(x-9y)$
$(x-9y)(x-2y)$

14. $-2\times -7=14$
$-2+-7=-9$
$u^2-2uv-7uv+14v^2$
$u(u-2v)-7v(u-2v)$
$(u-2v)(u-7v)$

15. $4\times -3=-12$
$4+-3=1$
$x^2+4xy-3xy-12y^2$
$x(x+4y)-3y(x+4y)$
$(x+4y)(x-3y)$

16. $5\times 9=45$
$5+9=14$
$x^2+5xy+9xy+45y^2$
$x(x+5y)+9y(x+5y)$
$(x+5y)(x+9y)$

Answer Key 7.4

1. $-21 \times 2 = -42$
   $-21 + 2 = -19$
   $7x^2 - 21x + 2x - 6$
   $7x(x-3) + 2(x-3)$
   $(x-3)(7x+2)$
2. $-6 \times 4 = -24$
   $-6 + 4 = -2$
   $3n^2 - 6n + 4n - 8$
   $3n(n-2) + 4(n-2)$
   $(n-2)(3n+4)$
3. $14 \times 1 = 14$
   $14 + 1 = 15$
   $7b^2 + 14b + b + 2$
   $7b(b+2) + 1(b+2)$
   $(b+2)(7b+1)$
4. $-14 \times 3 = -42$
   $-14 + 3 = -11$
   $21v^2 - 14v + 3v - 2$
   $7v(3v-2) + 1(3v-2)$
   $(3v-2)(7v+1)$
5. $15 \times -2 = -30$
   $15 + -2 = 13$
   $5a^2 + 15a - 2a - 6$
   $5a(a+3) - 2(a+3)$
   $(a+3)(5a-2)$
6. $-20 \times 2 = -40$
   $-20 + 2 = -18$
   $5n^2 - 20n + 2n - 8$
   $5n(n-4) + 2(n-4)$
   $(n-4)(5n+2)$
7. $-1 \times -4 = 4$
   $-1 + -4 = -5$
   $2x^2 - x - 4x + 2$
   $x(2x-1) - 2(2x-1)$
   $(2x-1)(x-2)$
8. $-6 \times 2 = -12$
   $-6 + 2 = -4$
   $3r^2 - 6r + 2r - 4$
   $3r(r-2) + 2(r-2)$
   $(r-2)(3r+2)$
9. $14 \times 5 = 70$
   $14 + 5 = 19$
   $2x^2 + 14x + 5x + 35$
   $2x(x+7) + 5(x+7)$

$(x+7)(2x+5)$

10. $9\times -5=-45$
$9+-5=4$
$3x^2+9x-5x-15$
$3x(x+3)-5(x+3)$
$(x+3)(3x-5)$

11. $-3\times 2=-6$
$-3+2=-1$
$2b^2-3b+2b-3$
$b(2b-3)+1(2b-3)$
$(2b-3)(b+1)$

12. $8\times -3=-24$
$8+-3=5$
$2k^2+8k-3k-12$
$2k(k+4)-3(k+4)$
$(k+4)(2k-3)$

13. $15\times 2=30$
$15+2=17$
$3x^2+15xy+2xy+10y^2$
$3x(x+5y)+2y(x+5y)$
$(x+5y)(3x+2y)$

14. $-7\times 5=-35$
$-7+5=-2$
$7x^2-7xy+5xy-5y^2$
$7x(x-y)+5y(x-y)$
$(x-y)(7x+5y)$

15. $15\times -4=-60$
$15+-4=11$
$3x^2+15xy-4xy-20y^2$
$3x(x+5y)-4y(x+5y)$
$(x+5y)(3x-4y)$

16. $18\times -2=-36$
$18+-2=16$
$12u^2+18uv-2uv-3v^2$
$6u(2u+3v)-v(2u+3v)$
$(2u+3v)(6u-v)$

17. $-16\times -1=16$
$-16+-1=-17$
$4k^2-16k-k+4$
$4k(k-4)-1(k-4)$
$(k-4)(4k-1)$

18. $7\times -4=-28$
$7+-4=3$
$4r^2+7r-4r-7$
$r(4r+7)-1(4r+7)$
$(4r+7)(r-1)$

19. $-12\times 3=-36$
$-12+3=-9$

$4m^2 - 12mn + 3mn - 9n^2$
$4m(m - 3n) + 3n(m - 3n)$
$(m - 3n)(4m + 3n)$

20. Cannot be factored.
21. $12 \times 1 = 12$
$12 + 1 = 13$
$4x^2 + 12xy + xy + 3y^2$
$4x(x + 3y) + y(x + 3y)$
$(x + 3y)(4x + y)$
22. $8 \times -3 = -24$
$8 + -3 = 5$
$6u^2 + 8uv - 3uv - 4v^2$
$2u(3u + 4v) - v(3u + 4v)$
$(3u + 4v)(2u - v)$
23. $20 \times -1 = -20$
$20 + -1 = 19$
$10x^2 + 20xy - xy - 2y^2$
$10x(x + 2y) - 1(x + 2y)$
$(x - 2y)(10x - y)$
24. $-15 \times 2 = -30$
$-15 + 2 = -13$
$6x^2 - 15xy + 2xy - 5y^2$
$3x(2x - 5y) + y(2x - 5y)$
$(2x - 5y)(3x + y)$

Answer Key 7.5

1. $(r-4)(r+4)$
2. $(x-3)(x+3)$
3. $(v-5)(v+5)$
4. $(x-1)(x+1)$
5. $(p-2)(p+2)$
6. $(2v-1)(2v+1)$
7. $3(x^2-9)$
   $3(x-3)(x+3)$
8. $5(n^2-4)$
   $5(n-2)(n+2)$
9. $4(4x^2-9)$
   $4(2x-3)(2x+3)$
10. $5(25x^2+9y^2)$
11. $(a-1)^2$
12. $(k+2)^2$
13. $(x+3)^2$
14. $(n-4)^2$
15. $(5p-1)^2$
16. $(x+1)^2$
17. $(5a+3b)^2$
18. $(x+4y)^2$
19. $2(4x^2-12xy+9y^2)$
    $2(2x-3y)^2$
20. $5(4x^2+4xy+y^2)$
    $5(2x+y)^2$
21. $(2-m)(4+2m+m^2)$
22. $(x+4)(x^2-4x+16)$
23. $(x-4)(x^2+4x+16)$
24. $(x+2)(x^2-2x+4)$
25. $(6-u)(36+6u+u^2)$
26. $(5x-6)(25x^2+30x+36)$
27. $(5a-4)(25a^2+20a+16)$
28. $(4x-3)(16x^2+12x+9)$
29. $(4x+3y)(16x^2-12xy+9y^2)$
30. $4(8m^3-27n^3)$
    $4(2m-3n)(4m^2+6mn+9n^2)$

Answer Key 7.6

1. $(x^2-4y^2)(x^2+4y^2)$
   $(x-2y)(x+2y)(x^2+4y^2)$
2. $(4x^2-9y^2)(4x^2+9y^2)$
   $(2x-3y)(2x+3y)(4x^2+9y^2)$
3. $(x^2-16y^2)(x^2+16y^2)$
   $(x-4y)(x+4y)(x^2+16y^2)$
4. $(25x^2-9y^2)(25x^2+9y^2)$
   $(5x-3y)(5x+3y)(25x^2+9y^2)$
5. $(9x^2-4y^2)(9x^2+4y^2)$
   $(3x+2y)(3x-2y)(9x^2+4y^2)$
6. $(x^2-9y^2)(x^2+9y^2)$
   $(x-3y)(x+3y)(x^2+9y^2)$
7. $(25x^2-16y^2)(25x^2+16y^2)$
   $(5x-4y)(5x+4y)(25x^2+16y^2)$
8. $(x^2-9y^2)(x^2+9y^2)$
   $(x-3y)(x+3y)(x^2+9y^2)$
9. $(x^3-y^3)(x^3+y^3)$
   $(x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)$
10. $(x^2)^3+(y^2)^3$
    $(x^2+y^2)(x^4-x^2y^2+y^4)$
11. $(x^3-8y^3)(x^3+8y^3)$
    $(x-2y)(x^2+2xy+4y^2)(x+2y)(x^2-2xy+4y^2)$
12. $(4x^2)^3+(y^2)^3$
    $(4x^2+y^2)(16x^4-4x^2y^2+y^4)$
13. $(27x^3-y^3)(27x^3+y^3)$
    $(3x-y)(9x^2+3xy+y^2)(3x+y)(9x^2-3xy+y^2)$
14. $(9x^2)^3+(y^2)^3$
    $(9x^2+y^2)(81x^4-9x^2y^2+y^4)$
15. $(9x^2)^3+(4y^2)^3$
    $(9x^2+4y^2)(81x^4-36x^2y^2+16y^4)$
16. $(8x^3-125y^3)(8x^3+125y^3)$
    $(2x-5y)(4x^2+10xy+25y^2)(2x+5y)(4x^2-10xy+25y^2)$
17. $[(a+b)-(c-d)][(a+b)+(c-d)]$
    $[a+b-c+d][a+b+c-d]$
18. $[(a+2b)-(3a-4b)][(a+2b)+(3a-4b)]$
    $[a+2b-3a+4b][a+2b+3a-4b]$
    $[-2a+6b][4a-2b]$
    $4[-a+3b][2a-b]$
19. $[(a+3b)-(2c-d)][(a+3b)+(2c-d)]$
    $[a+3b-2c+d][a+3b+2c-d]$
20. $[(3a+b)-(a-b)][(3a+b)+(a-b)]$
    $[2a+2b][4a]$

$2[a+b][4a]$
$8a(a+b)$

21. $[(a+b)-(c-d)][(a+b)^2+(a+b)(c-d)+(c-d)^2]$
$[a+b-c+d][a^2+2ab+b^2+ac-ad+bc-bd+c^2-2cd+d^2]$

22. $[(a+3b)+(4a-b)][(a+3b)^2-(a+3b)(4a-b)+(4a-b)^2]$
$[5a+2b][a^2+6ab+9b^2-4a^2+ab-12ab+3b^2+16a^2-8ab+b^2]$
$[5a+2b][13a^2-13ab+13b^2]$
$13[5a+2b][a^2-ab+b^2]$

Answer Key 7.7

1. $6a(4c - 3b) + 15d(4c - 3b)$
   $(4c - 3b)(6a + 15d)$
   $3(4c - 3b)(2a + 5d)$
2. $-6 \times -5 = 30$
   $-6 + -5 = -11$
   $2x^2 - 6x - 5x + 15$
   $2x(x - 3) - 5(x - 3)$
   $(x - 3)(2x - 5)$
3. $-5 \times -4 = 20$
   $-5 + -4 = -9$
   $5u^2 - 5uv - 4uv + 4v^2$
   $5u(u - v) - 4v(u - v)$
   $(u - v)(5u - 4v)$
4. $(4x + 6y)^2$
5. $-2(x^3 - 64y^3)$
   $-2(x - 4y)(x^2 + 4xy + 16y^2)$
6. $20u(v - 3u^2) - 5x(v - 3u^2)$
   $(v - 3u^2)(20u - 5x)$
7. $2(27u^3 - 8)$
   $2(3u - 2)(9u^2 + 6u + 4)$
8. $2(27 - 64x^3)$
   $2(3 - 4x)(9 + 12x + 16x^2)$
9. $n(n - 1)$
10. $-25 \times 3 = -75$
    $-25 + 3 = -22$
    $5x^2 - 25x + 3x - 15$
    $5x(x - 5) + 3(x - 5)$
    $(x - 5)(5x + 3)$
11. $x^2 - 3xy - xy + 3y^2$
    $x(x - 3y) - y(x - 3y)$
    $(x - 3y)(x - y)$
12. $-15 \times -15 = 225$
    $-15 + -15 = -30$
    $5(9u^2 - 30uv + 25v^2)$
    $5(9u^2 - 15uv - 15uv + 25v^2)$
    $5(3u(3u - 5v) - 5v(3u - 5v))$
    $5(3u - 5v)(3u - 5v)$
13. $(m - 2n)(m + 2n)$
14. $3(4ab - 6a + 2nb - 3n)$
    $3(2a(2b - 3) + n(2b - 3))$
    $3(2b - 3)(2a + n)$
15. $36b^2c - 24b^2d + 24ac - 16ad$
    $12b^2(3c - 2d) + 8a(3c - 2d)$

$(3c-2d)(12b^2+8a)$
$4(3c-2d)(3b^2+2a)$

16. $-4\times 2=-8$
$-4+2=-2$
$3m(m^2-2mn-8n^2)$
$3m(m^2-4mn+2mn-8n^2)$
$3m(m(m-4n)+2n(m-4n))$
$3m(m-4n)(m+2n)$
17. $2(64+27x^3)$
$2(4+3x)(16-12x+9x^2)$
18. $(4m+3n)(16m^2-12mn+9n^2)$
19. $5\times 2=10$
$5+2=7$
$n(n^2+7n+10)$
$n(n^2+5n+2n+10)$
$n(n(n+5)+2(n+5))$
$n(n+5)(n+2)$
20. $(4m-n)(16m^2+4mn+n^2)$
21. $(3x-4)(9x^2+12x+16)$
22. $(4a-3b)(4a+3b)$
23. $x(5x+2)$
24. $-6\times -4=24$
$-6+-4=-10$
$2x^2-6x-4x+12$
$2x(x-3)-4(x-3)$
$(x-3)(2x-4)$

Answer Key 7.8

1. $$\begin{array}{rrrrr} k & - & 7 & = & 0 \\ & + & 7 & & +7 \\ & & k & = & 7 \end{array} \qquad \begin{array}{rrrrr} k & + & 2 & = & 0 \\ & - & 2 & & -2 \\ & & k & = & -2 \end{array}$$

2. $$\begin{array}{rrrrr} a & + & 4 & = & 0 \\ & - & 4 & & -4 \\ & & a & = & -4 \end{array} \qquad \begin{array}{rrrrr} a & - & 3 & = & 0 \\ & + & 3 & & +3 \\ & & a & = & 3 \end{array}$$

3. $$\begin{array}{rrrrr} x & - & 1 & = & 0 \\ & + & 1 & & +1 \\ & & x & = & 1 \end{array} \qquad \begin{array}{rrrrr} x & + & 4 & = & 0 \\ & - & 4 & & -4 \\ & & x & = & -4 \end{array}$$

4. $$\begin{array}{rrrrr} 2x & + & 5 & = & 0 \\ & - & 5 & & -5 \\ & & \dfrac{2x}{2} & = & \dfrac{-5}{2} \\ & & x & = & -\dfrac{5}{2} \end{array} \qquad \begin{array}{rrrrr} x & - & 7 & = & 0 \\ & + & 7 & & +7 \\ & & x & = & 7 \end{array}$$

5. $$\begin{array}{rcl} 6(x^2-25) & = & 0 \\ 6(x-5)(x+5) & = & 0 \\ \\ x & = & 5 \\ x & = & -5 \end{array}$$

6. $$\begin{array}{rcl} (p+8)(p-4) & = & 0 \\ \\ p & = & -8 \\ p & = & 4 \end{array}$$

7. $$\begin{array}{rcl} 2(n^2+5n-14) & = & 0 \\ 2(n+7)(n-2) & = & 0 \\ \\ n & = & -7 \\ n & = & 2 \end{array}$$

8. $$\begin{array}{rcl} (m-6)(m+5) & = & 0 \\ \\ m & = & 6 \\ m & = & -5 \end{array}$$

9. $$\begin{array}{rcl} (x+3)(7x+5) & = & 0 \\ \\ x & = & -3 \\ x & = & -\dfrac{5}{7} \end{array}$$

10.
$$\begin{aligned}(2b+1)(b-2) &= 0\\ b &= -\frac{1}{2}\\ b &= 2\end{aligned}$$

11.
$$\begin{array}{rrrrrrr} x^2 & - & 4x & - & 8 & = & -8\\ & & & + & 8 & & +8\\ & & x^2 & - & 4x & = & 0\\ & & x(x & - & 4) & = & 0\\ & & & & x & = & 0\\ & & & & x & = & 4\end{array}$$

12.
$$\begin{array}{rrrrrrr} v^2 & - & 8v & - & 3 & = & -3\\ & & & + & 3 & & +3\\ & & v^2 & - & 8v & = & 0\\ & & v(v & - & 8) & = & 0\\ & & & & v & = & 0\\ & & & & v & = & 8\end{array}$$

13.
$$\begin{array}{rrrrrrrrr} & x^2 & - & 5x & - & 1 & = & -5\\ & & & & + & 5 & & +5\\ & x^2 & - & 5x & + & 4 & = & 0\\ (x & - & 4) & (x & - & 1) & = & 0\\ & & & & & x & = & 4\\ & & & & & x & = & 1\end{array}$$

14.
$$\begin{array}{rrrrrrrrr} & a^2 & - & 6a & + & 6 & = & -2\\ & & & & + & 2 & = & +2\\ & a^2 & - & 6a & + & 8 & = & 0\\ (a & - & 4) & (a & - & 2) & = & 0\\ & & & & & a & = & 4\\ & & & & & a & = & 2\end{array}$$

15.
$$\begin{array}{rrrrrrrrr} & 7x^2 & + & 17x & - & 20 & = & -8\\ & & & & + & 8 & & +8\\ & 7x^2 & + & 17x & - & 12 & = & 0\\ (7x & - & 4) & (x & + & 3) & = & 0\\ & & & & & x & = & \frac{4}{7}\\ & & & & & x & = & -3\end{array}$$

16.
$$\begin{array}{rrrrrrrrr} & 4n^2 & - & 13n & + & 8 & = & 5 \\ & & & & - & 5 & & -5 \\ & 4n^2 & - & 13n & + & 3 & = & 0 \\ (4n & - & 1) & (n & - & 3) & = & 0 \\ \\ & & & & & n & = & \frac{1}{4} \\ \\ & & & & & n & = & 3 \end{array}$$

17.
$$\begin{array}{rrrrrrrr} & x^2 & - & 6x & & & = & 16 \\ & & & & - & 16 & & -16 \\ & x^2 & - & 6x & - & 16 & = & 0 \\ (x & - & 8) & (x & + & 2) & = & 0 \\ \\ & & & & & x & = & 8 \\ & & & & & x & = & -2 \end{array}$$

18.
$$\begin{array}{rrrrr} 7n^2 & - & 28n & = & 0 \\ 7n(n & - & 4) & = & 0 \\ \\ & & n & = & 0 \\ & & n & = & 4 \end{array}$$

19.
$$\begin{array}{rrrrrrrrrrr} & 4k^2 & + & 22k & + & 23 & = & 6k & + & 7 \\ & & - & 6k & - & 7 & & -6k & - & 7 \\ & 4k^2 & + & 16k & + & 16 & = & 0 \\ & 4(k^2 & + & 4k & + & 4) & = & 0 \\ 4(k & + & 2) & (k & + & 2) & = & 0 \\ \\ & & & & & k & = & -2 \end{array}$$

20.
$$\begin{array}{rrrrrrrrrr} & a^2 & + & 7a & - & 9 & = & -3 & + & 6a \\ & & - & 6a & + & 3 & & +3 & - & 6a \\ & a^2 & + & a & - & 6 & = & 0 \\ (a & + & 3) & (a & - & 2) & = & 0 \\ \\ & & & & & a & = & -3 \\ & & & & & a & = & 2 \end{array}$$

21.
$$\begin{array}{rrrrrrrrrrrr} & 9x^2 & - & 46 & + & 7x & = & 7x & + & 8x^2 & + & 3 \\ & -8x^2 & - & 3 & - & 7x & & -7x & - & 8x^2 & - & 3 \\ & & & x^2 & - & 49 & = & 0 \\ (x & & - & 7) & (x & + & 7) & = & 0 \\ \\ & & & & & x & = & 7 \\ & & & & & x & = & -7 \end{array}$$

22.
$$\begin{array}{rrrrrrrr} & x^2 & + & 10x & + & 30 & = & 6 \\ & & & & - & 6 & = & -6 \\ & x^2 & + & 10x & + & 24 & = & 0 \\ (x & + & 6) & (x & + & 4) & = & 0 \\ \\ & & & & & x & = & -6 \\ & & & & & x & = & -4 \end{array}$$

23.
$$\begin{array}{rrrrrrrrrr} & 40p^2 & + & 183p & - & 168 & = & p & + & 5p^2 \\ & -5p^2 & - & p & & & & -p & - & 5p^2 \\ & 35p^2 & + & 182p & - & 168 & = & 0 & & \\ & 7(5p^2 & + & 26p & - & 24) & = & 0 & & \\ 7(p & + & 6) & (5p & - & 4) & = & 0 & & \\ \\ & & & & & p & = & -6 & & \\ \\ & & & & & p & = & \frac{4}{5} & & \end{array}$$

24.
$$\begin{array}{rrrrrrrr} & 24x^2 & + & 11x & - & 80 & = & 3x \\ & & - & 3x & & & & -3x \\ & 24x^2 & + & 8x & - & 80 & = & 0 \\ & 8(3x^2 & + & x & - & 10) & = & 0 \\ 8(3x & - & 5) & (x & + & 2) & = & 0 \\ \\ & & & & & x & = & \frac{5}{3} \\ \\ & & & & & x & = & -2 \end{array}$$

Answer Key 7.9

1. $$\begin{aligned} R &= J+10 \\ R+4 &= 2(J+4) \end{aligned}$$
2. $$\begin{aligned} F &= 4S \\ F+20 &= 2(S+20) \end{aligned}$$
3. $$\begin{aligned} P &= J+20 \\ P+2 &= 2(J+2) \end{aligned}$$
4. $$\begin{aligned} D &= 23+A \\ D+6 &= 2(A+6) \end{aligned}$$
5. $$\begin{aligned} F &= B+4 \\ (F-5)+(B-5) &= 48 \end{aligned}$$
6. $$\begin{aligned} J &= 4M \\ (J-5)+(M-5) &= 50 \end{aligned}$$
7. $$\begin{aligned} T &= 5+J \\ (T+6)+(J+6) &= 79 \end{aligned}$$
8. $$\begin{aligned} J &= 2L \\ (J+3)+(L+3) &= 54 \end{aligned}$$
9. $$\begin{array}{rrrrrrrrr} & & J & + & m & = & 32 & & \\ & & & - & m & & -m & & \\ & & & & J & = & 32 & - & m \\ \\ & & J & - & 4 & = & 2(m & - & 4) \\ (32 & - & m) & - & 4 & = & 2m & - & 8 \\ 32 & - & m & - & 4 & = & 2m & - & 8 \\ & & 28 & - & m & = & 2m & - & 8 \\ & & +8 & + & m & & +m & + & 8 \\ & & & & \dfrac{36}{3} & = & \dfrac{3m}{3} & & \\ \\ & & & & m & = & 12 & & \\ \\ & & & & \therefore J & = & 32 & - & m \\ & & & & J & = & 32 & - & 12 \\ & & & & J & = & 20 & & \end{array}$$

10.

$$\begin{array}{rrrrrrr} & F & + & S & = & 56 & & \\ & & - & S & = & & - & S \\ & & & F & = & 56 & - & S \end{array}$$

$$\begin{array}{rrrrrrrr} & & F & - & 4 & = & 3(S & - & 4) \\ 56 & - & S & - & 4 & = & 3S & - & 12 \\ & + & S & + & 12 & & +S & + & 12 \end{array}$$

$$\frac{64}{4} = \frac{4S}{4}$$

$$S = 16$$

$$\begin{array}{rrrrr} \therefore F & = & 56 & - & S \\ F & = & 56 & - & 16 \\ F & = & 40 & & \end{array}$$

11.

$$\begin{array}{rrrrrrr} w & + & B & = & 20 & & \\ -w & & & & & - & w \\ & & B & = & 20 & - & w \end{array}$$

$$B - 4 = \frac{1}{2}(w - 4)$$

$$20 - w - 4 = \frac{1}{2}(w - 4)$$

$$[16 - w = \frac{1}{2}(w - 4)] \quad (2)$$

$$\begin{array}{rrrrrrrr} 32 & - & 2w & = & w & - & 4 \\ +4 & + & 2w & & +2w & + & 4 \end{array}$$

$$\frac{36}{3} = \frac{3w}{3}$$

$$w = 12$$

$$\begin{array}{rrrrr} B & = & 20 & - & w \\ B & = & 20 & - & 12 \\ B & = & 8 & & \end{array}$$

12.

$$\begin{array}{rrr} m & = & 36 \\ D & = & 3 \end{array}$$

$$\begin{array}{rrrrrrr} m & + & x & = & 4(D & + & x) \\ 36 & + & x & = & 4(3 & + & x) \\ 36 & + & x & = & 12 & + & 4x \\ -12 & - & x & & -12 & - & x \end{array}$$

$$\frac{24}{3} = \frac{3x}{3}$$

$$x = 8 \qquad \text{years}$$

13.

$$
\begin{aligned}
B_{\mathrm{o}} &= 2B_{\mathrm{y}}
\end{aligned}
$$

$$
\begin{array}{rcrcrcr}
B_{\mathrm{o}} & - & 5 & = & 3(B_{\mathrm{y}} & - & 5) \\
2B_{\mathrm{y}} & - & 5 & = & 3B_{\mathrm{y}} & - & 15 \\
-3B_{\mathrm{y}} & + & 5 & & -3B_{\mathrm{y}} & + & 5 \\
& & -B_{\mathrm{y}} & = & -10 & & \\
& & B_{\mathrm{y}} & = & 10 & &
\end{array}
$$

$$
\begin{aligned}
\therefore B_{\mathrm{o}} &= 2B_{\mathrm{y}} \\
B_{\mathrm{o}} &= 2(10) \\
B_{\mathrm{o}} &= 20
\end{aligned}
$$

14.

$$
\begin{aligned}
P &= 30 \\
V &= 22
\end{aligned}
$$

$$
\begin{array}{rcrcrcr}
P & - & x & = & 2(V & - & x) \\
30 & - & x & = & 2(22 & - & x) \\
30 & - & x & = & 44 & - & 2x \\
-44 & + & x & & -44 & + & x \\
& & -14 & = & -x & & \\
& & x & = & 14 & &
\end{array}
$$

15.

$$
m = 2c
$$

$$
\begin{array}{rcrcrcrcr}
(m & - & 7) & + & (c & - & 7) & = & 13 \\
& & m & + & c & - & 14 & = & 13 \\
& & 2c & + & c & - & 14 & = & 13 \\
& & & & & + & 14 & & +14 \\
& & & & & & \dfrac{3c}{3} & = & \dfrac{27}{3} \\
& & & & & & c & = & 9
\end{array}
$$

$$
\begin{aligned}
\therefore m &= 2c \\
m &= 2(9) \\
m &= 18
\end{aligned}
$$

16.

$$\begin{array}{rrrrrrrr}
 & J & + & m & = & 35 & & \\
 & & - & m & & & - & m \\
 & & & J & = & 35 & - & m \\
\\
 & J & - & 10 & = & 2(m & - & 10) \\
35 - & m & - & 10 & = & 2m & - & 20 \\
 & 25 & - & m & = & 2m & - & 20 \\
 & -25 & - & 2m & & -2m & - & 25 \\
 & & & -3m & = & -45 & & \\
\\
 & & & m & = & \dfrac{-45}{-3} & \text{or} & 15 \\
\\
 & & & \therefore J & = & 35 & - & m \\
 & & & J & = & 35 & - & 15 \\
 & & & J & = & 20 & &
\end{array}$$

17.

$$\begin{array}{rrrrrrrr}
 & & & S & = & 28 & + & B \\
\\
 & S & + & 6 & = & 2(B & + & 6) \\
28 + & B & + & 6 & = & 2B & + & 12 \\
 & B & + & 34 & = & 2B & + & 12 \\
 & -B & - & 12 & = & -B & - & 12 \\
 & & & 22 & = & B & & \\
\\
 & & & S & = & 28 & + & B \\
 & & & S & = & 28 & + & 22 \\
 & & & S & = & 50 & &
\end{array}$$

18.

$$\begin{array}{rrrrrrrr}
 & c & + & w & = & 64 & & \\
 & -c & & & & & - & c \\
 & & & w & = & 64 & - & c \\
 & & & w & = & 64 & - & 14 \\
 & & & \therefore w & = & 50 & & \\
\\
 & w & + & 4 & = & 3(c & + & 4) \\
64 - & c & + & 4 & = & 3c & + & 12 \\
+ & c & - & 12 & & +c & - & 12 \\
 & & & \dfrac{56}{4} & = & \dfrac{4c}{4} & & \\
\\
 & & & c & = & 14 & &
\end{array}$$

19.

$$\begin{array}{rcl} S & = & 12 \\ T & = & 36 \end{array}$$

$$\begin{array}{rcrcrcrcr} T & + & x & = & 2(S & + & x) \\ 36 & + & x & = & 2(12 & + & x) \\ 36 & + & x & = & 24 & + & 2x \\ -24 & - & x & & -24 & - & x \\ & & x & = & 12 & & \end{array}$$

20.

$$\begin{array}{rcrcr} F & = & 3S & & \\ D & = & S & - & 3 \end{array}$$

$$\begin{array}{rcrcrcrcrcrcr} F & - & 3 & + & D & - & 3 & + & S & - & 3 & = & 63 \\ F & & & + & D & & & + & S & - & 9 & = & 63 \\ 3S & & & + & S & - & 3 & + & S & - & 9 & = & 63 \\ & & & & & & & & 5S & - & 12 & = & 63 \\ & & & & & & & & & + & 12 & & +12 \\ & & & & & & & & & & \frac{5S}{5} & = & \frac{75}{5} \end{array}$$

$$S = 15$$

$$\begin{array}{rcrcl} F & = & 3S & & \\ F & = & 3(15) & \text{or} & 45 \end{array}$$

$$\begin{array}{rcrcl} D & = & S & - & 3 \\ D & = & 15 & - & 3 \text{ or } 12 \end{array}$$

Answer Key 7.10

$$\begin{array}{rcl}
\text{2012 average age} & = & \text{2016 average age} \\
\\
10x & = & 9(x+4)+N \\
10x & = & 9x+36+N \\
-9x & & -9x \\
x & = & 36+N \\
\\
x & = & 36+N \\
-36 & & -36 \\
x-36 & = & N
\end{array}$$

This means the new member is 36 years younger than the average age in 2012.

Midterm 2: Prep Answer Key

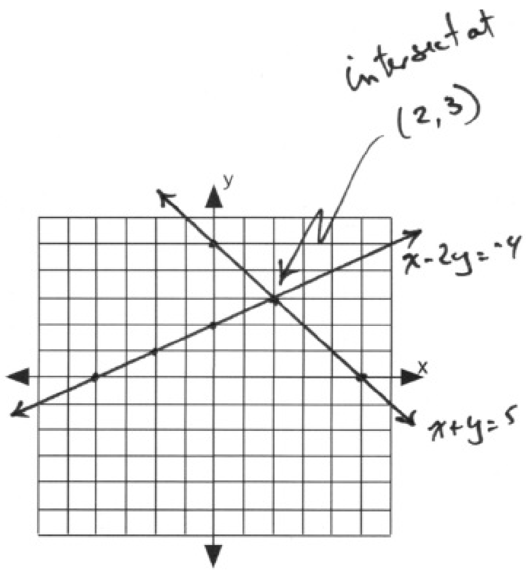
Midterm Two Review

1. $x - 2y = -4$

x	y
−4	0
0	2
−2	1

$x + y = 5$

x	y
0	5
5	0
2	3



2.
$$\begin{array}{rcrcl} 2x & - & y & = & 0 \quad \Rightarrow \quad y = 2x \\ 3x & + & 4y & = & -22 \end{array}$$

$$\begin{array}{rcrcl} \therefore 3x & + & 4(2x) & = & -22 \\ 3x & + & 8x & = & -22 \\ & & 11x & = & -22 \\ & & x & = & -2 \end{array}$$

$$\begin{array}{rcl} y & = & 2x \\ y & = & 2(-2) \quad = \quad -4 \end{array}$$

$(-2, -4)$

3.
$$\begin{array}{rrcrcl} & (2x & - & 5y & = & 15)(2) \\ & (3x & + & 2y & = & 13)(5) \\ & 4x & - & 10y & = & 30 \\ + & 15x & + & 10y & = & 65 \\ & & & 19x & = & 95 \\ & & & x & = & 5 \end{array}$$

$$\begin{array}{rcrcr} \therefore 3(5) & + & 2y & = & 13 \\ 15 & + & 2y & = & 13 \\ -15 & & & & -15 \\ & & 2y & = & -2 \\ & & y & = & -1 \end{array}$$

$(5, -1)$

4.
$$\begin{array}{rrcrcrcl} & & & (5x & + & 6z & = & -4)(-1) \\ & 5x & + & y & + & 6z & = & -2 \\ + & -5x & & & - & 6z & = & 4 \\ & & & & & y & = & 2 \end{array}$$

$$\begin{array}{rcrcr} \therefore 2y & - & 3z & = & 3 \\ 2(2) & - & 3z & = & 3 \\ -4 & & & & -4 \\ & & -3z & = & -1 \\ & & z & = & \frac{1}{3} \end{array}$$

$$\begin{array}{rcrcr} 5x & + & 6z & = & -4 \\ 5x & + & 6\left(\frac{1}{3}\right) & = & -4 \\ 5x & + & 2 & = & -4 \\ & - & 2 & & -2 \\ & & 5x & = & -6 \\ & & x & = & -\frac{6}{5} \end{array}$$

$-\frac{6}{5}, 2, \frac{1}{3}$

5.
$$\begin{array}{rrcrcr} & 4a^2 & - & 9a & + & 2 \\ & -a^2 & + & 4a & + & 5 \\ + & 3a^2 & - & a & + & 9 \\ & 6a^2 & - & 6a & + & 16 \end{array}$$

6. $8x^4 - 12x^2y^2 - 15x^2y^2 - 3x^4 \Rightarrow 5x^4 - 27x^2y^2$

7. $6 - 2\left[3x - 20x + 8 - 1\right]$

$6 - 2\left[-17x + 7\right]$

$6 + 34x - 14$

$34x - 8$

8. $25a^{-10}b^6$ or $\dfrac{25b^6}{a^{10}}$

9. $8a^2(a^2 + 10a + 25)$

$8a^4 + 80a^3 + 200a^2$

10. $4ab^2(a^2 - 4)$

$4a^3b^2 - 16ab^2$

11.
$$\begin{array}{rrrrrrrr}
 & x^2 & - & 4x & + & 7 & & \\
\times & & & x & - & 3 & & \\
 & x^3 & - & 4x^2 & + & 7x & & \\
+ & & - & 3x^2 & + & 12x & - & 21 \\
 & x^3 & - & 7x^2 & + & 19x & - & 21
\end{array}$$

12.
$$\begin{array}{rrrrrrrrrr}
 & 2x^2 & + & x & - & 3 & & & & \\
\times & 2x^2 & + & x & - & 3 & & & & \\
 & 4x^4 & + & 2x^3 & - & 6x^2 & & & & \\
 & & & 2x^3 & + & x^2 & - & 3x & & \\
+ & & & & & -6x^2 & - & 3x & + & 9 \\
 & 4x^4 & + & 4x^3 & - & 11x^2 & - & 6x & + & 9
\end{array}$$

13.
$$\begin{array}{rrrrrrrrrr}
 & x^2 & + & 5x & - & 2 & & & & \\
\times & 2x^2 & - & x & + & 3 & & & & \\
 & 2x^4 & + & 10x^3 & - & 4x^2 & & & & \\
 & & & -x^3 & - & 5x^2 & + & 2x & & \\
+ & & & & & 3x^2 & + & 15x & - & 6 \\
 & 2x^4 & + & 9x^3 & - & 6x^2 & + & 17x & - & 6
\end{array}$$

14.
$$(x+4)(x+4) \quad \Rightarrow \quad \begin{array}{rrrrrrrr}
 & x^2 & + & 8x & + & 16 & & \\
\times & & & x & + & 4 & & \\
 & x^3 & + & 8x^2 & + & 16x & & \\
+ & & & 4x^2 & + & 32x & + & 64 \\
 & x^3 & + & 12x^2 & + & 48x & + & 64
\end{array}$$

15. $r^{-4-3}s^{9+9} \Rightarrow r^{-7}s^{18} \Rightarrow \dfrac{s^{18}}{r^7}$

$\dfrac{s^{18}}{r^7}$

16. $(x^{-2--2}y^{-3-4})^{-1}$

$(1x^0y^{-7})^{-1}$

y^7y^7

17. $2x^3 - 7x^2 + 15x - 2$

18. $2^3 \cdot 11$

19. $2^5 \cdot 3 \cdot 7 \begin{cases} 84 = 2^2 \cdot 3 \cdot 7 \\ 96 = 2^5 \cdot 3 \end{cases}$

20. $x(5y+6z) - 3(5y+6z)$
$(5y+6z)(x-3)$

21. $-12 = 4 \times -3$
$1 = 4 + -3$

$x^2 + 4x - 3x - 12$
$x(x+4) - 3(x+4)$
$(x+4)(x-3)$

22. $x^2(x+1) - 4(x+1)$
$(x+1)(x^2-4)$
$x+1)(x-2)(x+2) < /li >< li > x^3 - (3y)^3$
$(x-3y)(x^2+3xy+9y^2)$

23. $(x^2-36)(x^2+1)$
$(x-6)(x+6)(x^2+1)$

24.

$$\begin{array}{rcrcr} (A & + & B & = & 70)(-4) \\ 4A & + & 7B & = & 430 \end{array} \Rightarrow \begin{array}{rrcrcr} & -4A & - & 4B & = & -280 \\ + & 4A & + & 7B & = & 430 \\ & & & 3B & = & 150 \\ & & & & & \\ & & & B & = & \dfrac{150}{3} \text{ or } 50 \end{array}$$

$$\begin{array}{rcrcr} \therefore A & + & B & = & 70 \\ & & & & \\ A & + & 50 & = & 70 \\ & & -50 & & -50 \\ & & A & = & 20 \end{array}$$

25.

$$\begin{array}{rcrcrcr} 5x & + & 21(2) & = & 11(x & + & 2) \\ & & & & & & \\ 5x & + & 42 & = & 11x & + & 22 \\ -5x & - & 22 & & -5x & - & 22 \\ & & 20 & = & 6x & & \\ & & & & & & \\ & & x & = & \dfrac{20}{6} & = & 3\dfrac{1}{3} \text{ litres} \end{array}$$

26.

$B + G = 16 \Rightarrow B = 16 - G \text{ or } G = 16 - B$

$$\begin{array}{rcrcrcr} G & - & 4 & = & 3(B & - & 4) \\ 16 - B & - & 4 & = & 3B & - & 12 \\ +B & + & 12 & & +B & + & 12 \\ & & 24 & = & 4B & & \\ & & & & & & \\ & & B & = & \dfrac{24}{4} & = & 6 \end{array}$$

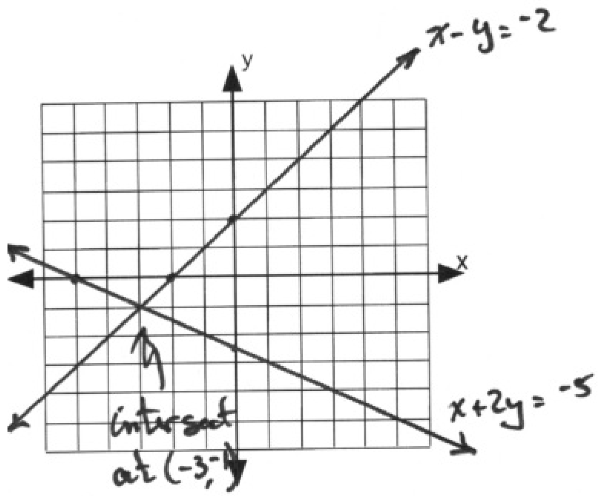
$$\begin{array}{rcrcr} \therefore G & = & 16 & - & B \\ G & = & 16 & - & 6 \\ G & = & 10 & & \end{array}$$

Midterm 2: Version A Answer Key

1.

$x + 2y = -5$	
x	y
0	$-\frac{5}{2}$
−5	0

$x - y = -2$	
x	y
0	2
−2	0



2.

$$\begin{array}{rrrrr} (4x & - & 3y & = & 13)(2) \\ (5x & - & 2y & = & 4)(-3) \\ \\ 8x & - & 6y & = & 26 \\ + \quad -15x & + & 6y & = & -12 \\ & & -7x & = & 14 \\ \\ & & x & = & \frac{14}{-7} \text{ or } -2 \end{array}$$

$$\begin{array}{rrrrr} 5x & - & 2y & = & 4 \\ 5(-2) & - & 2y & = & 4 \\ -10 & - & 2y & = & 4 \\ +10 & & & & +10 \\ & & -2y & = & 14 \\ \\ & & y & = & \frac{14}{-2} \text{ or } -7 \end{array}$$

$(-2, -7)$

3.
$$\begin{array}{rrrrrl} & x & - & 2y & = & -5 \\ & (2x & + & y & = & 5)(2) \end{array}$$

$$\begin{array}{rrrrrl} & x & - & 2y & = & -5 \\ + & 4x & + & 2y & = & 10 \\ & & & 5x & = & 5 \\ & & & x & = & 1 \end{array}$$

$$\begin{array}{rrrrrr} \therefore & 2(1) & + & y & = & 5 \\ & -2 & & & & -2 \\ & & & y & = & 3 \end{array}$$

4.
$$\begin{array}{rrrrrrrl} & (x & + & y & + & 2z & = & 0)(-2) \end{array}$$

$$\begin{array}{rrrrrrrl} & -2x & - & 2y & - & 4z & = & 0 \\ + & 2x & & & + & z & = & 1 \\ & & & -2y & - & 3z & = & 1 \end{array}$$

$$\begin{array}{rrrrl} (-2y & - & 3z & = & 1)(3) \\ (3y & + & 4z & = & 0)(2) \end{array}$$

$$\begin{array}{rrrrrrl} & -6y & - & 9z & = & 3 \\ + & 6y & + & 8z & = & 0 \\ & & & -z & = & 3 \\ & & & z & = & -3 \end{array}$$

$$\begin{array}{rrrrl} 3y & + & 4z & = & 0 \\ 3y & + & 4(-3) & = & 0 \\ 3y & - & 12 & = & 0 \\ & & 3y & = & 12 \\ & & y & = & 4 \end{array}$$

$$\begin{array}{rrrrl} 2x & + & z & = & 1 \\ 2x & + & (-3) & = & 1 \\ & & 2x & = & 4 \\ & & x & = & 2 \end{array}$$

$(2, 4, -3)$

5. $28 - \{5x - [6x - 3(5 - 2x)]^0 1\} + 5x^2$
$28 - \{5x - 1\} + 5x^2$
$28 - 5x + 1 + 5x^2$
$5x^2 - 5x + 29$

6.
$$\begin{array}{rrrrrr} & a & - & 3 & & \\ \times & a & - & 3 & & \\ & a^2 & - & 3a & & \\ + & & - & 3a & + & 9 \\ & a^2 & - & 6a & + & 9 \end{array}$$

$$\begin{array}{rrrrrr} & a^2 & - & 6a & + & 9 \\ \times & & & & & 4a^2 \\ & 4a^4 & - & 24a^3 & + & 36a^2 \end{array}$$

7.

$$\begin{array}{rrrrrrrrrr} & x^2 & + & 2x & + & 3 & & & & \\ \times & x^2 & + & 2x & + & 3 & & & & \\ & x^4 & + & 2x^3 & + & 3x^2 & & & & \\ & & & 2x^3 & + & 4x^2 & + & 6x & & \\ + & & & & & 3x^2 & + & 6x & + & 9 \\ \hline & x^4 & + & 4x^3 & + & 10x^2 & + & 12x & + & 9 \end{array}$$

8. $2x^3 - 7x^2 + 15x - 2$

9. $a(2b + 3c) - 2(2b + 3c)$
$(2b + 3c)(a - 2)$

10. $a^2 - 5ab + 3ab - 15b^2$
$a(a - 5b) + 3b(a - 5b)$
$(a - 5b)(a + 3b)$

11. $x^2(x + 1) - 9(x + 1)$
$(x^2 - 9)(x + 1)$
$(x + 3)(x - 3)(x + 1)$

12. $(x)^3 - (4y)^3$
$(x - 4y)(x^2 + 4xy + 16y^2)$

13.

$$B + S = 35 \Rightarrow B = 35 - S$$

$$\begin{array}{rrrrrrrr} & B & - & 10 & = & 2(S & - & 10) \\ & 35 - S & - & 10 & = & 2S & - & 20 \\ & 25 & - & S & = & 2S & - & 20 \\ + & 20 & + & S & & S & + & 20 \\ \hline & & & 45 & = & 3S & & \end{array}$$

$$\begin{array}{rcl} S & = & \frac{45}{3} \text{ or } 15 \\ \therefore B & = & 35 - S \\ B & = & 35 - 15 \\ B & = & 20 \end{array}$$

14.

$$D + Q = 20 \Rightarrow Q = 20 - D$$

$$\begin{array}{rrlrr} 10D & + & 25Q & = & 275 \\ 10D & + & 25(20 - D) & = & 275 \\ 10D & + & 500 - 25D & = & 275 \\ & - & 500 & & -500 \\ \hline & & -15D & = & -225 \end{array}$$

$$\begin{array}{rcl} D & = & \frac{-225}{-15} \text{ or } 15 \\ \therefore Q & = & 20 - D \\ Q & = & 20 - 15 \\ Q & = & 5 \end{array}$$

15.

$$A + B = 50 \Rightarrow A = 50 - B$$

$$
\begin{aligned}
(3.95A + 3.70(50 - A) &= 191.25)(100) \\
395A + 370(50 - A) &= 19125 \\
395A + 18500 - 370A &= 19125 \\
- 18500 \qquad & \quad -18500 \\
25A &= 625 \\
A &= \frac{625}{25} \text{ or } 25 \text{ kg} \\
B &= 50 - 25 \\
B &= 25 \text{ kg}
\end{aligned}
$$

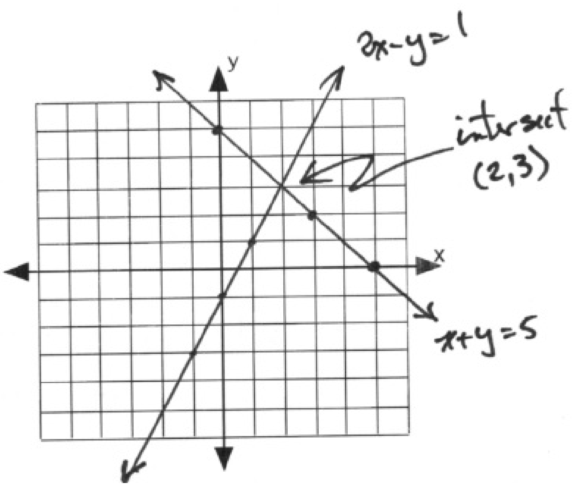
Midterm 2: Version B Answer Key

1. $x + y = 5$

x	y
5	0
0	5
3	2

$2x - y = 1$

x	y
1	1
0	−1
−1	−3



2. $$\begin{array}{rrrrrcr} 4(4y & + & 2) & + & 3y & = & 8 \\ 16y & + & 8 & + & 3y & = & 8 \\ & - & 8 & & & & -8 \\ & & & & 19y & = & 0 \\ & & & & y & = & 0 \end{array} \qquad \begin{array}{rcrrr} x & = & 4y & + & 2 \\ x & = & 4y0 & + & 2 \\ x & = & 2 & & \end{array}$$

$(2, 0)$

3.
$$\begin{array}{rcrcl} 5x & - & 3y & = & 2 \\ (3x & + & y & = & 4)(3) \end{array}$$

$$\begin{array}{crcrcr} & 5x & - & 3y & = & 2 \\ + & 9x & + & 3y & = & 12 \\ & & & 14x & = & 14 \\ & & & x & = & 1 \end{array}$$

$$\begin{array}{rcrcl} \therefore 3(1) & + & y & = & 4 \\ 3 & + & y & = & 4 \\ & & y & = & 1 \end{array}$$

4.
$$\begin{array}{crcrcrcl} & & & (x & - & 2z & = & -7)(-1) \\ & x & + & y & + & z & = & 3 \\ + & -x & & & + & 2z & = & 7 \\ & & & y & + & 3z & = & 10 \\ & & & (-2y & + & 4z & = & 20)(\div 2) \\ & & & & & \Downarrow & & \\ & & & y & + & 3z & = & 10 \\ & & + & -y & + & 2z & = & 10 \\ & & & & & 5z & = & 20 \\ & & & & & z & = & 4 \end{array}$$

$$\begin{array}{rcrcr} x & - & 2z & = & -7 \\ x & - & 2(4) & = & -7 \\ x & - & 8 & = & -7 \\ & + & 8 & & +8 \\ & & x & = & 1 \\ \\ y & + & 3z & = & 10 \\ y & + & 3(4) & = & 10 \\ y & + & 12 & = & 10 \\ & - & 12 & & -12 \\ & & y & = & -2 \end{array}$$

$(1, -2, 4)$

5. $5 - 3\left[4x - 2(6x-5)^0 1 - (7-2x)\right]$
$5 - 3\left[4x - 2(1) - (7-2x)\right]$
$5 - 3\left[6x - 9\right]$
$5 - 18x + 27 \Rightarrow -18x + 32$

6.
$$\begin{array}{crcrcr} & a & + & 3 & & \\ \times & a & + & 3 & & \\ & a^2 & + & 3a & & \\ + & & & 3a & + & 9 \\ & a^2 & + & 6a & + & 9 \\ \times & & & & & 3a^2 \\ & 3a^4 & + & 18a^3 & + & 27a^2 \end{array}$$

7.
$$\begin{array}{crcrcrcrcr} & x^2 & + & x & + & 5 & & & & \\ \times & x^2 & + & x & - & 5 & & & & \\ & x^4 & + & x^3 & + & 5x^2 & & & & \\ & & & x^3 & + & x^2 & + & 5x & & \\ + & & & & & -5x^2 & - & 5x & - & 25 \\ & x^4 & + & 2x^3 & + & x^2 & - & 25 & & \end{array}$$

8. $(x^{4n-3n}x^{-6})^{-1}$

$(x^n x^{-6})^{-1}$

$x^{-n}x^6$ or $\dfrac{x^6}{x^n}$

9. $2a(7xy-3z)-1(7xy-3z)$
$(7xy-3z)(2a-1)$
10. $a^2-3ab+5ab-15b^2$
$a(a-3b)+5b(a-3b)$
$(a-3b)(a+5b)$
11. $2x^2(x+4)-1(x+4)$
$(x+4)(2x^2-1)$
12. $(3x)^3+(2y)^3$
$(3x+2y)(9x^2-6xy+4y^2)$
13.

$$F+D=38 \Rightarrow F=38-D$$

$$\begin{array}{rrrrrrr} (F & + & 6) & = & 4(D & + & 6) \\ 38-D & + & 6 & = & 4D & + & 24 \\ -24+D & & & & +D & - & 24 \\ & & 20 & = & 5D & & \\ & & & & & & \\ & & D & = & \dfrac{20}{5} & = & 4 \\ & & & & & & \\ & & \therefore F & = & 38 & - & D \\ & & F & = & 38 & - & 4 \\ & & F & = & 34 & & \end{array}$$

14.

$$A+B=90 \Rightarrow B=90-A$$

$$\begin{array}{rrrrrrl} 3A & + & 5(90 & - & A) & = & 370 \\ 3A & + & 450 & - & 5A & = & 370 \\ & - & 450 & & & & -450 \\ & & & & -2A & = & -80 \\ & & & & & & \\ & & & & A & = & \dfrac{-80}{-2} \text{ or } 40 \text{ kg} \\ & & & & & & \\ & & & & \therefore B & = & 90-A \\ & & & & B & = & 90-40 \\ & & & & B & = & 50 \text{ kg} \end{array}$$

15.
$$\begin{array}{rcrcrcr} 10x & + & 25(40) & = & 15(x & + & 40) \\ \\ 10x & + & 1000 & = & 15x & + & 600 \\ -10x & - & 600 & & -10x & - & 600 \\ & & 400 & = & 5x & & \\ \\ & & x & = & \dfrac{400}{5} & = & 80 \end{array}$$

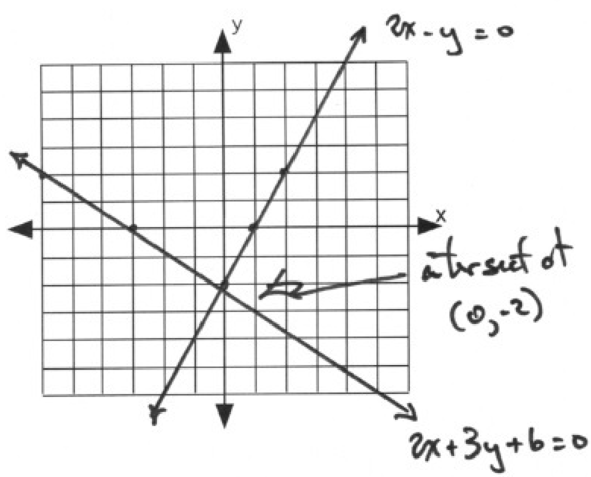
Midterm 2: Version C Answer Key

1. $$2x - y - 2 = 0$$

x	y
0	−2
1	0
2	2

$$2x + 3y + 6 = 0$$

x	y
0	−2
−3	0
−6	2



2. $$x + y = 2 \Rightarrow x = 2 - y$$

$$\begin{array}{rcrcrcr} 3(2 & - & y) & - & 4y & = & 13 \\ 6 & - & 3y & - & 4y & = & 13 \\ -6 & & & & & & -6 \\ & & & & -7y & = & 7 \\ & & & & y & = & -1 \\ \\ & & & & x & = & 2 - -1 \\ & & & & x & = & 3 \end{array}$$

$(3, -1)$

3.
$$\begin{array}{rrrrrr} & 4x & - & 3y & = & 6 \\ + & 4x & + & 3y & = & 2 \\ & & & 8x & = & 8 \\ & & & x & = & 1 \end{array}$$

$$\begin{array}{rrrrr} 3y & + & 4(1) & = & 2 \\ & - & 4 & & -4 \\ & & 3y & = & -2 \\ & & & & \\ & & y & = & -\dfrac{2}{3} \end{array}$$

$\left(1, -\dfrac{2}{3}\right)$

4.
$$\begin{array}{lrrrrrrr} [1] & (x & + & y & + & z & = & 6)(-2) \\ [2] & & & (-2x & + & z & = & -3)(-1) \end{array}$$

$$\begin{array}{lrrrrrrr} [1] & -2x & - & 2y & - & 2z & = & -12 \\ + & & & 2y & + & 4z & = & 10 \\ & & & -2x & + & 2z & = & -2 \\ + & & [2] & 2x & - & z & = & 3 \\ & & & & & z & = & 1 \end{array}$$

$$\begin{array}{rrrrr} 2y & + & 4z & = & 10 \\ 2y & + & 4(1) & = & 10 \\ & - & 4 & & -4 \\ & & 2y & = & 6 \\ & & y & = & 3 \end{array}$$

$$\begin{array}{rrrrr} -2x & + & z & = & -3 \\ -2x & + & 1 & = & -3 \\ & - & 1 & & -1 \\ & & -2x & = & -4 \\ & & x & = & 2 \end{array}$$

$(2, 3, 1)$

5. $36 + \{-2x - [6x - 3(5 - 2x)]\}^0 1 + 6x^3$
$36 + 1 + 6x^3 \Rightarrow 6x^3 + 37$

6. $6a^2b(a^2 - 9)$
$6a^4b - 54a^2b$

7.
$$\begin{array}{rrrrrrrrr} & x^2 & + & 3x & + & 5 & & & \\ \times & x^2 & + & 3x & + & 5 & & & \\ & x^4 & + & 3x^3 & + & 5x^2 & & & \\ & & & 3x^3 & + & 9x^2 & + & 15x & \\ + & & & & & 5x^2 & + & 15x & + \; 25 \\ & x^4 & + & 6x^3 & + & 19x^2 & + & 30x & + \; 25 \end{array}$$

8. $2x^4 + x^3 + 4x^2 - 4x - 52x + 1$

9. $x^2 - x + 18x - 18$
$x(x - 1) + 18(x - 1)$
$(x - 1)(x + 18)$

10. $2(a^2 - 2ab - 15b^2)$
$2(a^2 - 5ab + 3ab - 15b^2)$
$2\left[a(a - 5b) + 3b(a - 5b)\right]$
$2(a - 5b)(a + 3b)$

11. $(2x)^3 - y^3$
$(2x-y)(4x^2+2xy+y^2)$
12. $(4y^2-x^2)(4y^2+x^2)$
$(2y-x)(2y+x)(4y^2+x^2)$
13.
$$B+S=30 \Rightarrow B=30-S$$

$$\begin{array}{rrrrrrr} B & - & 10 & = & 4(S & - & 10) \\ 30-S & - & 10 & = & 4S & - & 40 \\ +S & + & 40 & & +S & + & 40 \\ & & 60 & = & 5S & & \\ \\ & & S & = & \dfrac{60}{5} & = & 12 \\ \\ & & \therefore B & = & 30 & - & S \\ & & B & = & 30 & - & 12 \\ & & B & = & 18 & & \end{array}$$

14.
$$\begin{array}{rrrrrl} & (D & + & N & = & 18)(-1) \\ & (10D & + & 5N & = & 120)(\div 5) \\ \\ & -D & - & N & = & -18 \\ + & 2D & + & N & = & 24 \\ & & & D & = & 6 \\ \\ \therefore & D & + & N & = & 18 \\ & 6 & + & N & = & 18 \\ & -6 & & & & -6 \\ & & & N & = & 12 \end{array}$$

15.
$$\text{if } x=5\%, \text{ then } 10-x=30\%$$

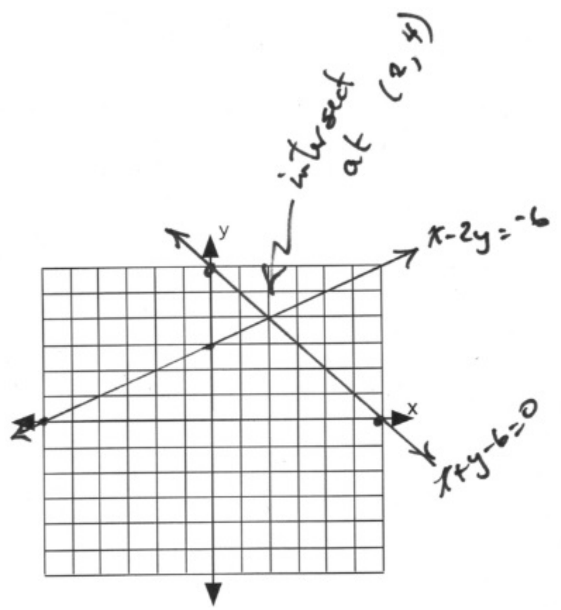
$$\begin{array}{rrrrrcl} 5x & + & 30(10 & - & x) & = & 25(10) \\ 5x & + & 300 & - & 30x & = & 250 \\ & - & 300 & & & & -300 \\ & & & & -25x & = & -50 \\ \\ & & & & x & = & \dfrac{-50}{-25} \text{ or } 2 \text{ L of } 5\% \\ \\ & & 10 & - & x & = & 8 \text{ L of } 30\% \end{array}$$

Midterm 2: Version D Answer Key

1.

$x - 2y = -6$	
x	y
0	3
−6	0

$x + y = 6$	
x	y
0	6
6	0



2.

$$\begin{array}{rrcrcl} & (3x & - & 2y & = & 0)(5) \\ & (2x & + & 5y & = & 0)(2) \end{array}$$

$$\begin{array}{rrcrcl} & 15x & - & 10y & = & 0 \\ + & 4x & + & 10y & = & 0 \\ & & & 19x & = & 0 \\ & & & x & = & 0 \end{array}$$

$$\begin{array}{rcrcl} \therefore 2x0 & + & 5y & = & 0 \\ & & 5y & = & 0 \\ & & y & = & 0 \end{array}$$

$(0, 0)$

3.

$$\begin{array}{rrcrcr} & 2x & - & 3y & = & 8 \\ + & -2x & + & 3y & = & 4 \\ & & & 0 & = & 12 \end{array}$$

$\therefore$ No solution. Parallel lines.

4.

$$\begin{array}{rcrcrcl}
(2x & + & y & - & 3z & = & -7)(2) \\
\\
4x & + & 2y & - & 6z & = & -14 \\
+ & & -2y & + & 3z & = & 9 \\
 & & 4x & - & 3z & = & -5 \\
\\
 & & (3x & + & z & = & 6)(3) \\
\\
 & & 4x & - & 3z & = & -5 \\
+ & & 9x & + & 3z & = & 18 \\
 & & & & 13x & = & 13 \\
 & & & & x & = & 1
\end{array}$$

$$\begin{array}{rcrcr}
3x & + & z & = & 6 \\
3(1) & + & z & = & 6 \\
-3 & & & & -3 \\
 & & z & = & 3 \\
\\
-2y & + & 3z & = & 9 \\
-2y & + & 3(3) & = & 9 \\
 & & -9 & & -9 \\
 & & -2y & = & 0 \\
 & & y & = & 0
\end{array}$$

$(1, 0, 3)$

5. $36 - \{-2x - [6x - 3(5 - 2x)]\}^0 1 + 3x^2$
$36 - 1 + 3x^2$
$35 + 3x^2$

6. $3a^2(a^2 - 4a + 4)$
$3a^4 - 12a^3 + 12a^2$

7.

$$\begin{array}{rrcrcrcrcr}
 & x^2 & + & 2x & - & 4 \\
\times & x^2 & + & 2x & - & 4 \\
 & x^4 & + & 2x^3 & - & 4x^2 \\
 & & & 2x^3 & + & 4x^2 & - & 8x \\
+ & & & & - & 4x^2 & - & 8x & + & 16 \\
 & x^4 & + & 4x^3 & - & 4x^2 & - & 16x & + & 16
\end{array}$$

8. $x^4 + 4x^3 + 4x^2 + 10x + 20x + 2$

9. $x^2 - 3x + 6x - 18$
$x(x - 3) + 6(x - 3)$
$(x - 3)(x + 6)$

10. $3x^2 + xy + 24xy + 8y^2$
$x(3x + y) + 8y(3x + y)$
$(3x + y)(x + 8y)$

11. $(5x)^3 - y^3$
$(5x - y)(25x^2 + 5xy + y^2)$

12. $(9y^2 - 4x^2)(9y^2 + 4x^2)$
$(3y - 2x)(3y + 2x)(9y^2 + 4x^2)$

13.

$B + G = 18 \Rightarrow G = 18 - B$

$$\begin{array}{rrrrrrr}
G & - & 4 & = & 4(B & - & 4) \\
18-B & - & 4 & = & 4B & - & 16 \\
+\quad 16+B & & & & +B & + & 16 \\
 & & 30 & = & 5B & &
\end{array}$$

$$B = \frac{30}{5} = 6$$

$$\begin{array}{rrrrr}
\therefore G & = & 18 & - & B \\
G & = & 18 & - & 6 \\
G & = & 12 & &
\end{array}$$

14.

$$\begin{array}{rrrrr}
(D & + & Q & = & 20)(-10) \\
10D & + & 25Q & = & 350
\end{array}$$

$$\begin{array}{rrrrrr}
 & -10D & - & 10Q & = & -200 \\
+ & 10D & + & 25Q & = & 350 \\
 & & & 15Q & = & 150
\end{array}$$

$$Q = \frac{150}{15} \text{ or } 10$$

$$\begin{array}{rrrrrr}
\therefore & D & + & Q & = & 20 \\
 & D & + & 10 & = & 20 \\
 & & - & 10 & & -10 \\
 & & & D & = & 10
\end{array}$$

15.

$$A+B=60 \Rightarrow B=60-A$$

$$\begin{array}{rrrrrrr}
 & & 3.80A & + & 3.55B & = & 218.50 \\
3.80A & + & 3.55(60 & - & A) & = & 218.50 \\
3.80A & + & 213 & - & 3.55A & = & 218.50 \\
 & - & 213 & & & = & -213 \\
 & & & & 0.25A & = & 5.50
\end{array}$$

$$A = \frac{5.50}{0.25} \text{ or } 22 \text{ kg}$$

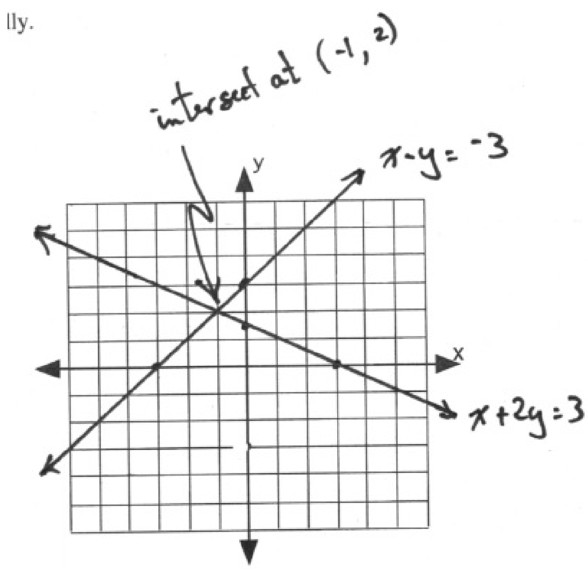
$$\begin{array}{rcl}
B & = & 60-A \\
B & = & 60-22 \\
B & = & 38 \text{ kg}
\end{array}$$

Midterm 2: Version E Answer Key

1.

$x - y = -3$	
x	y
0	3
−3	0

$x + 2y = 3$	
x	y
3	0
0	$\frac{3}{2}$



2.

$$\begin{array}{rcrcl}
(2x & - & 5y & = & -2)(-3) \\
(3x & - & 4y & = & 4)(2)
\end{array}$$

$$\begin{array}{crcrcl}
 & -6x & + & 15y & = & 6 \\
+ & 6x & - & 8y & = & 8 \\
 & & & 7y & = & 14 \\
 & & & y & = & 2
\end{array}$$

$$\begin{array}{rcrcl}
2x & - & 5(2) & = & -2 \\
2x & - & 10 & = & -2 \\
 & + & 10 & & +10 \\
 & & 2x & = & 8 \\
 & & x & = & 4
\end{array}$$

$(4,2)$

3.
$$\begin{array}{rrrrl} (4x & + & 3y & = & -29)(-2) \\ (3x & + & 2y & = & -21)(3) \end{array}$$

$$\begin{array}{rrrrrl} & -8x & - & 6y & = & 58 \\ + & 9x & + & 6y & = & -63 \\ & & & x & = & -5 \end{array}$$

$$\begin{array}{rrrrrl} & 3(-5) & + & 2y & = & -21 \\ & -15 & + & 2y & = & -21 \\ + & 15 & & & & +15 \\ & & & 2y & = & -6 \\ & & & y & = & -3 \end{array}$$

$(-5,-3)$

4.
$$\begin{array}{rrrrrrrl} & (x & + & y & - & 3z & = & 0)(-2) \\ & -2x & - & 2y & + & 6z & = & 0 \\ + & 2x & - & 3y & & & = & 16 \\ & & & -5y & + & 6z & = & 16 \end{array}$$

$$\begin{array}{rrrrrl} (2y & - & 2z & = & -12)(3) \end{array}$$

$$\begin{array}{rrrrrrl} & 6y & - & 6z & = & -36 \\ + & -5y & + & 6z & = & 16 \\ & & & y & = & -20 \end{array}$$

$$\begin{array}{rrrrl} 2x & - & 3y & = & 16 \\ 2x & - & 3(-20) & = & 16 \\ 2x & + & 60 & = & 16 \\ & & -60 & & -60 \\ & & 2x & = & -44 \\ & & x & = & -22 \end{array}$$

$$\begin{array}{rrrrl} 2y & - & 2z & = & -12 \\ 2(-20) & - & 2z & = & -12 \\ -40 & - & 2z & = & -12 \\ +40 & & & & +40 \\ & & -2z & = & 28 \\ & & z & = & -14 \end{array}$$

$(-22,-20,-14)$

5. $5-4\left[2x-2(6x-5)^0 1-(7-2x)\right]$
$5-4\left[2x-2(1)-7+2x\right]$
$5-4\left[4x-9\right]$
$5-16x+36$
$-16x+41$

6. $3ab^4(a^2-25)$
$3a^3b^4-75ab^4$

7.
$$\begin{array}{rrrrrrrrrr} & x^2 & + & 3x & - & 6 \\ \times & x^2 & + & 3x & - & 6 \\ & x^4 & + & 3x^3 & - & 6x^2 \\ & & & 3x^3 & + & 9x^2 & - & 18x \\ + & & & & - & 6x^2 & - & 18x & + & 36 \\ & x^4 & + & 6x^3 & - & 3x^2 & - & 36x & + & 36 \end{array}$$

8. $3x^3+18+7x^2x+3$

9. $x^2 - 3z + 7x - 21$
$x(x-3) + 7(x-3)$
$(x-3)(x+7)$

10. $4x^2(x+1) - 9(x+1)$
$(x+1)(4x^2-9)$
$(x+1)(2x-3)(2x+3)$

11. $(2x)^3 - (3y)^3$
$(2x-3y)(4x^2+6xy+9y^2)$

12. $x^4 - 625x^2 + x^2 - 625$
$x^2(x^2-625) + 1(x^2-625)$
$(x^2+1)(x^2-625)$
$(x^2+1)(x-25)(x+25)$

13.

$B + G = 20 \Rightarrow B = 20 - G$

$$\begin{array}{rrrrrrrrr} & G & - & 4 & = & 2(B & - & 4) & \\ & G & - & 4 & = & 2B & - & 8 & \\ \\ & G & - & 4 & = & 2(20 & - & G) & - & 8 \\ & G & - & 4 & = & 40 & - & 2G & - & 8 \\ & G & - & 4 & = & 32 & - & 2G & \\ + & 2G & + & 4 & & 4 & + & 2G & \\ & & & 3G & = & 36 & & & \\ & & & G & = & 12 & & & \\ \\ & & & B & = & 20 & - & G & \\ & & & B & = & 20 & - & 12 & = & 8 \end{array}$$

14.

$x = 16\%$ solution

$$\begin{array}{rrrrrrrr} & 16x & + & 6(20) & = & 12(x & + & 20) \\ & 16x & + & 120 & = & 12x & + & 240 \\ - & 12x & - & 120 & & -12x & - & 120 \\ & & & 4x & = & 120 & & \\ \\ & & & x & = & \dfrac{120}{4} & = & 30 \text{ ml} \end{array}$$

15.

$$
\begin{array}{rrcrcl}
 & (C & + & R & = & 60)(-3.40) \\
 & & & & & \\
 & -3.40C & - & 3.40R & = & -204 \\
+ & 3.40C & + & 3.90R & = & 213 \\
 & & & 0.50R & = & 9 \\
 & & & & & \\
 & & & R & = & \dfrac{9}{0.50} \text{ or } 18 \text{ kg} \\
 & & & & & \\
 & C & + & R & = & 60 \\
 & C & + & 18 & = & 60 \\
 & & - & 18 & & -18 \\
 & & & C & = & 42 \text{ kg}
\end{array}
$$

Answer Key 8.1

1. $\dfrac{4(4)+2}{6} \Rightarrow \dfrac{16+2}{6} \Rightarrow \dfrac{18}{6} \Rightarrow 3$
2. $\dfrac{-2-3}{3(-2)-9} \Rightarrow \dfrac{-5}{-6-9} \Rightarrow \dfrac{-5}{-15} \Rightarrow \dfrac{1}{3}$
3. $\dfrac{-4-3}{(-4)^2-4(-4)+3} \Rightarrow \dfrac{-7}{16+16+3} \Rightarrow -\dfrac{7}{35} \Rightarrow -\dfrac{1}{5}$
4. $\dfrac{-1+2}{(-1)^2+3(-1)+2} \Rightarrow \dfrac{1}{1-3+2} \Rightarrow \dfrac{1}{0} \Rightarrow \text{Undefined}$
5. $\dfrac{b+2}{b^2+4b+4} \Rightarrow \dfrac{2}{4} \Rightarrow \dfrac{1}{2}$
6. $\dfrac{(4)^2-4-6}{4-3} \Rightarrow \dfrac{16-10}{1} \Rightarrow \dfrac{6}{1} \Rightarrow 6$
7. $$\begin{array}{rrrrr} k & + & 10 & \neq & 0 \\ & - & 10 & & -10 \\ & & k & \neq & -10 \end{array}$$
8. $$\begin{array}{rrrrr} & & 18p(p & - & 2) \\ \\ & & 18p & \neq & 0 \\ & & p & \neq & 0 \\ \\ p & - & 2 & \neq & 0 \\ & + & 2 & & +2 \\ & & p & \neq & 2 \end{array}$$
9. $$\begin{array}{rrr} 10m & \neq & 0 \\ m & \neq & 0 \end{array}$$
10. $2(3x+10) \Rightarrow$ $$\begin{array}{rrrrr} 3x & + & 10 & \neq & 0 \\ & - & 10 & & -10 \\ & & 3x & \neq & -10 \\ \\ & & x & \neq & -\dfrac{10}{3} \end{array}$$
11. $$\begin{array}{rrr} 5(r & + & 2) \\ r & \neq & -2 \end{array}$$
12. $$\begin{array}{rrr} 3n(2n & + & 1) \\ n & \neq & 0 \\ \\ n & \neq & -\dfrac{1}{2} \end{array}$$

13. $(b-4)(b+8)$

$$\begin{aligned} b &\neq 4 \\ b &\neq -8 \end{aligned}$$

14. $5v(7v - 1)$

$$\begin{aligned} v &\neq 0 \\ v &\neq \frac{1}{7} \end{aligned}$$

15. $\dfrac{21x^2}{18x} \Rightarrow \dfrac{3\cdot 7\cdot x\cdot x}{3\cdot 6\cdot x} \Rightarrow \dfrac{7x}{6}$

16. $\dfrac{12n}{4n^2} \Rightarrow \dfrac{3\cdot 4\cdot n}{4\cdot n\cdot n} \Rightarrow \dfrac{3}{n}$

17. $\dfrac{24a}{40a^2} \Rightarrow \dfrac{3\cdot 8\cdot a}{5\cdot 8\cdot a\cdot a} \Rightarrow \dfrac{3}{5a}$

18. $\dfrac{21k}{24k^2} \Rightarrow \dfrac{3\cdot 7\cdot k}{3\cdot 8\cdot k} \Rightarrow \dfrac{7}{8k}$

19. $\dfrac{18m-24}{60} \Rightarrow \dfrac{6(3m-4)}{6(10)} \Rightarrow \dfrac{3m-4}{10}$

20. $\dfrac{n-9}{9n-81} \Rightarrow \dfrac{n-9}{9(n-9)} \Rightarrow \dfrac{1}{9}$

21. $\dfrac{x+1}{x^2+8x+7} \Rightarrow \dfrac{x+1}{(x+1)(x+7)} \Rightarrow \dfrac{1}{x+7}$

22. $\dfrac{28m+12}{36} \Rightarrow \dfrac{4(7m+3)}{4(9)} \Rightarrow \dfrac{7m+3}{9}$

23. $\dfrac{n^2+4n-12}{n^2-7n+10} \Rightarrow \dfrac{(n+6)(n-2)}{(n-5)(n-2)} \Rightarrow \dfrac{n+6}{n-5}$

24. $\dfrac{b^2+14b+48}{b^2+15b+56} \Rightarrow \dfrac{(b+8)(b+6)}{(b+8)(b+7)} \Rightarrow \dfrac{b+6}{b+7}$

25. $\dfrac{9v+54}{v^2-4v-60} \Rightarrow \dfrac{9(v-6)}{(v-10)(v+6)} \Rightarrow \dfrac{9}{v-10}$

26. $\dfrac{k^2-12k+32}{k^2-64} \Rightarrow \dfrac{(k-8)(k-4)}{(k-8)(k+8)} \Rightarrow \dfrac{k-4}{k+8}$

27. $\dfrac{2n^2+19n-10}{9n+90} \Rightarrow \dfrac{(2n-1)(n+10)}{9(n+10)} \Rightarrow \dfrac{2n-1}{9}$

28. $\dfrac{3x^2-29x+40}{5x^2-30x-80} \Rightarrow \dfrac{(3x-5)(x-8)}{5(x+2)(x-8)} \Rightarrow \dfrac{3x-5}{5(x+2)}$

29. $\dfrac{2x^2-10x+8}{3x^2-7x+4} \Rightarrow \dfrac{2(x-4)(x-1)}{(3x-4)(x-1)} \Rightarrow \dfrac{2(x-4)}{3x-4}$

30. $\dfrac{7n^2-32n+16}{4n-16} \Rightarrow \dfrac{(7n-4)(n-4)}{4(n-4)} \Rightarrow \dfrac{7n-4}{4}$

31. $\dfrac{7a^2-26a-45}{6a^2-34a+20} \Rightarrow \dfrac{(a-5)(7a+9)}{2(3a-2)(a-5)} \Rightarrow \dfrac{7a+9}{2(3a-2)}$

32. $\dfrac{4k^3-2k^2-2k}{k^3-18k^2+9k} \Rightarrow \dfrac{2k(2k^2-k-1)}{k(k^2-18k+9)} \Rightarrow \dfrac{2(2k^2-k-1)}{k^2-18k+9}$

Answer Key 8.2

1. $\frac{8x^2}{9}\cdot\frac{9}{2}\Rightarrow\frac{2\cdot4\cdot x^2}{9}\cdot\frac{9}{2}\Rightarrow 4x^2$
2. $\frac{8x}{3}\div\frac{4x}{7}\Rightarrow\frac{8x}{3}\cdot\frac{7}{4x}\Rightarrow\frac{2\cdot4\cdot x}{3}\cdot\frac{7}{4\cdot x}\Rightarrow\frac{14}{3}$
3. $\frac{5x^2}{4}\cdot\frac{6}{5}\Rightarrow\frac{5\cdot x^2}{2\cdot2}\cdot\frac{2\cdot3}{5}\Rightarrow\frac{3x^2}{2}$
4. $\frac{10p}{5}\div\frac{8}{10}\Rightarrow\frac{10p}{5}\cdot\frac{10}{8}\Rightarrow\frac{2\cdot5\cdot p}{5}\cdot\frac{2\cdot5}{2\cdot2\cdot2}\Rightarrow\frac{5p}{2}$
5. $\frac{(m-6)}{7(7m-5)}\cdot\frac{5m(7m-5)}{m-6}\Rightarrow\frac{5m}{7}$
6. $\frac{7(n-2)}{10(n+3)}\div\frac{n-2}{(n+3)}\Rightarrow\frac{7(n-2)}{10(n+3)}\cdot\frac{(n+3)}{n-2}\Rightarrow\frac{7}{10}$
7. $\frac{7r}{7r(r+10)}\div\frac{r-6}{(r-6)^2}\Rightarrow\frac{7r}{7r(r+10)}\cdot\frac{(r-6)^2}{r-6}\Rightarrow\frac{7r}{7r(r+10)}\cdot\frac{(r-6)(r-6)}{r-6}\Rightarrow$
$\frac{r-6}{r+10}$
8. $\frac{6x(x+4)}{(x-3)}\cdot\frac{(x-3)(x-6)}{6x(x-6)}\Rightarrow\frac{6x(x+4)}{(x-3)}\cdot\frac{(x-3)(x-6)}{6x(x-6)}\Rightarrow x+4$
9. $\frac{x-10}{35x+21}\div\frac{7}{35x+21}\Rightarrow\frac{x-10}{7(5x+3)}\cdot\frac{7(5x+3)}{7}\Rightarrow\frac{x-10}{7}$
10. $\frac{v-1}{4}\cdot\frac{4}{v^2-11v+10}\Rightarrow\frac{v-1}{4}\Rightarrow\frac{4}{(v-1)(v-10)}\Rightarrow\frac{1}{v-10}$
11. $\frac{x^2-6x-7}{x+5}\cdot\frac{x+5}{x-7}\Rightarrow\frac{(x-7)(x+1)}{(x+5)}\cdot\frac{(x+5)}{(x-7)}\Rightarrow x+1$
12. $\frac{1}{a-6}\cdot\frac{8a+80}{8}\Rightarrow\frac{1}{a-6}\cdot\frac{8(a+10)}{8}\Rightarrow\frac{a+10}{a-6}$
13. $\frac{4m+36}{m+9}\cdot\frac{m-5}{5m^2}\Rightarrow\frac{4(m+9)}{m+9}\cdot\frac{m-5}{5m^2}\Rightarrow\frac{4(m-5)}{5m^2}$
14. $\frac{2r}{r+6}\div\frac{2r}{74+42}\Rightarrow\frac{2r}{r+6}\cdot\frac{7(r+6)}{2r}\Rightarrow 7$
15. $\frac{n-7}{6n-12}\cdot\frac{12-6n}{n^2-13n+42}\Rightarrow\frac{(n-7)}{6(n-2)}\cdot\frac{6(2-n)}{(n-6)(n-7)}\Rightarrow\frac{-1(n-2)}{(n-2)(n-6)}\Rightarrow$
$\frac{-1}{n-6}$
16. $\frac{x^2+11x+24}{6x^3+18x^2}\cdot\frac{6x^3+6x^2}{x^2+5x-24}\Rightarrow\frac{(x+3)(x+8)}{6x^2(x+3)}\cdot\frac{6x^2(x+1)}{(x+8)(x-3)}\Rightarrow\frac{x+1}{x-3}$
17. $\frac{27a+36}{9a+63}\div\frac{6a+8}{2}\Rightarrow\frac{9(3a+4)}{9(a+7)}\cdot\frac{2}{2(3a+4)}\Rightarrow\frac{1}{a+7}$
18. $\frac{k-7}{k^2-k-12}\cdot\frac{7k^2-28k}{8k^2-56k}\Rightarrow\frac{k-7}{(k-4)(k+3)}\cdot\frac{7\cdot k(k-4)}{8\cdot k(k-7)}\Rightarrow\frac{7}{8(k+3)}$

19. $\dfrac{x^2-12x+32}{x^2-6x-16} \cdot \dfrac{7x^2+14x}{7x^2+21x} \Rightarrow \dfrac{(x-8)(x-4)}{(x-8)(x+2)} \cdot \dfrac{7x(x+2)}{7x(x+3)} \Rightarrow \dfrac{x-4}{x+3}$

20. $\dfrac{9x^3+54x^2}{x^2+5x-14} \cdot \dfrac{x^2+5x-14}{10x^2} \Rightarrow \dfrac{9x^2(x+6)}{10x^2} \Rightarrow \dfrac{9(x+6)}{10}$

21. $(10m^2+100m) \cdot \dfrac{18m^3-36m^2}{20m^2-40m} \Rightarrow 10m(m+10) \cdot \dfrac{2 \cdot 9m^2(m-2)}{2 \cdot 10m(m-2)} \Rightarrow$
$9m^2(m+10)$

22. $\dfrac{n-7}{n^2-2n-35} \div \dfrac{9n+54}{10n+50} \Rightarrow \dfrac{n-7}{(n-7)(n+5)} \cdot \dfrac{10(n+5)}{9(n+6)} \Rightarrow \dfrac{10}{9(n+6)}$

23. $\dfrac{x^2-1}{2x-4} \cdot \dfrac{x^2-4}{x^2-x-2} \div \dfrac{x^2+x-2}{3x-6} \Rightarrow$
$\dfrac{(x-1)(x+1)}{2(x-2)} \cdot \dfrac{(x+2)(x-2)}{(x-2)(x+1)} \cdot \dfrac{3(x-2)}{(x+2)(x-1)} \Rightarrow \dfrac{3}{2}$

24. $\dfrac{a^3+b^3}{a^2+3ab+2b^2} \cdot \dfrac{3a-6b}{3a^2-3ab+3b^2} \div \dfrac{a^2-4b^2}{a+2b} \Rightarrow$
$\dfrac{(a+b)(a^2-ab+b^2)}{(a+2b)(a+b)} \cdot \dfrac{3(a-2b)}{3(a^2-ab+b^2)} \cdot \dfrac{a+2b}{(a-2b)(a+2b)} \Rightarrow \dfrac{1}{a+2b}$

Answer Key 8.3

1. $12a^4b^5$
2. $25x^3y^5z$
3. $x(x-3)$
4. $4(x-2)$
5. $(x+2)(x-4)$
6. $x(x-7)(x+1)$
7. $(x+5)(x-5)$
8. $(x+3)(x-3)^2$
9. $(x+1)(x+2)(x+3)$
10. $(x-5)(x-2)(x+3)$
11. $$\text{LCD} = 10a^3b^2$$
$$\frac{3a}{5b^2}\cdot\frac{2a^3}{2a^3} \Rightarrow \frac{6a^4}{10a^3b^2}$$
$$\frac{2}{10a^3b}\cdot\frac{b}{b} \Rightarrow \frac{2b}{10a^3b^2}$$
12. $$\text{LCD} = (x-4)(x+2)$$
$$\frac{3x}{(x-4)}\cdot\frac{(x+2)}{(x+2)} \Rightarrow \frac{3x^2+6x}{(x-4)(x+2)}$$
$$\frac{2}{(x+2)}\cdot\frac{(x-4)}{(x-4)} \Rightarrow \frac{2x-8}{(x-4)(x+2)}$$
13. $$\text{LCD} = (x-3)(x+2)$$
$$\frac{(x+2)}{(x-3)}\cdot\frac{(x+2)}{(x+2)} \Rightarrow \frac{x^2+4x+4}{(x-3)(x+2)}$$
$$\frac{(x-3)}{(x+2)}\cdot\frac{(x-3)}{(x-3)} \Rightarrow \frac{x^2-6x+9}{(x-3)(x+2)}$$

14.

$$\begin{aligned} \text{LCD} &= x(x-6) \\ \frac{5}{x^2-6x} &\Rightarrow \frac{5}{x(x-6)} \\ \frac{2}{x}\cdot\frac{(x-6)}{(x-6)} &\Rightarrow \frac{2x-12}{x(x-6)} \\ \frac{-3}{(x-6)}\cdot\frac{x}{x} &\Rightarrow \frac{-3x}{x(x-6)} \end{aligned}$$

15.

$$\begin{aligned} \text{LCD} &= (x-4)^2(x+4) \\ \frac{x}{x^2-16}\cdot\frac{(x-4)}{(x-4)} &\Rightarrow \frac{x^2-4x}{(x-4)^2(x+4)} \\ \frac{3x}{(x^2-8x+16)}\cdot\frac{(x+4)}{(x+4)} &\Rightarrow \frac{3x^2+12}{(x-4)^2(x+4)} \end{aligned}$$

16.

$$\begin{aligned} \text{LCD} &= (x-5)(x+2) \\ \frac{5x+1}{x^2-3x-10} &\Rightarrow \frac{5x+1}{(x-5)(x+2)} \\ \frac{4}{(x-5)}\cdot\frac{(x+2)}{(x+2)} &\Rightarrow \frac{4x+8}{(x-5)(x+2)} \end{aligned}$$

17.

$$\begin{aligned} \text{LCD} &= (x+6)^2(x-6) \\ \frac{x+1}{x^2-36}\cdot\frac{(x+6)}{(x+6)} &\Rightarrow \frac{x^2+7x+6}{(x+6)^2(x-6)} \\ \frac{(2x+3)}{(x^2+12x+36)}\cdot\frac{(x-6)}{(x-6)} &\Rightarrow \frac{2x^2-9x-18}{(x+6)^2(x-6)} \end{aligned}$$

18.

$$\begin{aligned} \text{LCD} &= (x-4)(x+3)(x+1) \\ \frac{(3x+1)}{(x^2-x-12)}\cdot\frac{(x+1)}{(x+1)} &\Rightarrow \frac{3x^2+4x+1}{(x-4)(x+3)(x+1)} \\ \frac{2x}{(x^2+4x+3)}\cdot\frac{(x-4)}{(x-4)} &\Rightarrow \frac{2x^2-8x}{(x-4)(x+3)(x+1)} \end{aligned}$$

19.
$$\begin{aligned} \text{LCD} &= (x-3)(x+2) \end{aligned}$$

$$\frac{4x}{x^2-x-6} \Rightarrow \frac{4x}{(x-3)(x+2)}$$

$$\frac{(x+2)}{(x-3)} \cdot \frac{(x+2)}{(x+2)} \Rightarrow \frac{x^2+4x+4}{(x-3)(x+2)}$$

20.
$$\text{LCD} = (x-4)(x-2)(x+5)$$

$$\frac{3x}{x^2-6x+8} \cdot \frac{(x+5)}{(x+5)} \Rightarrow \frac{3x^2+15x}{(x-4)(x-2)(x+5)}$$

$$\frac{(x-2)}{(x^2+x-20)} \cdot \frac{(x-2)}{(x-2)} \Rightarrow \frac{x^2-4x+4}{(x-4)(x-2)(x+5)}$$

$$\frac{5}{(x^2+3x-10)} \cdot \frac{(x-4)}{(x-4)} \Rightarrow \frac{5x-20}{(x-4)(x-2)(x+5)}$$

Answer Key 8.4

1. $\dfrac{2+4}{a+3}=\dfrac{6}{a+3}$
2. $\dfrac{x^2-(6x-8)}{x-2}\Rightarrow\dfrac{x^2-6x+8}{x-2}\Rightarrow\dfrac{(x-4)(x-2)}{(x-2)}\Rightarrow x-4$
3. $\dfrac{t^2+4t+2t-7}{t-1}\Rightarrow\dfrac{t^2+6t-7}{t-1}\Rightarrow\dfrac{(t+7)(t-1)}{(t-1)}\Rightarrow t+7$
4. $\dfrac{a^2+3a-4}{a^2+5a-6}\Rightarrow\dfrac{(a+4)(a-1)}{(a+6)(a-1)}\Rightarrow\dfrac{a+4}{a+6}$
5. $\text{LCD}=24r$ $\quad\dfrac{5}{6r}\cdot\dfrac{4}{4}-\dfrac{5}{8r}\cdot\dfrac{3}{3}\Rightarrow\dfrac{20}{24r}-\dfrac{15}{24r}\Rightarrow\dfrac{5}{24r}$
6. $\text{LCD}=x^2y^2$ $\quad\dfrac{7}{xy^2}\cdot\dfrac{x}{x}+\dfrac{3}{x^2y}\cdot\dfrac{y}{y}\Rightarrow\dfrac{7x+3y}{x^2y^2}$
7. $\text{LCD}=18t^3$ $\quad\dfrac{8}{9t^3}\cdot\dfrac{2}{2}+\dfrac{5}{6t^2}\cdot\dfrac{3t}{3t}\Rightarrow\dfrac{15t+16}{18t^3}$
8. $\text{LCD}=24$ $\quad\dfrac{(x+5)(3)}{(8)(3)}+\dfrac{(x-3)(2)}{(12)(2)}\Rightarrow\dfrac{3x+15+2x-6}{24}\Rightarrow\dfrac{5x+9}{24}$
9. $\text{LCD}=4x$ $\quad\dfrac{x-1}{4x}-\dfrac{4(2x+3)}{4\cdot x}\Rightarrow\dfrac{x-1-8x-12}{4x}\Rightarrow\dfrac{-7x-13}{4x}$
10. $\text{LCD}=c^2d^2$ $\quad\dfrac{(2c-d)(d)}{c^2d(d)}-\dfrac{(c+d)(c)}{cd^2(c)}\Rightarrow\dfrac{2cd-d^2-c^2-cd}{c^2d^2}\Rightarrow\dfrac{cd-c^2-d^2}{c^2d^2}$
11. $\text{LCD}=2x^2y^2$ $\quad\dfrac{(5x+3y)(y)}{(2x^2y)(y)}-\dfrac{(3x+4y)(2x)}{(xy^2)(2x)}\Rightarrow\dfrac{5xy+3y^2-6x^2-8xy}{2x^2y^2}\Rightarrow$
$\dfrac{3y^2-3xy-6x^2}{2x^2y^2}$
12. $\text{LCD}=(x-1)(x+1)$ $\quad\dfrac{2(x+1)}{(x-1)(x+1)}+\dfrac{2(x-1)}{(x+1)(x-1)}\Rightarrow\dfrac{2x+2+2x-2}{(x+1)(x-1)}\Rightarrow$
$\dfrac{4x}{(x+1)(x-1)}$
13. $\text{LCD}=(x+3)(x+2)(x+1)$ $\quad\dfrac{x(x+1)}{(x+3)(x+2)(x+1)}-\dfrac{2(x+3)}{(x+3)(x+2)(x+1)}\Rightarrow$
$\dfrac{x^2+x-2x-6}{(x+3)(x+2)(x+1)}\Rightarrow\dfrac{x^2-x-6}{(x+3)(x+2)(x+1)}\Rightarrow\dfrac{(x-3)(x+2)}{(x+3)(x+2)(x+1)}\Rightarrow$
$\dfrac{x-3}{(x+3)(x+1)}$
14. $\text{LCD}=(x-1)(x+1)(x+4)$ $\quad\dfrac{2x(x+4)}{(x-1)(x+1)(x+4)}-\dfrac{3(x-1)}{(x-1)(x+1)(x+4)}\Rightarrow$

$$\frac{2x^2+8x-3x+3}{(x-1)(x+1)(x+4)} \Rightarrow \frac{2x^2+5x+3}{(x-1)(x+1)(x+4)} \Rightarrow \frac{(2x+3)(x+1)}{(x-1)(x+1)(x+4)} \Rightarrow$$

$$\frac{2x+3}{(x-1)(x+4)}$$

15. LCD $= (x+7)(x+8)(x+6)$ $\quad \frac{x(x+6)}{(x+7)(x+8)(x+6)} - \frac{7(x+8)}{(x+7)(x+8)(x+6)} \Rightarrow$

$$\frac{x^2+6x-7x-56}{(x+7)(x+8)(x+6)} \Rightarrow \frac{x^2-x-56}{(x+7)(x+8)(x+6)} \Rightarrow \frac{(x-8)(x+7)}{(x+7)(x+8)(x+6)} \Rightarrow$$

$$\frac{x-8}{(x+8)(x+6)}$$

16. LCD $= (x-3)(x+3)(x-2)$ $\quad \frac{2x(x-2)}{(x-3)(x+3)(x-2)} + \frac{5(x-3)}{(x-3)(x+3)(x-2)} \Rightarrow$

$$\frac{2x^2-4x+5x-15}{(x-3)(x+3)(x-2)} \Rightarrow \frac{2x^2+x-15}{(x-3)(x+3)(x-2)} \Rightarrow \frac{(x+3)(2x-5)}{(x-3)(x+3)(x-2)} \Rightarrow$$

$$\frac{2x-5}{(x-3)(x-2)}$$

17. LCD $= (x-3)(x+2)(x+3)$ $\quad \frac{5x(x+3)}{(x-3)(x+2)(x+3)} - \frac{18(x+2)}{(x-3)(x+2)(x+3)} \Rightarrow$

$$\frac{5x^2+15x-18x-36}{(x-3)(x+2)(x+3)} \Rightarrow \frac{5x^2-3x-36}{(x-3)(x+2)(x+3)} \Rightarrow \frac{(x-3)(5x+12)}{(x-3)(x+2)(x+3)} \Rightarrow$$

$$\frac{5x+12}{(x+2)(x+3)}$$

18. LCD $= (x-3)(x+1)(x-2)$ $\quad \frac{4x(x-2)}{(x-3)(x+1)(x-2)} - \frac{3(x+1)}{(x-3)(x+1)(x-2)} \Rightarrow$

$$\frac{4x^2-8x-3x-3}{(x-3)(x+1)(x-2)} \Rightarrow \frac{4x^2-11x-3}{(x-3)(x+1)(x-2)} \Rightarrow \frac{(4x+1)(x-3)}{(x-3)(x+1)(x-2)} \Rightarrow$$

$$\frac{4x+1}{(x+1)(x-2)}$$

Answer Key 8.5

1. $$\frac{\left(1+\frac{1}{x}\right)x^2}{\left(1-\frac{1}{x^2}\right)x^2} \Rightarrow \frac{x^2+x}{x^2-1} \Rightarrow \frac{x(x+1)}{(x+1)(x-1)} \Rightarrow \frac{x}{x-1}$$

2. $$\frac{\left(1-\frac{1}{y^2}\right)y^2}{\left(1+\frac{1}{y}\right)y^2} \Rightarrow \frac{y^2-1}{y^2+y} \Rightarrow \frac{(y-1)(y+1)}{y(y+1)} \Rightarrow \frac{y-1}{y}$$

3. $$\frac{\left(\frac{a}{b}+2\right)b^2}{\left(\frac{a^2}{b^2}-4\right)b^2} \Rightarrow \frac{ab+2b^2}{a^2-4b^2} \Rightarrow \frac{b(a+2b)}{(a+2b)(a-2b)} \Rightarrow \frac{b}{a-2b}$$

4. $$\frac{\left(\frac{1}{y^2}-9\right)y^2}{\left(\frac{1}{y}+3\right)y^2} \Rightarrow \frac{1-9y^2}{y+3y^2} \Rightarrow \frac{(1-3y)(1+3y)}{y(1+3y)} \Rightarrow \frac{1-3y}{y}$$

5. $$\frac{\left(\frac{1}{a^2}-\frac{1}{a}\right)a^2}{\left(\frac{1}{a^2}+\frac{1}{a}\right)a^2} \Rightarrow \frac{1-a}{1+a}$$

6. $$\frac{\left(\frac{1}{b}+\frac{1}{2}\right)2b(b^2-1)}{\left(\frac{4}{b^2-1}\right)2b(b^2-1)} \Rightarrow \frac{2b(b^2-1)+b(b^2-1)}{4(2b)} \Rightarrow \frac{2b^2-2+b^3-b}{8b} \Rightarrow$$
$$\frac{b^3+2b^2-b-2}{8b} \Rightarrow \frac{(b-1)(b+1)(b+2)}{8b}$$

7. $$\frac{\left(x+2-\frac{9}{x+2}\right)(x+2)}{\left(x+1+\frac{x-7}{x+2}\right)(x+2)} \Rightarrow \frac{(x+2)(x+2)-9}{(x+1)(x+2)+x-7} \Rightarrow \frac{x^2+4x+4-9}{x^2+3x+2+x-7} \Rightarrow$$
$$\frac{x^2+4x-5}{x^2+4x-5} \Rightarrow 1$$

8. $$\frac{\left(a-3+\frac{a-3}{a+2}\right)(a+2)}{\left(a+4-\frac{4a+5}{a+2}\right)(a+2)} \Rightarrow \frac{(a-3)(a+2)+a-3}{(a+4)(a+2)-4a+5} \Rightarrow \frac{a^2-a-6+a-3}{a^2+6a+8-4a+5} \Rightarrow$$

$$\frac{a^2-9}{a^2+2a+13} \Rightarrow \frac{(a-3)(a+3)}{a^2+2a+13}$$

$$\frac{\left(\dfrac{x+y}{y}+\dfrac{y}{x-y}\right)y(x-y)}{\left(\dfrac{y}{x-y}\right)y(x-y)} \Rightarrow \frac{(x+y)(x-y)+y(y)}{y(y)} \Rightarrow$$

9.
$$\frac{x^2-y^2+y^2}{y^2} \Rightarrow \frac{x^2}{y^2}$$

$$\frac{\left(\dfrac{a-b}{a}-\dfrac{a}{a+b}\right)a(a+b)}{\left(\dfrac{b^2}{a+b}\right)a(a+b)} \Rightarrow \frac{(a-b)(a+b)-a(a)}{b^2(a)} \Rightarrow$$

10.
$$\frac{a^2-b^2-a^2}{ab^2} \Rightarrow \frac{-b^2}{ab^2}$$

$$\frac{\left(\dfrac{x-y}{y}+\dfrac{x+y}{x-y}\right)y(x-y)}{\left(\dfrac{y}{x-y}\right)y(x-y)} \Rightarrow \frac{(x-y)(x-y)+(x+y)(y)}{y(y)} \Rightarrow$$

11.
$$\frac{x^2-2xy+y^2+xy+y^2}{y^2} \Rightarrow \frac{x^2-xy+2y^2}{y^2}$$

$$\frac{\left(\dfrac{x-2}{x+2}-\dfrac{x+2}{x-2}\right)(x+2)(x-2)}{\left(\dfrac{x-2}{x+2}+\dfrac{x+2}{x-2}\right)(x+2)(x-2)} \Rightarrow \frac{x^2-4x+4-(x^2+4x+4)}{x^2-4x+4+x^2+4x+4} \Rightarrow \frac{-8x}{2x^2+8} \Rightarrow$$

12.
$$\frac{2(-4x)}{2(x^2+4)} \Rightarrow \frac{-4x}{x^2+4}$$

Answer Key 8.6

1. $$\mathrm{LCD} = 5(2)$$

$$\begin{aligned} 2(m - 1) &= 5(8) \\ 2m - 2 &= 40 \\ +2 &\quad +2 \\ \frac{2m}{2} &= \frac{42}{2} \\ m &= 21 \end{aligned}$$

2. $$\mathrm{LCD} = 2(x - 8)$$

$$\begin{aligned} 8(x - 8) &= 2(8) \\ 8x - 64 &= 16 \\ +64 &\quad +64 \\ \frac{8x}{8} &= \frac{80}{8} \\ x &= 10 \end{aligned}$$

3. $$\mathrm{LCD} = 9(p - 4)$$

$$\begin{aligned} 2(p - 4) &= 9(10) \\ 2p - 8 &= 90 \\ +8 &\quad +8 \\ \frac{2p}{2} &= \frac{98}{2} \\ p &= 49 \end{aligned}$$

4. $$\mathrm{LCD} = 9(n + 2)$$

$$\begin{aligned} 9(9) &= 3(n + 2) \\ 81 &= 3n + 6 \\ -6 &\quad -6 \\ \frac{75}{3} &= \frac{3n}{3} \\ n &= 25 \end{aligned}$$

5. $\text{LCD} = 10(a + 2)$

$$\begin{aligned} 3(a + 2) &= 10(a) \\ 3a + 6 &= 10a \\ -3a \qquad &\quad -3a \\ \frac{6}{7} &= \frac{7a}{7} \\ a &= \frac{6}{7} \end{aligned}$$

6. $\text{LCD} = 3(4)$

$$\begin{aligned} 4(x + 1) &= 3(x + 3) \\ 4x + 4 &= 3x + 9 \\ -3x - 4 &\quad -3x - 4 \\ x &= 5 \end{aligned}$$

7. $\text{LCD} = 3(p + 4)$

$$\begin{aligned} 2(3) &= (p + 4)(p + 5) \\ 6 &= p^2 + 9p + 20 \\ -6 &\qquad\qquad\quad - 6 \\ 0 &= p^2 + 9p + 14 \\ 0 &= (p + 7)(p + 2) \\ p &= -2, \ -7 \end{aligned}$$

8. $\text{LCD} = 10(n + 1)$

$$\begin{aligned} 5(10) &= (n - 4)(n + 1) \\ 50 &= n^2 - 3n - 4 \\ -50 &\qquad\qquad\quad - 50 \\ 0 &= n^2 - 3n - 54 \\ 0 &= (n - 9)(n + 6) \\ n &= 9, \ -6 \end{aligned}$$

9. $\text{LCD} = 5(x-2)$

$$\begin{aligned}(x+5)(x-2) &= 5(6)\\ x^2+3x-10 &= 30\\ -30 &\quad -30\\ x^2+3x-40 &= 0\\ (x-5)(x+8) &= 0\\ x &= 5,-7\end{aligned}$$

10. $\text{LCD} = 5(x-3)$

$$\begin{aligned}20 &= (x-3)(x+5)\\ 20 &= x^2+2x-15\\ -20 &\quad\quad -20\\ 0 &= x^2+2x-35\\ 0 &= (x-5)(x+7)\\ x &= 5,\ -7\end{aligned}$$

11. $\text{LCD} = 4(m-4)$

$$\begin{aligned}(m+3)(m-4) &= 4(11)\\ m^2-m-12 &= 44\\ -44 &\quad -44\\ (m^2-m-56) &= 0\\ (m-8)(m+7) &= 0\\ m &= 8,-7\end{aligned}$$

12. $\text{LCD} = 8(x-1)$

$$\begin{aligned}(x-5)(x-1) &= 4(8)\\ x^2-6x+5 &= 32\\ -32 &\quad -32\\ x^2-6x-27 &= 0\\ (x-9)(x+3) &= 0\\ x &= 9,-3\end{aligned}$$

Answer Key 8.7

1. $\text{LCD} = 2(x)$

$$\begin{array}{rcl} 3x(2x) - x - 2 &=& 0 \\ 6x^2 - x - 2 &=& 0 \\ (3x - 2)(2x + 1) &=& 0 \\ \\ x &=& \dfrac{2}{3}, -\dfrac{1}{2} \end{array}$$

2. $\text{LCD} = x + 1$

$$\begin{array}{rcl} (x + 1)(x + 1) &=& 4 \\ x^2 + 2x + 1 &=& 4 \\ -4 && -4 \\ x^2 + 2x - 3 &=& 0 \\ (x - 1)(x + 3) &=& 0 \\ \\ x &=& 1, -3 \end{array}$$

3. $\text{LCD} = x - 4$

$$\begin{array}{rcl} x(x - 4) + 20 &=& 5x - 2(x - 4) \\ x^2 - 4x + 20 &=& 5x - 2x + 8 \\ -3x - 8 && -3x - 8 \\ x^2 - 7x + 12 &=& 0 \\ (x - 4)(x - 3) &=& 0 \\ \\ x &=& 3, \quad 4 \end{array}$$

4. $\text{LCD} = x - 1$

$$\begin{array}{rcl} x^2 + 6 + x - 2 &=& 2x(x - 1) \\ x^2 + x + 4 &=& 2x^2 - 2x \\ -2x^2 + 2x && -2x^2 + 2x \\ -x^2 + 3x + 4 &=& 0 \\ x^2 - 3x - 4 &=& 0 \\ (x - 4)(x + 1) &=& 0 \\ \\ x &=& 4, 1 \end{array}$$

5. $\text{LCD} = x - 3$

$$\begin{aligned} x(x-3)+6 &= 2x \\ x^2-3x+6 &= 2x \\ -2x \quad &\quad -2x \\ x^2-5x+6 &= 0 \\ (x-3)(x-2) &= 0 \\ \\ x &= 2,3 \end{aligned}$$

6. $\text{LCD} = (x-1)(3-x)$

$$\begin{aligned} (x-4)(3-x) &= 12(x-1)+(x-1)(3-x) \\ -x^2+7x-12 &= 12x-12-x^2+4x-3 \\ +x^2-16x+15 &\quad -12x+12+x^2-4x+3 \\ -9x+3 &= 0 \\ 3 &= 9x \\ \\ x &= \frac{3}{9} \text{ or } \frac{1}{3} \end{aligned}$$

7. $\text{LCD} = (2m-5)(3m+1)(2)$

$$\begin{aligned} 3m(3m+1)(2)-7(2m-5)(2) &= 3(2m-5)(3m+1) \\ 18m^2+6m-28m+70 &= 18m^2-39m-15 \\ -18m^2 \quad +39m+15 &\quad -18m^2+39m+15 \\ 17m+85 &= 0 \\ -85 &\quad -85 \\ \frac{17m}{17} &= \frac{-85}{17} \\ \\ m &= -5 \end{aligned}$$

8. $\text{LCD} = (1-x)(3-x)$

$$\begin{aligned} (4-x)(3-x) &= 12(1-x) \\ 12-7x+x^2 &= 12-12x \\ -12+12x \quad &\quad -12+12x \\ x^2+5x &= 0 \\ x(x+5) &= 0 \\ \\ x &= 0, \ -5 \end{aligned}$$

9. $$\text{LCD} = 2(y-3)(y-4)$$

$$\begin{aligned}
7(2)(y-4) - 1(y-3)(y-4) &= (y-2)(2)(y-3)\\
14y - 56 - y^2 + 7y - 12 &= 2y^2 - 10y + 12\\
-y^2 + 21y - 68 &= 2y^2 - 10y + 12\\
-2y^2 + 10y - 12 & \quad -2y^2 + 10y - 12\\
-3y^2 + 31y - 80 &= 0\\
3y^2 - 31y + 80 &= 0\\
(y-5)(3y-16) &= 0\\
y &= 5, \frac{16}{3}
\end{aligned}$$

10. $$\text{LCD} = (x+2)(x-2)$$

$$\begin{aligned}
1(x-2) + 1(x+2) &= 3x + 8\\
x - 2 + x + 2 &= 3x + 8\\
-2x \quad & \quad -2x\\
0 &= x + 8\\
-8 \quad & \quad -8\\
x &= -8
\end{aligned}$$

11. $$\text{LCD} = (x+1)(x-1)(6)$$

$$\begin{aligned}
(x+1)(x+1)(6) - (x-1)(x-1)(6) &= 5(x+1)(x-1)\\
6(x^2+2x+1) - 6(x^2-2x+1) &= 5(x^2-1)\\
6x^2 + 12x + 6 - 6x^2 + 12x - 6 &= 5x^2 - 5\\
24x &= 5x^2 - 5\\
-24x \quad & \quad -24x\\
0 &= 5x^2 - 24x - 5\\
0 &= (5x+1)(x-5)\\
x &= 5, -\frac{1}{5}
\end{aligned}$$

12. $$\text{LCD} = (x+3)(x-2)$$

$$\begin{aligned}
(x-2)(x-2) - 1(x+3) &= 1\\
x^2 - 4x + 4 - x - 3 &= 1\\
-1 \quad & \quad -1\\
x^2 - 5x &= 0\\
x(x-5) &= 0\\
x &= 0, 5
\end{aligned}$$

13. $\text{LCD} = (x-1)(x+1)$

$$\begin{array}{rcl} x(x+1)-2(x-1) &=& 4x^2 \\ x^2+x-2x+2 &=& 4x^2 \\ -x^2+x-2 & & -x^2+x-2 \\ 0 &=& 3x^2+x-2 \\ 0 &=& (3x-2)(x+1) \\ 0 &=& \frac{2}{3},\ -1 \end{array}$$

14. $\text{LCD} = (x+2)(x-4)$

$$\begin{array}{rcl} 2x(x-4)+2(x+2) &=& 3x \\ 2x^2-8x+2x+4 &=& 3x \\ -3x & & -3x \\ 2x^2-9x+4 &=& 0 \\ (2x-1)(x-4) &=& 0 \\ x &=& \frac{1}{2}, 4 \end{array}$$

15. $\text{LCD} = (x+1)(x+5)$

$$\begin{array}{rcl} 2x(x+5)-3(x+1) &=& -8x^2 \\ 2x^2+10x-3x-3 &=& -8x^2 \\ +8x^2 & & +8x^2 \\ 10x^2+7x-3 &=& 0 \\ (10x-3)(x+1) &=& 0 \\ x &=& \frac{3}{10}, -1 \end{array}$$

Answer Key 8.8

1.

Who or What	Rate	Time	Equation
A	20	t	$20t$
B	25	t	$25t$

$$20t + 25t = 60$$

2.

Who or What	Rate	Time	Equation
A_1	r	6	$6r$
A_1	$r + s$	6	$6(r + s)$

$$6r + 6(r + s) = 276$$

3.

Who or What	Rate	Time	Equation
T_1	25	t	$25t$
T_2	40	t	$40t$

$$25t + 40t = 195$$

4.

Who or What	Rate	Time	Equation
J	20	5	$20(5)$
S	r	5	$r(5)$

$$20(5) + 5r = 150$$

5.

Who or What	Rate	Time	Equation
P	$r + 15$	4	$4(r + 15)$
F	r	4	$4r$

$$4(r + 15) + 4r = 300$$

6.

Who or What	Rate	Time	Equation
A_1	25	t	$25t$
A_2	35	t	$35t$

$$25t + 35t = 180$$

7.

Who or What	Rate	Time	Equation
Away	10	t	$10(t)$
Return	3	$10 - t$	$3(10 - t)$

$10t = 3(10 - t)$

8.

Who or What	Rate	Time	Equation
w	4	t	$4t$
r	20	$(3 - t)$	$20(3 - t)$

$4t = 20(3 - t)$

9.

Who or What	Rate	Time	Equation
Away	28	t	$28t$
Back	4	$2 - t$	$4(2 - t)$

$$\begin{array}{rcrcr} 28t & = & 4(2 & - & t) \\ 28t & = & 8 & - & 4t \\ +4t & & & + & 4t \\ \dfrac{32t}{32} & = & \dfrac{8}{32} & & \\ & & & & \\ t & = & \dfrac{1}{4} & & \\ & & & & \\ d & = & rt & & \\ d & = & 28(\dfrac{1}{4}) & & \\ d & = & 7\text{ km} & & \end{array}$$

10.

Who or What	Rate	Time	Equation
Leave	15	t	$15t$
Return	10	$5 - t$	$10(5 - t)$

$$\begin{array}{rcrcr} 15t & = & 10(5 & - & t) \\ 15t & = & 50 & - & 10t \\ +10t & & & + & 10t \\ \dfrac{25t}{25} & = & \dfrac{50}{25} & & \\ & & & & \\ t & = & 2 & & \\ & & & & \\ d & = & rt & & \\ d & = & 15(2) & & \\ d & = & 30\text{ km} & & \end{array}$$

11.

Who or What	Rate	Time	Equation
To resort	30	t	$30t$
Return	50	$8-t$	$50(8-t)$

$$\begin{array}{rcrcr} 30t & = & 50(8 & - & t) \\ 30t & = & 400 & - & 50t \\ +50t & & & + & 50t \\ \frac{80t}{80} & = & \frac{400}{80} & & \\ t & = & 5 & & \end{array}$$

$$\begin{array}{rcl} d & = & rt \\ d & = & 30(5) \\ d & = & 150 \text{ km} \end{array}$$

12.

Who or What	Rate	Time	Equation
To airport	90	t	$90t$
Return	120	$7-t$	$120(7-t)$

$$\begin{array}{rcrcr} 90t & = & 120(7 & - & t) \\ 90t & = & 840 & - & 120t \\ +120t & & & + & 120t \\ \frac{210t}{210} & = & \frac{840}{210} & & \\ t & = & 4 & & \end{array}$$

$$\begin{array}{rcl} d & = & rt \\ d & = & 90(4) \\ d & = & 360 \text{ km} \end{array}$$

13.

Who or What	Rate	Time	Equation
Sam	4	t	$4t$
Sue	6	$t-2$	$6(t-2)$

$$\begin{array}{rcrcr} 4t & = & 6(t & - & 2) \\ 4t & = & 6t & - & 12 \\ -6t & & -6t & & \\ \frac{-2t}{-2} & = & \frac{-12}{-2} & & \\ t & = & 6 & & \\ t-2 & = & 4 & & \end{array}$$

14.

Who or What	Rate	Time	Equation
M₁	5	t	$5t$
M₂	8	$t-6$	$8(t-6)$

$$\begin{array}{rcrcr} 5t & = & 8(t & - & 6) \\ 5t & = & 8t & - & 48 \\ -8t & & -8t & & \\ \frac{-3t}{-3} & = & \frac{-48}{-3} & & \\ t & = & 16 & & \\ t-6 & = & 10 & & \end{array}$$

15.

Who or What	Rate	Time	Equation
MB	8	t	$8t$
CC	16	$t-2$	$16(t-2)$

$$\begin{array}{rcrcr} 8t & = & 16(t & - & 2) \\ 8t & = & 16t & - & 32 \\ -16t & & -16t & & \\ \frac{-8t}{-8} & = & \frac{-32}{-8} & & \\ t & = & 4 & & \\ t-2 & = & 2 & & \end{array}$$

16.

Who or What	Rate	Time	Equation
R₁	6	t	$6t$
R₂	8	$t-1$	$8(t-1)$

$$\begin{aligned}
6t &= 8(t-1)\\
6t &= 8t-8\\
-8t & \quad -8t\\
\frac{-2t}{-2} &= \frac{-8}{-2}\\
t &= 4\\
t-1 &= 3
\end{aligned}$$

17.

Who or What	Rate	Time	Equation
M_1	20	t	$20t$
M_2	30	t	$30t$

$$\begin{aligned}
20t + 30t &= 300\\
\frac{50t}{50} &= \frac{300}{50}\\
t &= 6
\end{aligned}$$

18.

Who or What	Rate	Time	Equation
T_1	r	4	$4r$
T_2	$r+6$	4	$4(r+6)$

$$\begin{aligned}
4r + 4(r + 6) &= 168\\
4r + 4r + 24 &= 168\\
-24 & \quad -24\\
\frac{8r}{8} &= \frac{144}{8}\\
r &= 18\\
r+6 &= 24
\end{aligned}$$

19.

Who or What	Rate	Time	Equation
C_1	r	3	$3r$
C_2	$2r$	3	$3(2r)$

$$\begin{array}{rcrcl}
3r & + & 3(2r) & = & 72 \\
3r & + & 6r & = & 72 \\
 & & \dfrac{9r}{9} & = & \dfrac{72}{9} \\
 & & r & = & 8 \\
 & & C_1 & = & 8 \text{ km/h} \\
 & & C_2 & = & 16 \text{ km/h}
\end{array}$$

20.

Who or What	Rate	Time	Equation
P_1	$r-25$	2	$2(r-25)$
P_2	r	2	$2r$

$$\begin{array}{rcrcrcl}
2(r & - & 25) & + & 2r & = & 430 \\
2r & - & 50 & + & 2r & = & 430 \\
 & + & 50 & & & & +50 \\
 & & & & \dfrac{4r}{4} & = & \dfrac{480}{4} \\
 & & & & r & = & 120 \\
 & & & & P_1 & = & 120-25=95 \\
 & & & & P_2 & = & 120
\end{array}$$

21.

Who or What	Rate	Time	Equation
S_1	55	t	$55t$
S_2	40	$2.5-t$	$40(2.5-t)$

$$\begin{array}{rcrcrcr}
55t & + & 40(2.5 & - & t) & = & 130 \\
55t & + & 100 & - & 40t & = & 130 \\
 & - & 100 & & & & -100 \\
 & & & & \dfrac{15t}{15} & = & \dfrac{30}{15} \\
 & & & & t & = & 2
\end{array}$$

22.

Who or What	Rate	Time	Equation
To end	8	t	$8t$
Return	3	$55-t$	$3(55-t)$

$$
\begin{array}{rcrcr}
8t & = & 3(55 & - & t) \\
8t & = & 165 & - & 3t \\
+3t & & & + & 3t \\
\dfrac{11t}{11} & = & \dfrac{165}{11} & & \\
& & & & \\
t & = & 15 & &
\end{array}
$$

$$
\begin{array}{rcl}
d & = & r \cdot t \\
d & = & 8(15) \\
d & = & 120 \text{ m}
\end{array}
$$

Answer Key 9.1

1. $\sqrt{5 \cdot 49}$
   $\pm 7\sqrt{5}$
2. $\sqrt{5 \cdot 25}$
   $\pm 5\sqrt{5}$
3. $2 \cdot (\pm 6)$
   ± 12
4. $5 \cdot (\pm 14)$
   ± 70
5. $\sqrt{4 \cdot 3}$
   $\pm 2\sqrt{3}$
6. $\sqrt{36 \cdot 2}$
   $\pm 6\sqrt{2}$
7. $3\sqrt{4 \cdot 3}$
   $3 \cdot 2\sqrt{3}$
   $\pm 6\sqrt{3}$
8. $5\sqrt{16 \cdot 2}$
   $5 \cdot 4\sqrt{2}$
   $\pm 20\sqrt{2}$
9. $6\sqrt{64 \cdot 2}$
   $6 \cdot 8\sqrt{2}$
   $\pm 48\sqrt{2}$
10. $7\sqrt{64 \cdot 2}$
    $7 \cdot 8\sqrt{2}$
    $\pm 56\sqrt{2}$
11. $-7 \cdot 8x^2$
    $\pm 56x^2$
12. $-2\sqrt{64 \cdot 2 \cdot n}$
    $-2 \cdot 8\sqrt{2n}$
    $\pm 16\sqrt{2n}$
13. $-5 \cdot 6\sqrt{m}$
    $\pm 30\sqrt{m}$
14. $8\sqrt{7 \cdot 16 \cdot p^2}$
    $8 \cdot 4 \cdot p\sqrt{7}$
    $\pm 32p\sqrt{7}$
15. $\sqrt{5 \cdot 9 \cdot x^2 \cdot y^2}$
    $\pm 3xy\sqrt{5}$
16. $\sqrt{2 \cdot 36 \cdot a \cdot a^2 \cdot b^4}$
    $\pm 6ab^2\sqrt{2a}$
17. $\sqrt{16 \cdot x^2 \cdot x \cdot y^2 \cdot y}$
    $\pm 4xy\sqrt{xy}$

18. $\sqrt{2 \cdot 256 \cdot a^4 \cdot b^2}$
$\pm 16a^2b\sqrt{2}$
19. $\sqrt{5 \cdot 64 \cdot x^4 \cdot y^4}$
$\pm 8x^2y^2\sqrt{5}$
20. $\sqrt{2 \cdot 256 \cdot m^4 \cdot n^2 \cdot n}$
$\pm 16m^2n\sqrt{2n}$

Answer Key 9.2

1. 4
2. −5
3. $\sqrt[3]{5 \cdot 125}$
   $5\sqrt[3]{5}$
4. $\sqrt[3]{2 \cdot 125}$
   $5\sqrt[3]{2}$
5. $\sqrt[3]{3 \cdot 64}$
   $4\sqrt[3]{3}$
6. $\sqrt[3]{3 \cdot 8}$
   $-2\sqrt[3]{3}$
7. $-4\sqrt[4]{6 \cdot 16}$
   $-4 \cdot 2\sqrt[4]{6}$
   $\pm 8\sqrt[4]{6}$
8. $-8\sqrt[4]{3 \cdot 16}$
   $-8 \cdot 2\sqrt[4]{3}$
   $\pm 16\sqrt[4]{3}$
9. $6 \cdot \sqrt[4]{7 \cdot 16}$
   $6 \cdot 2\sqrt[4]{7}$
   $\pm 12\sqrt[4]{7}$
10. $5 \cdot \sqrt[4]{3 \cdot 81}$
    $5 \cdot 3\sqrt[4]{3}$
    $\pm 15\sqrt[4]{3}$
11. $6\sqrt[4]{8 \cdot 81 \cdot x^4 \cdot x \cdot y^4 \cdot y^3 \cdot z^2}$
    $6 \cdot 3 \cdot x \cdot y\sqrt[4]{8xy^3z^2}$
    $\pm 18xy\sqrt[4]{8xy^3z^2}$
12. $-6\sqrt[4]{5 \cdot 81 \cdot a^4 \cdot a \cdot b^8 \cdot c}$
    $-6 \cdot 3 \cdot a \cdot b^2\sqrt[4]{5ac}$
    $\pm 18ab^2\sqrt[4]{5ac}$
13. $\sqrt[5]{7 \cdot 32 \cdot n^3 \cdot p^2 \cdot p^5 \cdot q^5}$
    $2pq\sqrt[5]{7n^3p^2}$
14. $\sqrt[5]{3 \cdot -32 \cdot x^3 \cdot y^5 \cdot y \cdot z^5}$
    $-2yz\sqrt[5]{3x^3y}$
15. $\sqrt[5]{7 \cdot 32 \cdot p^5 \cdot q^{10} \cdot r^{15}}$
    $2pq^2r^3\sqrt[5]{7}$
16. $\sqrt[6]{4 \cdot 64 \cdot x^6 \cdot y^6 \cdot z^6 \cdot z}$
    $\pm 2xyz\sqrt[6]{4z}$
17. $-3\sqrt[7]{7 \cdot 128 \cdot r \cdot s^7 \cdot t^{14}}$
    $-3 \cdot 2 \cdot s \cdot t^2\sqrt[7]{7r}$
    $-6st^2\sqrt[7]{7r}$
18. $-8\sqrt[7]{3 \cdot 128 \cdot b^7 \cdot b \cdot c^7 \cdot d^6}$

$-8 \cdot 2 \cdot b \cdot c\sqrt[7]{3bd^6}$
$-16bc\sqrt[7]{3bd^6}$

Answer Key 9.3

1. $(2+2+2)\sqrt{5}$
   $6\sqrt{5}$
2. $-5\sqrt{3}-3\sqrt{6}$
3. $-3\sqrt{2}+6\sqrt{5}$
4. $-\sqrt{3}-5\sqrt{6}$
5. $\sqrt{2}-3\sqrt{9\cdot 2}$
   $\sqrt{2}-3\cdot 3\sqrt{2}$
   $-8\sqrt{2}$
6. $-\sqrt{6\cdot 9}-3\sqrt{6}+3\sqrt{9\cdot 3}$
   $-3\sqrt{6}-3\sqrt{6}+3\cdot 3\sqrt{3}$
   $-6\sqrt{6}+9\sqrt{3}$
7. $-3\sqrt{6}-\sqrt{4\cdot 3}+3\sqrt{3}$
   $-3\sqrt{6}-2\sqrt{3}+3\sqrt{3}$
   $-3\sqrt{6}+\sqrt{3}$
8. $-2\sqrt{5}-2\sqrt{6\cdot 9}$
   $-2\sqrt{5}-2\cdot 3\sqrt{6}$
   $-2\sqrt{5}-6\sqrt{6}$
9. $3\sqrt{2}+2\sqrt{4\cdot 2}-3\sqrt{2\cdot 9}$
   $3\sqrt{2}+2\cdot 2\sqrt{2}-3\cdot 3\sqrt{2}$
   $3\sqrt{2}+4\sqrt{2}-9\sqrt{2}$
   $-2\sqrt{2}$
10. $4\sqrt{20}-\sqrt{3}$
    $4\sqrt{4\cdot 5}-\sqrt{3}$
    $4\cdot 2\sqrt{5}-\sqrt{3}$
    $8\sqrt{5}-\sqrt{3}$
11. $3\sqrt{9\cdot 2}-4\sqrt{2}$
    $3\cdot 3\sqrt{2}-4\sqrt{2}$
    $9\sqrt{2}-4\sqrt{2}\Rightarrow 5\sqrt{2}$
12. $-3\sqrt{9\cdot 3}+2\sqrt{3}-\sqrt{4\cdot 3}$
    $-3\cdot 3\sqrt{3}+2\sqrt{3}-2\sqrt{3}$
    $-9\sqrt{3}$
13. $-3\sqrt{6}-\sqrt{3}$
14. $-3\sqrt{2}+3\sqrt{4\cdot 2}+3\sqrt{6}$
    $-3\sqrt{2}+3\cdot 2\sqrt{2}+3\sqrt{6}$
    $3\sqrt{2}+3\sqrt{6}$
15. $-2\sqrt{9\cdot 2}-3\sqrt{4\cdot 2}-\sqrt{4\cdot 5}+2\sqrt{4\cdot 5}$
    $-6\sqrt{2}-6\sqrt{2}-2\sqrt{5}+4\sqrt{5}$
    $-12\sqrt{2}+2\sqrt{5}$
16. $-3\sqrt{9\cdot 2}+3\sqrt{8}$
    $-9\sqrt{2}+3\sqrt{4\cdot 2}$

$-9\sqrt{2}+6\sqrt{2}$
$-3\sqrt{2}$

17. $-2\sqrt{4\cdot 6}+2\sqrt{5\cdot 4}$
$-4\sqrt{6}+4\sqrt{5}$

18. $-3\sqrt{4\cdot 2}-\sqrt{5}-3\sqrt{6}+2\sqrt{9\cdot 2}$
$-6\sqrt{2}-\sqrt{5}-3\sqrt{6}+6\sqrt{2}$
$-\sqrt{5}-3\sqrt{6}$

19. $3\sqrt{6\cdot 4}-3\sqrt{3\cdot 9}+2\sqrt{6}+2\sqrt{2\cdot 4}$
$6\sqrt{6}-9\sqrt{3}+2\sqrt{6}+4\sqrt{2}$
$4\sqrt{2}-9\sqrt{3}+8\sqrt{6}$

20. $2\sqrt{6}-\sqrt{9\cdot 6}-3\sqrt{9\cdot 3}-\sqrt{3}$
$2\sqrt{6}-3\sqrt{6}-9\sqrt{3}-\sqrt{3}$
$-\sqrt{6}-10\sqrt{3}$

Answer Key 9.4

1. $12\sqrt{5\cdot 16}$
   $12\cdot 4\sqrt{5}$
   $48\sqrt{5}$
2. $-5\sqrt{10\cdot 15}$
   $-5\sqrt{150}$
   $-5\sqrt{25\cdot 6}\Rightarrow -25\sqrt{6}$
3. $\sqrt{15\cdot 12\cdot m^2}$
   $\sqrt{3\cdot 5\cdot 3\cdot 4\cdot m^2}$
   $3\cdot 2m\sqrt{5}$
   $6m\sqrt{5}$
4. $-5\sqrt{5r^3\cdot 10r^2}$
   $-5\sqrt{25\cdot 2\cdot r^4\cdot r}$
   $-25r^2\sqrt{2r}$
5. $\sqrt[3]{8x^7}$
   $\sqrt[3]{8\cdot x^6\cdot x}$
   $2x^2\sqrt[3]{x}$
6. $3\sqrt[3]{40a^7}$
   $3\sqrt[3]{5\cdot 8\cdot a^6\cdot a}$
   $3\cdot 2a^2\sqrt[3]{5a}\Rightarrow 6a^2\sqrt[3]{5a}$
7. $\sqrt{12}+2\sqrt{6}$
   $\sqrt{4\cdot 3}+2\sqrt{6}$
   $2\sqrt{3}+2\sqrt{6}$
8. $\sqrt{50}+\sqrt{20}$
   $\sqrt{25\cdot 2}+\sqrt{4\cdot 5}$
   $5\sqrt{2}+2\sqrt{5}$
9. $-15\sqrt{45}-10\sqrt{15}$
   $-15\sqrt{9\cdot 5}-10\sqrt{15}$
   $-15\cdot 3\sqrt{5}-10\sqrt{15}$
   $-45\sqrt{5}-10\sqrt{15}$
10. $15\sqrt{45}+10\sqrt{15}$
    $15\sqrt{9\cdot 5}+10\sqrt{15}$
    $15\cdot 3\sqrt{5}+10\sqrt{15}$
    $45\sqrt{5}+10\sqrt{15}$
11. $25n\sqrt{10}+5\sqrt{20}$
    $25n\sqrt{10}+5\sqrt{4\cdot 5}$
    $25n\sqrt{10}+10\sqrt{5}$
12. $\sqrt{75}-3\sqrt{45v}$
    $\sqrt{25\cdot 3}-3\sqrt{9\cdot 5v}$
    $5\sqrt{3}-9\sqrt{5v}$
13. $-6+2\sqrt{2}-6\sqrt{2}+2(\sqrt{2})(\sqrt{2})$

$-6+2\sqrt{2}-6\sqrt{2}+2(2)$
$-6+4+2\sqrt{2}-6\sqrt{2}$
$-2-4\sqrt{2}$

14. $10-4\sqrt{3}-5\sqrt{3}+2(\sqrt{3})(\sqrt{3})$
$10-4\sqrt{3}-5\sqrt{3}+2(3)$
$10+6-4\sqrt{3}-5\sqrt{3}$
$16-9\sqrt{3}$

15. $(2\sqrt{5})(\sqrt{5})-\sqrt{5}-10\sqrt{5}+5$
$2(5)-\sqrt{5}-10\sqrt{5}+5$
$10+5-\sqrt{5}-10\sqrt{5}$
$15-11\sqrt{5}$

16. $10(3)+4\sqrt{12}+5\sqrt{15}+2\sqrt{20}$
$30+4\sqrt{4\cdot 3}+5\sqrt{15}+2\sqrt{5\cdot 4}$
$30+5\sqrt{15}+8\sqrt{3}+4\sqrt{5}$

17. $3(2a)+6\sqrt{6a^2}+\sqrt{10a^2}+2\sqrt{15a^2}$
$6a+6a\sqrt{6}+a\sqrt{10}+2a\sqrt{15}$

18. $(-2\sqrt{2p}+5\sqrt{5})(2\sqrt{5p})$
$-4\sqrt{10p^2}+10\sqrt{25p}$
$-4p\sqrt{10}+50\sqrt{p}$

19. $15+12\sqrt{3}+20\sqrt{3}+16(3)$
$63+32\sqrt{3}$

20. $-5\sqrt{4m}+\sqrt{2m}+25\sqrt{2}-5$
$-10\sqrt{m}+\sqrt{2m}+25\sqrt{2}-5$

21.
$$\frac{\sqrt{12}}{5\sqrt{100}}\div\sqrt{4}$$
$$\frac{\sqrt{3}}{5\sqrt{25}}\Rightarrow\frac{\sqrt{3}}{5\cdot 5}\Rightarrow\frac{\sqrt{3}}{25}$$

22. $\frac{\sqrt{15}}{2\cdot 2}\Rightarrow\frac{\sqrt{15}}{4}$

23.
$$\frac{\sqrt{5}}{4\sqrt{125}}\div\sqrt{5}$$
$$\frac{\sqrt{1}}{4\sqrt{25}}\Rightarrow\frac{1}{4\cdot 5}\Rightarrow\frac{1}{20}$$

24.
$$\frac{\sqrt{12}}{\sqrt{3}}\div\sqrt{3}$$
$$\frac{\sqrt{4}}{\sqrt{1}}\Rightarrow\frac{2}{1}\Rightarrow 2$$

25.

$$\frac{\sqrt{10}}{\sqrt{6}} \div \sqrt{2}$$

$$\frac{\sqrt{5}}{\sqrt{3}}$$

26. Does not reduce

27. $\dfrac{5x^2}{4\sqrt{3 \cdot x^2 \cdot x \cdot y^2 \cdot y}} \Rightarrow \dfrac{5x^2}{4xy\sqrt{3xy}} \Rightarrow \dfrac{5x}{4y\sqrt{3xy}}$

28. $\dfrac{4}{5y^2\sqrt{3x}}$

29.

$$\frac{\sqrt{2p^2}}{\sqrt{3p}} \div \sqrt{p}$$

$$\frac{\sqrt{2p}}{\sqrt{3}}$$

30.

$$\frac{\sqrt{8n^2}}{\sqrt{10n}} \div \sqrt{2n}$$

$$\frac{\sqrt{4n}}{\sqrt{5}} \Rightarrow \frac{2\sqrt{n}}{\sqrt{5}}$$

Answer Key 9.5

1. $\dfrac{4+2\sqrt{3}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} \Rightarrow \dfrac{4\sqrt{3}+2(3)}{3} \Rightarrow \dfrac{4\sqrt{3}+6}{3}$
2. $\dfrac{-4+\sqrt{3}}{4\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} \Rightarrow \dfrac{-4\sqrt{3}+3}{4(3)} \Rightarrow \dfrac{-4\sqrt{3}+3}{12}$
3. $\dfrac{4+2\sqrt{3}}{5\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} \Rightarrow \dfrac{4\sqrt{6}+2\sqrt{18}}{5(6)} \Rightarrow \dfrac{4\sqrt{6}+6\sqrt{2}}{30} \Rightarrow \dfrac{2\sqrt{6}+3\sqrt{2}}{15}$
4. $\dfrac{2\sqrt{3}-2}{2\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} \Rightarrow \dfrac{2(3)-2\sqrt{3}}{2(3)} \Rightarrow \dfrac{6-2\sqrt{3}}{6} \Rightarrow \dfrac{3-\sqrt{3}}{3}$
5. $\dfrac{2-5\sqrt{5}}{4\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} \Rightarrow \dfrac{2\sqrt{3}-5\sqrt{15}}{4(3)} \Rightarrow \dfrac{2\sqrt{3}-5\sqrt{15}}{12}$
6. $\dfrac{\sqrt{5}+4}{4\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} \Rightarrow \dfrac{5+4\sqrt{5}}{4(5)} \Rightarrow \dfrac{5+4\sqrt{5}}{20}$
7. $\dfrac{\sqrt{2}-3\sqrt{3}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} \Rightarrow \dfrac{\sqrt{6}-3(3)}{3} \Rightarrow \dfrac{\sqrt{6}-9}{3}$
8. $\dfrac{\sqrt{5}-\sqrt{2}}{3\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} \Rightarrow \dfrac{\sqrt{30}-\sqrt{12}}{3(6)} \Rightarrow \dfrac{\sqrt{30}-2\sqrt{3}}{18}$
9. $\dfrac{5}{3\sqrt{5}+\sqrt{2}} \cdot \dfrac{3\sqrt{5}-\sqrt{2}}{3\sqrt{5}-\sqrt{2}} \Rightarrow \dfrac{15\sqrt{5}-5\sqrt{2}}{9(5)-2} \Rightarrow \dfrac{15\sqrt{5}-5\sqrt{2}}{43}$
10. $\dfrac{5}{\sqrt{3}+4\sqrt{5}} \cdot \dfrac{\sqrt{3}-4\sqrt{5}}{\sqrt{3}-4\sqrt{5}} \Rightarrow \dfrac{5\sqrt{3}-20\sqrt{5}}{3-16(5)} \Rightarrow \dfrac{5\sqrt{3}-20\sqrt{5}}{-77} \Rightarrow \dfrac{20\sqrt{5}-5\sqrt{3}}{77}$
11. $\dfrac{2}{5+\sqrt{2}} \cdot \dfrac{5-\sqrt{2}}{5-\sqrt{2}} \Rightarrow \dfrac{10-2\sqrt{2}}{25-2} \Rightarrow \dfrac{10-2\sqrt{2}}{23}$
12. $\dfrac{5}{2\sqrt{3}-\sqrt{2}} \cdot \dfrac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}+\sqrt{2}} \Rightarrow \dfrac{10\sqrt{3}+5\sqrt{2}}{4(3)-2} \Rightarrow \dfrac{10\sqrt{3}+5\sqrt{2}}{10} \Rightarrow \dfrac{2\sqrt{3}+\sqrt{2}}{2}$
13. $\dfrac{3}{4-\sqrt{3}} \cdot \dfrac{4+\sqrt{3}}{4+\sqrt{3}} \Rightarrow \dfrac{12+3\sqrt{3}}{16-3} \Rightarrow \dfrac{12+3\sqrt{3}}{13}$
14. $\dfrac{4}{\sqrt{2}-2} \cdot \dfrac{\sqrt{2}+2}{\sqrt{2}+2} \Rightarrow \dfrac{4\sqrt{2}+8}{2-4} \Rightarrow \dfrac{4\sqrt{2}+8}{-2} \Rightarrow -2\sqrt{2}-4$
15. $\dfrac{4}{3+\sqrt{5}} \cdot \dfrac{3-\sqrt{5}}{3-\sqrt{5}} \Rightarrow \dfrac{12-4\sqrt{5}}{9-5} \Rightarrow \dfrac{12-4\sqrt{5}}{4} \Rightarrow 3-\sqrt{5}$
16. $\dfrac{2}{\sqrt{5}+2\sqrt{3}} \cdot \dfrac{\sqrt{5}-2\sqrt{3}}{\sqrt{5}-2\sqrt{3}} \Rightarrow \dfrac{2\sqrt{5}-4\sqrt{3}}{5-4(3)} \Rightarrow \dfrac{2\sqrt{5}-4\sqrt{3}}{-7}$
17. $\dfrac{-3+2\sqrt{3}}{\sqrt{3}+2} \cdot \dfrac{\sqrt{3}-2}{\sqrt{3}-2} \Rightarrow \dfrac{-3\sqrt{3}+6+2(3)-4\sqrt{3}}{3-4} \Rightarrow \dfrac{12-7\sqrt{3}}{-1} \Rightarrow$
$-12+7\sqrt{3}$
18. $\dfrac{4+\sqrt{5}}{2+2\sqrt{5}} \cdot \dfrac{2-2\sqrt{5}}{2-2\sqrt{5}} \Rightarrow \dfrac{8-8\sqrt{5}+2\sqrt{5}-2(5)}{4-4(5)} \Rightarrow \dfrac{-2-6\sqrt{5}}{-16} \Rightarrow \dfrac{1+3\sqrt{5}}{8}$

19. $\dfrac{2-\sqrt{3}}{1+\sqrt{2}} \cdot \dfrac{1-\sqrt{2}}{1-\sqrt{2}} \Rightarrow \dfrac{2-2\sqrt{2}-\sqrt{3}+\sqrt{6}}{1-2} \Rightarrow \dfrac{2-2\sqrt{2}-\sqrt{3}+\sqrt{6}}{-1} \Rightarrow$
$2\sqrt{2}+\sqrt{3}-\sqrt{6}-2$

20. $\dfrac{-1+\sqrt{3}}{\sqrt{3}-1} \cdot \dfrac{\sqrt{3}+1}{\sqrt{3}+1} \Rightarrow \dfrac{-\sqrt{3}-1+3+\sqrt{3}}{3-1} \Rightarrow \dfrac{2}{2} \Rightarrow 1$

Answer Key 9.6

1. $\sqrt[5]{m^3}$
2. $\dfrac{1}{\sqrt[4]{(10r)^3}}$
3. $\sqrt{(7x)^3}$
4. $\dfrac{1}{\sqrt[3]{(6b)^4}}$
5. $\dfrac{1}{\sqrt{(2x+3)^3}}$
6. $\sqrt[4]{(x-3y)^3}$
7. $5^{\frac{1}{3}}$
8. $2^{\frac{3}{5}}$
9. $(ab^5)^{\frac{1}{3}}$ or $a^{\frac{1}{3}}b^{\frac{5}{3}}$
10. $x^{\frac{3}{5}}$
11. $(a+5)^{\frac{2}{3}}$
12. $(a-2)^{\frac{3}{5}}$
13. $8^{\frac{2}{3}} \Rightarrow (2^3)^{\frac{2}{3}} \Rightarrow 2^2 \text{ or } 4$
14. $16^{\frac{1}{4}} \Rightarrow (2^4)^{\frac{1}{4}} \Rightarrow 2$
15. $\sqrt[3]{4^6} \Rightarrow (2^2)^{\frac{6}{3}} \Rightarrow 2^4 \text{ or } 16$
16. $\sqrt[5]{32^2} \Rightarrow (2^5)^{\frac{2}{5}} \Rightarrow 2^2 \text{ or } 4$
17. $x^2y^{\frac{1}{3}+\frac{2}{3}} \Rightarrow x^2y$
18. $4v^{\frac{2}{3}-1} \Rightarrow 4v^{-\frac{1}{3}} \text{ or } \dfrac{4}{v^{\frac{1}{3}}}$
19. $a^{-\frac{1}{2}}b^{-\frac{1}{2}} \Rightarrow \dfrac{1}{a^{\frac{1}{2}}b^{\frac{1}{2}}}$
20. 1
21. $\dfrac{a^2b^01}{3a^4a^2} \Rightarrow \dfrac{1}{3a^2}$
22. $\dfrac{2x^{\frac{1}{2}}y^{\frac{1}{3}}}{2x^{\frac{4}{3}}y^{\frac{7}{4}}} \Rightarrow x^{\frac{1}{2}-\frac{4}{3}}y^{\frac{1}{3}-\frac{7}{4}} \Rightarrow x^{-\frac{5}{6}}y^{-\frac{17}{12}} \Rightarrow \dfrac{1}{x^{\frac{5}{6}}y^{\frac{17}{12}}}$
23. $\dfrac{a^{\frac{3}{4}}b^{-1}b^{\frac{7}{4}}}{3b^{-1}} \Rightarrow \dfrac{a^{\frac{3}{4}}b^{\frac{7}{4}}}{3}$
24. $2x^{-2+\frac{5}{4}-1}y^{\frac{5}{3}+\frac{5}{3}-\frac{1}{2}} \Rightarrow 2x^{-\frac{7}{4}}y^{\frac{17}{6}} \Rightarrow \dfrac{2y^{\frac{17}{6}}}{x^{\frac{7}{4}}}$
25. $\dfrac{3}{2}y^{-\frac{5}{4}--1--\frac{1}{3}} \Rightarrow \dfrac{3}{2}y^{\frac{1}{12}}$
26. $\dfrac{ab^{\frac{1}{3}}2b^{-\frac{5}{4}}}{42a^{-\frac{1}{2}}b^{-\frac{2}{3}}} \Rightarrow \dfrac{1}{2}a^{1--\frac{1}{2}}b^{\frac{1}{3}-\frac{5}{4}--\frac{2}{3}} \Rightarrow \dfrac{1}{2}a^{\frac{3}{2}}b^{-\frac{1}{4}} \Rightarrow \dfrac{a^{\frac{3}{2}}}{2b^{\frac{1}{4}}}$

Answer Key 9.7

1. $(x^{-2--2}y^{-6-4})^2$
   $(1x^0y^{-10})^2$
   y^{-20} or $\dfrac{1}{y^{20}}$
2. $(x^{-3--1}y^{-3-6})^3$
   $(x^{-2}y^{-9})^3$
   $x^{-6}y^{-27}$ or $\dfrac{1}{x^6y^{27}}$
3. $(x^{-2-2}y^{-4--4})^2$
   $(x^{-4}y^01)^2$
   x^{-8} or $\dfrac{1}{x^8}$
4. $(x^{-5--4}y^{-3-2})^4$
   $(x^{-1}y^{-5})^4$
   $x^{-4}y^{-20}$ or $\dfrac{1}{x^4y^{20}}$
5. $(x^{-2--3}y^{-2-3})^8$
   $(x^1y^{-5})^8$
   x^8y^{-40} or $\dfrac{x^8}{y^{40}}$
6. $(x^{-4--3}y^{-3-2})^5$
   $(x^{-1}y^{-5})^5$
   $x^{-5}y^{-25}$ or $\dfrac{1}{x^5y^{25}}$
7. $(x^{-2--2}y^{-4-4})^{-2}$
   $(1x^0y^{-8})^{-2}$
   y^{16}
8. $(x^{-2--5}y^{-3-3})^{-3}$
   $(x^3y^{-6})^{-3}$
   $x^{-9}y^{18}$ or $\dfrac{y^{18}}{x^9}$
9. $(x^{-2--2}y^{-3--3})^{-1}$
   $(x^0y^01)^{-1}$
   1
10. $(x^{-2--2}y^{-3--4})^{-2}$
    $(x^0y^{-7})^{-2}$
    y^{14}
11. $\left(\dfrac{1x^0y^{-3}}{x^{-2}y^01}\right)^{-5}$

$$\left(\frac{x^2}{y^3}\right)^{-5}$$

$$\frac{x^{-10}}{y^{-15}}$$

$$\frac{y^{15}}{x^{10}}$$

12. 1
13. $$\left(\frac{1a^0b^3}{a^6b^{-12}}\right)^{-\frac{1}{3}}$$

$$\left(\frac{b^{15}}{a^6}\right)^{-\frac{1}{3}}$$

$$\frac{b^{-5}}{a^{-2}} \text{ or } \frac{a^2}{b^5}$$

14. $(a^4b^4c^{12})^{\frac{1}{4}}$

abc^3

15. $$\left(\frac{a^5c^{10}d^{15}}{b^5}\right)^{\frac{2}{5}}$$

$$\frac{a^2c^4d^6}{b^2}$$

16. $$\left(\frac{b^{20}}{a^4}\right)^{-\frac{3}{4}}$$

$$\frac{b^{-15}}{a^{-3}} \text{ or } \frac{a^3}{b^{15}}$$

17. 1
18. Undefined

Answer Key 9.8

1. $(2^4x^4y^6)^{\frac{1}{8}} \Rightarrow 2^{\frac{1}{2}}x^{\frac{1}{2}}y^{\frac{3}{4}}$
2. $(3^2x^2y^6)^{\frac{1}{4}} \Rightarrow 3^{\frac{1}{2}}x^{\frac{1}{2}}y^{\frac{3}{2}}$
3. $(2^6x^4y^6z^8)^{\frac{1}{12}} \Rightarrow 2^{\frac{1}{2}}x^{\frac{1}{3}}y^{\frac{1}{2}}z^{\frac{2}{3}}$
4. $\left(\dfrac{5^2x^3}{2^4x^5}\right)^{\frac{1}{8}} \Rightarrow \left(\dfrac{5^2}{2^4x^2}\right)^{\frac{1}{8}} \Rightarrow \dfrac{5^{\frac{1}{4}}}{2^{\frac{1}{2}}x^{\frac{1}{4}}}$
5. $\left(\dfrac{2^4x}{3^2y^4}\right)^{\frac{1}{6}} \Rightarrow \dfrac{2^{\frac{2}{3}}x^{\frac{1}{6}}}{3^{\frac{1}{3}}y^{\frac{2}{3}}}$
6. $(x^9y^{12}z^6)^{\frac{1}{15}} \Rightarrow x^{\frac{3}{5}}y^{\frac{4}{5}}z^{\frac{2}{5}}$
7. $(x^6y^9)^{\frac{1}{12}} \Rightarrow x^{\frac{1}{2}}y^{\frac{3}{4}}$
8. $(2^6x^8y^4)^{\frac{1}{10}} \Rightarrow 2^{\frac{3}{5}}x^{\frac{4}{5}}y^{\frac{2}{5}}$
9. $(x^6y^4z^2)^{\frac{1}{8}} \Rightarrow x^{\frac{3}{4}}y^{\frac{1}{2}}z^{\frac{1}{4}}$
10. $(5^2y^2)^{\frac{1}{4}} \Rightarrow 5^{\frac{1}{2}}y^{\frac{1}{2}}$
11. $(2^3x^3y^6)^{\frac{1}{9}} \Rightarrow 2^{\frac{1}{3}}x^{\frac{1}{3}}y^{\frac{2}{3}}$
12. $(3^4x^8y^{12})^{\frac{1}{16}} \Rightarrow 3^{\frac{1}{4}}x^{\frac{1}{2}}y^{\frac{3}{4}}$
13. $5^{\frac{1}{3}} \cdot 5^{\frac{1}{2}} \Rightarrow 5^{\frac{3}{6}} \cdot 5^{\frac{2}{6}} \Rightarrow 5^{\frac{5}{6}}$
14. $7^{\frac{1}{3}} \cdot 7^{\frac{1}{4}} \Rightarrow 7^{\frac{4}{12}} \cdot 7^{\frac{3}{12}} \Rightarrow 7^{\frac{7}{12}}$
15. $x^{\frac{1}{2}} \cdot 7^{\frac{1}{3}}x^{\frac{1}{3}} \Rightarrow 7^{\frac{1}{3}}x^{\frac{5}{6}}$
16. $y^{\frac{1}{3}} \cdot 3^{\frac{1}{5}}y^{\frac{1}{5}} \Rightarrow 3^{\frac{1}{5}}y^{\frac{8}{15}}$
17. $x^{\frac{1}{2}} \cdot x^{\frac{2}{3}} \Rightarrow x^{\frac{7}{6}}$
18. $3^{\frac{1}{4}}x^{\frac{1}{4}}x^{\frac{4}{2}} \Rightarrow 3^{\frac{1}{4}}x^{\frac{9}{4}}$
19. $x^{\frac{2}{5}}y^{\frac{1}{5}}x^{\frac{2}{2}} \Rightarrow x^{\frac{7}{5}}y^{\frac{1}{5}}$
20. $a^{\frac{1}{2}}b^{\frac{1}{2}}2^{\frac{1}{5}}a^{\frac{2}{5}}b^{\frac{2}{5}} \Rightarrow 2^{\frac{1}{5}}a^{\frac{9}{10}}b^{\frac{9}{10}}$
21. $x^{\frac{1}{4}}y^{\frac{2}{4}} \cdot x^{\frac{2}{3}}y^{\frac{1}{3}} \Rightarrow x^{\frac{11}{12}}y^{\frac{5}{6}}$
22. $3^{\frac{1}{5}}a^{\frac{2}{5}}b^{\frac{3}{5}}3^{\frac{2}{4}}a^{\frac{2}{4}}b^{\frac{1}{4}} \Rightarrow 3^{\frac{7}{10}}a^{\frac{9}{10}}b^{\frac{17}{20}}$
23. $a^{\frac{2}{4}}b^{\frac{1}{4}}c^{\frac{2}{4}}a^{\frac{2}{5}}b^{\frac{3}{5}}c^{\frac{1}{5}} \Rightarrow a^{\frac{9}{10}}b^{\frac{17}{20}}c^{\frac{7}{10}}$
24. $x^{\frac{2}{6}}y^{\frac{1}{6}}z^{\frac{3}{6}}x^{\frac{2}{5}}y^{\frac{1}{5}}z^{\frac{2}{5}} \Rightarrow x^{\frac{11}{15}}y^{\frac{11}{30}}z^{\frac{9}{10}}$

Answer Key 9.9

1. $3 + 8 - 4i$
   $11 - 4i$
2. $3i - 7i$
   $-4i$
3. $7i - 3 + 2i$
   $9i - 3$
4. $5 - 6 - 6i$
   $-1 - 6i$
5. $-6i - 3 - 7i$
   $-13i - 3$
6. $-8i - 7i - 5 + 3i$
   $-12i - 5$
7. $3 - 3i - 7 - 8i$
   $-4 - 11i$
8. $-4 - i + 1 - 5i$
   $-3 - 6i$
9. $i - 2 - 3i - 6$
   $-2i - 8$
10. $5 - 4i + 8 - 4i$
    $13 - 8i$
11. $-48i^2$
    $-48(-1)$
    48
12. $-24i^2$
    $-24(-1)$
    24
13. $-40i^2$
    $-40(-1)$
    40
14. $-32i^2$
    $-32(-1)$
    32
15. $49i^2$
    $49(-1)$
    -49
16. $-7i^2(4 - 3i)$
    $-28i^2 + 21i^3$
    $-28(-1) + 21(-1)i$
    $28 - 21i$
17. $36 + 60i + 25i^2$
    $36 + 60i + 25(-1)$
    $36 + 60i - 25$
    $11 + 60i$
18. $16i^2(-2 - 8i)$

$32i^2 + 128i^3$
$32(-1) + 128(-1)i$
$-32 - 128i$

19.
$$\begin{array}{rrrrrr} & 56 & - & 42i & & \\ & & + & 32i & - & 24i^2 \\ & 56 & - & 10i & - & 24(-1) \\ + & 24 & & & \Longleftarrow & \\ & 80 & - & 10i & & \end{array}$$

20. $9i^2(4 - 4i)$
$-36i^2 + 36i^3$
$-36(-1) + 36(-1)i$
$36 - 36i$

21.
$$\begin{array}{rrrrrr} & -8 & + & 10i & & \\ & & + & 28i & - & 35i^2 \\ & -8 & + & 38i & - & 35(-1) \\ + & 35 & & & \Longleftarrow & \\ & 27 & + & 38i & & \end{array}$$

22. $-32i + 64i + 4 + 12i$
$-28 + 76i$

23.
$$\begin{array}{rrrrrr} & 32 & + & 24i & & \\ & & - & 16i & - & 12i^2 \\ & 32 & + & 8i & - & 12(-1) \\ + & 12 & & & \Longleftarrow & \\ & 44 & + & 8i & & \end{array}$$

24. $-18i + 12i^2 - 28i^2$
$-18i + 12(-1) - 28(-1)$
$-18i - 12 + 28$
$-18i + 16$

25.
$$\begin{array}{rrrrrr} & 2 & + & 10i & & \\ & & + & i & + & 5i^2 \\ & 2 & + & 11i & + & 5(-1) \\ - & 5 & & & \Longleftarrow & \\ & -3 & + & 11i & & \end{array}$$

26.
$$\begin{array}{rrrrrr} & -6 & + & 3i & & \\ & & + & 10i & - & 5i^2 \\ & -6 & + & 13i & - & 5(-1) \\ + & 5 & & & \Longleftarrow & \\ & -1 & + & 13i & & \end{array}$$

27. $\dfrac{-9+5i}{i} \cdot \dfrac{i}{i} \Rightarrow \dfrac{-9i+5i^2}{i^2} \Rightarrow \dfrac{-9i+5(-1)}{(-1)} \Rightarrow 9i + 5$

28. $\dfrac{-3+2i}{-3i} \cdot \dfrac{i}{i} \Rightarrow \dfrac{-3i+2i^2}{-3i^2} \Rightarrow \dfrac{-3i+2(-1)}{-3(-1)} \Rightarrow \dfrac{-3i-2}{3}$

29. $\dfrac{-10i-9i}{6i}\cdot\dfrac{i}{i}\Rightarrow\dfrac{-10i-9i^2}{6i^2}\Rightarrow\dfrac{-10i-9(-1)}{6(-1)}\Rightarrow\dfrac{-10i+9}{-6}$

30. $\dfrac{-4+2i}{3i}\cdot\dfrac{i}{i}\Rightarrow\dfrac{-4i+2i^2}{3i^2}\Rightarrow\dfrac{-4i+2(-1)}{3(-1)}\Rightarrow\dfrac{-4i-2}{-3}$

31. $\dfrac{-3-6i}{4i}\cdot\dfrac{i}{i}\Rightarrow\dfrac{-3i-6i^2}{4i^2}\Rightarrow\dfrac{-3i-6(-1)}{4(-1)}\Rightarrow\dfrac{-3i+6}{4}$

32. $\dfrac{-5+9i}{9i}\cdot\dfrac{i}{i}\Rightarrow\dfrac{-5i+9i^2}{9i^2}\Rightarrow\dfrac{-5i+9(-1)}{9(-1)}\Rightarrow\dfrac{-5i-9}{-9}\Rightarrow\dfrac{5i+9}{9}$

33. $\dfrac{10-i}{-i}\cdot\dfrac{i}{i}\Rightarrow\dfrac{10i-i^2}{-i^2}\Rightarrow\dfrac{10i-(-1)}{-(-1)}\Rightarrow 10i+1$

34. $\dfrac{10}{5i}\cdot\dfrac{i}{i}\Rightarrow\dfrac{10i}{5i^2}\Rightarrow\dfrac{10i}{5(-1)}\Rightarrow\dfrac{10i\div-5}{-5\div-5}\Rightarrow -2i$

35. $\dfrac{4i}{-10+i}\cdot\dfrac{-10-i}{-10-i}\Rightarrow\dfrac{-40i-4i^2}{(-10)^2-(i)^2}\Rightarrow\dfrac{-40i-4(-1)}{100--1}\Rightarrow\dfrac{-40i+4}{101}$

36. $\dfrac{9i}{1-5i}\cdot\dfrac{1+5i}{1+5i}\Rightarrow\dfrac{9i+45i^2}{(1)^2-(5i)^2}\Rightarrow\dfrac{9i+45(-1)}{1-25i^2}\Rightarrow\dfrac{9i-45}{1-25(-1)}\Rightarrow\dfrac{9i-45}{1+25}\Rightarrow$
$\dfrac{9i-45}{26}$

37. $\dfrac{8}{7-6i}\cdot\dfrac{7+6i}{7+6i}\Rightarrow\dfrac{56+48i}{7^2-36i^2}\Rightarrow\dfrac{56+48i}{49-36(-1)}\Rightarrow\dfrac{56+48i}{49+36}\Rightarrow\dfrac{56+48i}{85}$

38. $\dfrac{4}{4+6i}\cdot\dfrac{4-6i}{4-6i}\Rightarrow\dfrac{16-24i}{16-36i^2}\Rightarrow\dfrac{16-24i}{16+36}\Rightarrow\dfrac{(16-24i)\div 4}{52\div 4}\Rightarrow\dfrac{4-6i}{13}$

39. $\dfrac{7}{10-7i}\cdot\dfrac{10+7i}{10+7i}\Rightarrow\dfrac{70+49i}{100-49i^2}\Rightarrow\dfrac{70+49i}{100+49}\Rightarrow\dfrac{70+49i}{149}$

40. $\dfrac{9}{-8-6i}\cdot\dfrac{-8+6i}{-8+6i}\Rightarrow\dfrac{-72+54i}{64-36i^2}\Rightarrow\dfrac{-72+54i}{64+36}\Rightarrow\dfrac{(-72+54i)\div 2}{100\div 2}\Rightarrow$
$\dfrac{-36+27i}{50}$

41. $\dfrac{5i}{-6-i}\cdot\dfrac{-6+i}{-6+i}\Rightarrow\dfrac{-30i+5i^2}{36-i^2}\Rightarrow\dfrac{-30i-5}{36+1}\Rightarrow\dfrac{-30i-5}{37}$

42. $\dfrac{8i}{6-7i}\cdot\dfrac{6+7i}{6+7i}\Rightarrow\dfrac{48i+56i^2}{36-49i^2}\Rightarrow\dfrac{48i-56}{36+49}\Rightarrow\dfrac{48i-56}{85}$

43. $\pm 9i$

44. $\sqrt{-5\cdot 9}\Rightarrow\pm 3\sqrt{5}i$

45. $\sqrt{-20}\Rightarrow\sqrt{-4\cdot 5}\Rightarrow\pm 2\sqrt{5}i$

46. $\sqrt{24}\Rightarrow\sqrt{4\cdot 6}\Rightarrow\pm 2\sqrt{6}$

47. $\dfrac{3+\sqrt{3\cdot -9}}{6}\Rightarrow\dfrac{3+3\sqrt{3}i}{6}\Rightarrow\dfrac{1+\sqrt{3}i}{2}$

48. $\dfrac{-4-\sqrt{-2\cdot 4}}{-4}\Rightarrow\dfrac{-4-2\sqrt{2}i}{-4}\Rightarrow\dfrac{2+\sqrt{2}i}{2}$

49. $\dfrac{8-4i}{4}\Rightarrow 2-i$

50. $\dfrac{6+\sqrt{2\cdot -16}}{4} \Rightarrow \dfrac{6+4\sqrt{2}i}{4} \Rightarrow \dfrac{3+2\sqrt{2}i}{2}$
51. i
52. $i^3 \Rightarrow -i$
53. 1
54. 1
55. $i^2 \Rightarrow -1$
56. i
57. $i^2 \Rightarrow -1$
58. $i^3 \Rightarrow -i$

Answer Key 9.10

1. $\text{Father} = \text{Bill} - 2\text{ h}$

$$\therefore \frac{1}{B - 2\text{ h}} + \frac{1}{B} = \frac{1}{2\text{ h }24\text{ min}}$$

2. $\text{Smaller} = \text{Larger} + 4\text{ h}$

$$\therefore \frac{1}{L + 4\text{ h}} + \frac{1}{L} = \frac{1}{3\text{ h }45\text{ min}}$$

3. $\text{Jack} = \text{Bob} - 1\text{ h}$

$$\therefore \frac{1}{B - 1\text{ h}} + \frac{1}{B} = \frac{1}{1.2\text{ h}}$$

4. $\frac{1}{Y} + \frac{1}{B} = \frac{1}{T}$

$Y = 6\text{ d}$
$B = 4\text{ d}$

$$\therefore \frac{1}{6} + \frac{1}{4} = \frac{1}{T}$$

5. $\text{John} = \text{Carlos} + 8\text{ h}$

$$\frac{1}{C + 8\text{ h}} + \frac{1}{C} = \frac{1}{3\text{ h}}$$

6. $M = 3\text{ d}$
$N = 4\text{ d}$
$E = 5\text{ d}$

$$\frac{1}{M} + \frac{1}{N} + \frac{1}{E} = \frac{1}{T}$$

$$\frac{1}{3\text{ d}} + \frac{1}{4\text{ d}} + \frac{1}{5\text{ d}} = \frac{1}{T}$$

7. $\text{Raj} = 4\text{ d}$

$$\text{Rubi} = \frac{1}{2}\text{ Raj or }2\text{ d}$$

$$\therefore \frac{1}{4\text{ d}} + \frac{1}{2\text{ d}} = \frac{1}{T}$$

8. $\frac{1}{20\text{ min}} + \frac{1}{30\text{ min}} = \frac{1}{T}$

9. $$\begin{aligned} \frac{1}{24\text{ d}} + \frac{1}{I} &= \frac{1}{6\text{ d}} \\ \frac{1}{I} &= \frac{1}{6\text{ d}} - \frac{1}{24\text{ d}} \\ \frac{1}{I} &= \frac{4}{24\text{ d}} - \frac{1}{24\text{ d}} \\ \frac{1}{I} &= \frac{1}{8\text{ d}} \\ \therefore I &= 8\text{ days} \end{aligned}$$

10. $$\begin{aligned} \frac{1}{C} + \frac{1}{A} &= \frac{1}{3.75\text{ d}} \\ \frac{1}{5\text{ d}} + \frac{1}{A} &= \frac{1}{3.75\text{ d}} \\ \frac{1}{A} &= \frac{1}{3.75\text{ d}} - \frac{1}{5\text{ d}} \\ \frac{1}{A} &= \frac{4}{15\text{ d}} - \frac{3}{15\text{ d}} \\ \frac{1}{A} &= \frac{1}{15\text{ d}} \\ \therefore A &= 15\text{ days} \end{aligned}$$

11. $$\begin{aligned} \frac{1}{S} + \frac{1}{F} &= \frac{1}{\text{job}} \\ \frac{1}{3\text{ d}} + \frac{1}{6\text{ d}} &= \frac{1}{\text{job}} \\ \frac{2}{6\text{ d}} + \frac{1}{6\text{ d}} &= \frac{1}{\text{job}} \\ \frac{3}{6\text{ d}} &= \frac{1}{\text{job}} \\ \frac{1}{2\text{ d}} &= \frac{1}{\text{job}} \\ \therefore \text{job} &= 2\text{ days} \end{aligned}$$

12. $$\begin{aligned} \frac{1}{T} + \frac{1}{J} &= \frac{1}{\text{job}} \\ \frac{1}{10\text{ h}} + \frac{1}{8\text{ h}} &= \frac{1}{\text{job}} \\ \frac{4}{40\text{ h}} + \frac{5}{40\text{ h}} &= \frac{1}{\text{job}} \\ \frac{9}{40\text{ h}} &= \frac{1}{\text{job}} \\ \therefore \text{job} &= \frac{40\text{ h}}{9} \\ \text{job} &= 4\frac{4}{9}\text{ h} = 4.\bar{4}\text{ h} \end{aligned}$$

13. $$\begin{aligned} \text{fast} &= 2 \times \text{slow} \\ \frac{1}{F} + \frac{1}{S} &= \frac{1}{6\text{ h}} \\ \frac{1}{2S} + \frac{1}{S} &= \frac{1}{6\text{ h}} \\ \frac{1}{2S} + \frac{2}{2S} &= \frac{1}{6\text{ h}} \\ \frac{3}{2S} &= \frac{1}{6\text{ h}} \\ \frac{2S}{3} &= 6\text{ h} \\ S &= \frac{6(3)}{2} \\ S &= 9\text{ h} \end{aligned}$$

14.
$$\begin{aligned}
\text{slower} &= 3 \times \text{faster} \\
\frac{1}{F} + \frac{1}{S} &= \frac{1}{3\text{ h}} \\
\frac{1}{F} + \frac{1}{3F} &= \frac{1}{3\text{ h}} \\
\frac{3}{3F} + \frac{1}{3F} &= \frac{1}{3\text{ h}} \\
\frac{4}{3F} &= \frac{1}{3\text{ h}} \\
\therefore \frac{4}{3} &= \frac{F}{3\text{ h}} \\
F &= 4\text{ h}
\end{aligned}$$

15.
$$\begin{aligned}
\text{full} &= 8\text{ h} \\
\text{empty} &= 2 \times \text{full or } 16\text{ h} \\
\frac{1}{F} - \frac{1}{E} &= \frac{1}{T} \\
\frac{1}{8\text{ h}} - \frac{1}{16\text{ h}} &= \frac{1}{T} \\
\frac{2}{16\text{ h}} - \frac{1}{16\text{ h}} &= \frac{1}{T} \\
\therefore \frac{1}{16\text{ h}} &= \frac{1}{T} \\
T &= 16\text{ h}
\end{aligned}$$

16. $$\begin{aligned} \frac{1}{E} - \frac{1}{F} &= \frac{1}{T} \\ \frac{1}{3} - \frac{1}{5} &= \frac{1}{T} \\ \frac{5}{15} - \frac{3}{15} &= \frac{1}{T} \\ \frac{2}{15\ \text{min}} &= \frac{1}{T} \\ \therefore T &= \frac{15\ \text{min}}{2} = 7.5\ \text{min} \end{aligned}$$

17. $$\begin{aligned} \frac{1}{\text{full}} - \frac{1}{\text{empty}} &= \frac{1}{2T} \\ \frac{1}{10\ \text{h}} - \frac{1}{15\ \text{h}} &= \frac{1}{2T} \\ \frac{3}{30\ \text{h}} - \frac{2}{30\ \text{h}} &= \frac{1}{2T} \\ \frac{1}{30\ \text{h}} &= \frac{1}{2T} \\ 2T &= 30\ \text{h} \\ T &= \frac{30\ \text{h}}{2} \\ T &= 15\ \text{h} \end{aligned}$$

18.
$$\begin{aligned}
\frac{1}{\text{full}} - \frac{1}{\text{empty}} &= \frac{3}{4T} \\
\frac{1}{6\text{ min}} - \frac{1}{8\text{ min}} &= \frac{3}{4T} \\
\frac{4}{24\text{ min}} - \frac{3}{24\text{ min}} &= \frac{3}{4T} \\
\therefore \frac{1}{24\text{ min}} &= \frac{3}{4T} \\
T &= \frac{3}{4}(24\text{ min}) \\
T &= 18\text{ min}
\end{aligned}$$

19.
$$\begin{aligned}
\frac{1}{H} + \frac{1}{C} &= \frac{1}{T} \\
\frac{1}{H} + \frac{1}{3.5\text{ min}} &= \frac{1}{2.1\text{ min}} \\
\frac{1}{H} &= \frac{1}{2.1\text{ min}} - \frac{1}{3.5\text{ min}} \\
\frac{1}{H} &= \frac{50}{105\text{ min}} - \frac{30}{105\text{ min}} \\
\frac{1}{H} &= \frac{20}{105\text{ min}} \\
\frac{1}{H} &= \frac{4}{21\text{ min}} \\
H &= \frac{21}{4}\text{ min} \\
H &= 5.25\text{ min}
\end{aligned}$$

20.
$$\begin{aligned}
\frac{1}{A} + \frac{1}{B} &= \frac{1}{T} \\
\frac{1}{4.5\text{ h}} + \frac{1}{B} &= \frac{1}{2\text{ h}} \\
\frac{1}{B} &= \frac{1}{2\text{ h}} - \frac{1}{4.5\text{ h}} \\
\frac{1}{B} &= \frac{9}{18\text{ h}} - \frac{4}{18\text{ h}} \\
\frac{1}{B} &= \frac{5}{18\text{ h}} \\
B &= \frac{18\text{ h}}{5} \\
B &= 3.6\text{ h}
\end{aligned}$$

Answer Key 9.11

First, the roots:

3 **44** 8 4	9 **32** 7 2	8 **75** 7 $\sqrt{x}$

Check for pattern in the first box:

1. $3 \cdot 8 + 4 = 28$
2. $4 \cdot 8 \cdot 3 = 35$
3. $(8+3) \cdot 4 = 44 \checkmark$

Check #3 pattern with the next box:

$(7+9) \cdot 2 = 32 \checkmark$

Finally:

$$\begin{aligned}
(7+8)\sqrt{x} &= 75 \\
15\sqrt{x} &= 75 \\
\frac{15}{15}\sqrt{x} &= \frac{75}{15} \\
\sqrt{x} &= 5 \\
\therefore (\sqrt{x})^2 &= (5)^2 \\
x &= 25
\end{aligned}$$

Answer Key 10.1

1.
$$\begin{array}{rcrcr} \sqrt{2x+3} & - & 3 & = & 0 \\ & + & 3 & & +3 \\ (\sqrt{2x+3})^2 & & & = & (3)^2 \\ \\ 2x & + & 3 & = & 9 \\ & - & 3 & & -3 \\ & & \dfrac{2x}{2} & = & \dfrac{6}{2} \\ \\ & & x & = & 3 \end{array}$$

2.
$$\begin{array}{rcrcr} \sqrt{5x+1} & - & 4 & = & 0 \\ & + & 4 & = & +4 \\ (\sqrt{5x+1})^2 & & & = & (4)^2 \\ \\ 5x & + & 1 & = & 16 \\ & - & 1 & & -1 \\ & & \dfrac{5x}{5} & = & \dfrac{15}{5} \\ \\ & & x & = & 3 \end{array}$$

3.
$$\begin{array}{rcrcrcrcr} \sqrt{6x-5} & - & x & = & 0 \\ & + & x & & +x \\ (\sqrt{6x-5})^2 & & & = & (x)^2 \\ \\ 6x & - & 5 & = & x^2 \\ -6x & + & 5 & & & - & 6x & + & 5 \\ & & 0 & = & x^2 & - & 6x & + & 5 \\ & & 0 & = & (x & - & 5)(x & - & 1) \\ \\ & & x & = & 5, & & 1 \end{array}$$

4. $(\sqrt{7x+8})^2 = (x)^2$

$$\begin{array}{rcrcrcrcr} 7x & + & 8 & = & x^2 \\ -7x & - & 8 & & & - & 7x & - & 8 \\ & & 0 & = & x^2 & - & 7x & - & 8 \\ & & 0 & = & (x & - & 8)(x & + & 1) \\ \\ & & x & = & -1, & & 8 \end{array}$$

5. $(\sqrt{3+x})^2 = (\sqrt{6x+13})^2$

$$\begin{array}{rcrcrcr} 3 & + & x & = & 6x & + & 13 \\ -3 & - & 6x & & -6x & - & 3 \\ & & \dfrac{-5x}{-5} & = & \dfrac{10}{-5} & & \\ & & x & = & -2 & & \end{array}$$

6. $(\sqrt{x-1})^2 = (\sqrt{7-x})^2$

$$\begin{array}{rcrcrcr} x & - & 1 & = & 7 & - & x \\ +x & + & 1 & & +1 & + & x \\ & & \dfrac{2x}{2} & = & \dfrac{8}{2} & & \\ & & x & = & 4 & & \end{array}$$

7. $(\sqrt[3]{3-3x})^3 = (\sqrt[3]{2x-5})^3$

$$\begin{array}{rcrcrcr} 3 & - & 3x & = & 2x & - & 5 \\ -3 & - & 2x & & -2x & - & 3 \\ & & \dfrac{-5x}{-5} & = & \dfrac{-8}{-5} & & \\ & & x & = & \dfrac{8}{5} & & \end{array}$$

8. $(\sqrt[4]{3x-2})^4 = (\sqrt[4]{x+4})^4$

$$\begin{array}{rcrcrcr} 3x & - & 2 & = & x & + & 4 \\ -x & + & 2 & & -x & + & 2 \\ & & \dfrac{2x}{2} & = & \dfrac{6}{2} & & \\ & & x & = & 3 & & \end{array}$$

9. $(\sqrt{x+7})^2 \geq (2)^2$

$$\begin{array}{rcrcr} x & + & 7 & \geq & 4 \\ & - & 7 & & -7 \\ & & x & \geq & -3 \end{array}$$

10. $(\sqrt{x-2})^2 \leq (4)^2$

$$\begin{array}{rcrcr} x & - & 2 & \leq & 16 \\ & + & 2 & & +2 \\ & & x & \leq & 18 \end{array}$$

11. $(3)^2 < (\sqrt{3x+6})^2 \leq (6)^2$

$$\begin{array}{rcccccc} 9 & < & 3x & + & 6 & \leq & 36 \\ -6 & & & - & 6 & & -6 \\ \dfrac{3}{3} & < & & \dfrac{3x}{3} & & \leq & \dfrac{30}{3} \\ 1 & < & & x & & \leq & 10 \end{array}$$

12. $(0)^2 < (\sqrt{x+5})^2 < (5)^2$

$$\begin{array}{rcccccc} 0 & < & x & + & 5 & < & 25 \\ -5 & & & - & 5 & & -5 \\ -5 & < & & x & & < & 20 \end{array}$$

Answer Key 10.2

1. $$\begin{aligned} \sqrt{x^2} &= \sqrt{75} \\ x &= \pm\sqrt{25\cdot 3} \\ x &= \pm 5\sqrt{3} \end{aligned}$$

2. $$\begin{aligned} \sqrt[3]{x^3} &= \sqrt[3]{-8} \\ x &= -2 \end{aligned}$$

3. $$\begin{array}{rrrcl} x^2 & + & 5 & = & 13 \\ & - & 5 & & -5 \\ & & \sqrt{x^2} & = & \sqrt{8} \\ & & x & = & \pm\sqrt{4\cdot 2} \\ & & x & = & \pm 2\sqrt{2} \end{array}$$

4. $$\begin{array}{rrrcl} 4x^3 & - & 2 & = & 106 \\ & + & 2 & & +2 \\ & & \dfrac{4x^3}{4} & = & \dfrac{108}{4} \\ & & x^3 & = & 27 \\ & & \sqrt[3]{x^3} & = & \sqrt[3]{27} \\ & & x & = & 3 \end{array}$$

5. $$\begin{array}{rrrcl} 3x^2 & + & 1 & = & 73 \\ & - & 1 & & -1 \\ & & \dfrac{3x^2}{3} & = & \dfrac{72}{3} \\ & & x^2 & = & 24 \\ & & \sqrt{x^2} & = & \pm\sqrt{24} \\ & & x & = & \pm\sqrt{4\cdot 6} \\ & & x & = & \pm 2\sqrt{6} \end{array}$$

6. $$\sqrt{(x-4)^2} = \sqrt{49}$$
$$\begin{array}{rrrcrrr} x & - & 4 & = & \pm 7 & & \\ & & x & = & 4 & \pm & 7 \\ & & x & = & 11, & & -3 \end{array}$$

7. $$\sqrt[5]{(x+2)^5} = \sqrt[5]{-3^5}$$
$$\begin{array}{rrrcr} x & + & 2 & = & -3 \\ & - & 2 & & -2 \\ & & x & = & -5 \end{array}$$

8. $$\sqrt[4]{(5x+1)^4} = \pm\sqrt[4]{2^4}$$

$$\begin{array}{rrrrrr} 5x & + & 1 & = & \pm & 2 \\ & - & 1 & & - & 1 \\ & & 5x & = & -1 & \pm \quad 2 \end{array}$$

$$x = -\frac{3}{5} \quad \text{or} \quad \frac{1}{5}$$

9. $$\begin{array}{rrrrrrr} (2x & + & 5)^3 & - & 6 & = & 21 \\ & & & + & 6 & & +6 \\ & & (2x & + & 5)^3 & = & 27 \end{array}$$

$$\sqrt[3]{(2x+5)^3} = \sqrt[3]{27}$$

$$\begin{array}{rrrrr} 2x & + & 5 & = & 3 \\ & - & 5 & & -5 \\ & & 2x & = & -2 \\ & & x & = & -1 \end{array}$$

10. $$\begin{array}{rrrrrrr} (2x & + & 1)^2 & + & 3 & = & 21 \\ & & & - & 3 & & -3 \\ & & (2x & + & 1)^2 & = & 18 \end{array}$$

$$\sqrt{(2x+1)^2} = \sqrt{18} \Rightarrow \sqrt{9 \cdot 2} \Rightarrow \pm 3\sqrt{2}$$

$$\begin{array}{rrrrr} 2x & + & 1 & = & \pm 3\sqrt{2} \\ & - & 1 & & -1 \\ & & \dfrac{2x}{2} & = & \dfrac{-1 \pm 3\sqrt{2}}{2} \\ & & x & = & \dfrac{-1 \pm 3\sqrt{2}}{2} \end{array}$$

11. $$\begin{array}{rrrrl} (x & - & 1)^{\frac{2}{3}} & = & 2^4 \\ (x & - & 1)^{\frac{2}{3} \cdot \frac{3}{2}} & = & 2^{4 \cdot \frac{3}{2}} \\ x & - & 1 & = & \pm 2^6 \\ & + & 1 & & +1 \\ & & x & = & 1 \pm 2^6 \\ & & x & = & 65 \text{ or } -63 \end{array}$$

12. $$\begin{array}{rrrrl} (x & - & 1)^{\frac{3}{2}} & = & 2^3 \\ (x & - & 1)^{\frac{3}{2} \cdot \frac{2}{3}} & = & 2^{3 \cdot \frac{2}{3}} \\ x & - & 1 & = & 2^2 \\ & + & 1 & = & +1 \\ & & x & = & 5 \end{array}$$

13.
$$\begin{array}{rcrcl}
(2 & - & x)^{\frac{3}{2}} & = & 3^3 \\
(2 & - & x)^{\frac{3}{2}\cdot\frac{2}{3}} & = & 3^{3\cdot\frac{2}{3}} \\
2 & - & x & = & 3^2 \\
-2 & & & & -2 \\
 & & -x & = & 7 \\
 & & x & = & -7
\end{array}$$

14.
$$\begin{array}{rclcl}
(2x & + & 3)^{\frac{4}{3}} & = & 2^4 \\
(2x & + & 3)^{\frac{4}{3}\cdot\frac{3}{4}} & = & 2^{4\cdot\frac{3}{4}} \\
2x & + & 3 & = & \pm 2^3 \\
 & - & 3 & & -3 \\
 & & 2x & = & 5 \\
 & & 2x & = & -11 \\
 & & x & = & \frac{5}{2}, -\frac{11}{2}
\end{array}$$

15.
$$\begin{array}{rclcl}
(2x & - & 3)^{\frac{2}{3}} & = & 2^2 \\
(2x & - & 3)^{\frac{2}{3}\cdot\frac{3}{2}} & = & 2^{2\cdot\frac{3}{2}} \\
2x & - & 3 & = & \pm 2^3 \\
 & + & 3 & & +3 \\
 & & 2x & = & 11 \\
 & & 2x & = & -5 \\
 & & x & = & \frac{11}{2}, -\frac{5}{2}
\end{array}$$

16.
$$\begin{array}{rclcl}
(3x & - & 2)^{\frac{4}{5}} & = & 2^4 \\
(3x & - & 2)^{\frac{4}{5}\cdot\frac{5}{4}} & = & 2^{4\cdot\frac{5}{4}} \\
3x & - & 2 & = & \pm 2^5 \\
 & + & 2 & & +2 \\
 & & \frac{3x}{3} & = & \frac{34}{3} \\
 & & \frac{3x}{3} & = & \frac{-30}{3} \\
 & & x & = & \frac{34}{3}, -10
\end{array}$$

Answer Key 10.3

1.
$$\begin{aligned}\frac{30}{2} &= 15\\ 15^2 &= 225\\ \therefore x^2 &- 30x + 225 \text{ or } (x-15)^2\end{aligned}$$

2.
$$\begin{aligned}\frac{24}{2} &= 12\\ 12^2 &= 144\\ \therefore a^2 &- 24a + 144 \text{ or } (a-12)^2\end{aligned}$$

3.
$$\begin{aligned}\frac{36}{2} &= 18\\ 18^2 &= 324\\ \therefore m^2 &- 36m + 324 \text{ or } (m-18)^2\end{aligned}$$

4.
$$\begin{aligned}\frac{34}{2} &= 17\\ 17^2 &= 289\\ \therefore x^2 &- 34x + 289 \text{ or } (x-17)^2\end{aligned}$$

5.
$$\begin{aligned}\frac{15}{2} &= 7.5\\ 7.5^2 &= 56.25\\ \therefore x^2 &- 15x + 56.25 \text{ or } \left(x-\frac{15}{2}\right)^2\end{aligned}$$

6.
$$\begin{aligned}\frac{19}{2} &= \frac{19}{2}\\ \left(\frac{19}{2}\right)^2 &= \frac{361}{4}\\ \therefore r^2 &- 19r + \frac{361}{4} \text{ or } \left(r-\frac{19}{2}\right)^2\end{aligned}$$

7.
$$\begin{aligned}&\frac{1}{2}\\ \left(\frac{1}{2}\right)^2 &= \frac{1}{4}\\ \therefore y^2 &- y + \frac{1}{4} \text{ or } \left(y-\frac{1}{2}\right)^2\end{aligned}$$

8. $$\frac{17}{2}$$

$$\left(\frac{17}{2}\right)^2 = \frac{289}{4}$$

$$\therefore p^2 - 17p + \frac{289}{4} \text{ or } \left(p - \frac{17}{2}\right)^2$$

9.
$$\begin{array}{rrrrrrrr} x^2 & - & 16x & + & 55 & = & 0 & \\ & & & - & 55 & & -55 & \\ & & x^2 & - & 16x & = & -55 & \\ \\ x^2 & - & 16x & + & 64 & = & 64 & - \quad 55 \\ & & (x & - & 8)^2 & = & 9 & \end{array}$$

$$\sqrt{(x-8)^2} = \sqrt{9}$$

$$\begin{array}{rrrrrrr} x & - & 8 & = & \pm & 3 \\ & + & 8 & & + & 8 \\ & & x & = & 8 & \pm & 3 \\ & & x & = & 5, & 11 \end{array}$$

10.
$$\begin{array}{rrrrrrrr} n^2 & - & 4n & - & 12 & = & 0 & \\ & & & + & 12 & & +12 & \\ & & n^2 & - & 4n & = & 12 & \\ \\ n^2 & - & 4n & + & 4 & = & 12 & + \quad 4 \\ & & (n & - & 2)^2 & = & 16 & \end{array}$$

$$\sqrt{(n-2)^2} = \pm\sqrt{16}$$

$$\begin{array}{rrrrrrr} n & - & 2 & = & \pm & 4 \\ & + & 2 & & + & 2 \\ & & n & = & 2 & \pm & 4 \\ & & n & = & 6, & -2 \end{array}$$

11.
$$\begin{array}{rrrrrrrr} v^2 & - & 4v & - & 21 & = & 0 & \\ & & & + & 21 & & +21 & \\ & & v^2 & - & 4v & = & 21 & \\ \\ v^2 & - & 4v & + & 4 & = & 21 & + \quad 4 \\ & & (v & - & 2)^2 & = & 25 & \end{array}$$

$$\sqrt{(v-2)^2} = \sqrt{25}$$

$$\begin{array}{rrrrrrr} v & - & 2 & = & \pm & 5 \\ & + & 2 & & + & 2 \\ & & v & = & 2 & \pm & 5 \\ & & v & = & 7, & -3 \end{array}$$

12. $$\begin{array}{cccccccccc} b^2 & + & 8b & + & 7 & = & 0 \\ & & & - & 7 & & -7 \\ & & b^2 & + & 8b & = & -7 \end{array}$$

$$\begin{array}{ccccccccc} b^2 & + & 8b & + & 16 & = & -7 & + & 16 \\ & & (b & + & 4)^2 & = & 9 \end{array}$$

$$\sqrt{(b+4)^2} = \sqrt{9}$$

$$\begin{array}{cccccccc} b & + & 4 & = & \pm & 3 \\ & - & 4 & & - & 4 \\ & & b & = & -4 & \pm & 3 \\ & & b & = & -7, & -1 \end{array}$$

13. $$\begin{array}{ccccccccc} x^2 & - & 8x & + & 16 & = & -6 & + & 16 \\ & & (x & - & 4)^2 & = & 10 \end{array}$$

$$\sqrt{(x-4)^2} = \sqrt{10}$$

$$\begin{array}{ccccccc} x & - & 4 & = & \pm & \sqrt{10} \\ & + & 4 & & + & 4 \\ & & x & = & 4 & \pm & \sqrt{10} \end{array}$$

14. $$\begin{array}{ccccccccc} x^2 & & & - & 13 & = & 4x \\ & - & 4x & + & 13 & & -4x & + & 13 \\ x^2 & - & 4x & + & 4 & = & 13 & + & 4 \\ & & (x & - & 2)^2 & = & 17 \end{array}$$

$$\sqrt{(x-2)^2} = \sqrt{17}$$

$$\begin{array}{ccccccc} x & - & 2 & = & \pm & \sqrt{17} \\ & + & 2 & & + & 2 \\ & & x & = & 2 & \pm & \sqrt{17} \end{array}$$

15. $$\begin{array}{ccccccccc} & & \frac{3}{3}(k^2 & + & 8k) & = & \frac{-1}{3} \\ & & k^2 & + & 8k & = & -\frac{1}{3} \\ k^2 & + & 8k & + & 16 & = & -\frac{1}{3} & + & 16 \\ & & (k & + & 4)^2 & = & 15\frac{2}{3} \end{array}$$

$$\sqrt{(k+4)^2} = \sqrt{15\frac{2}{3}}$$

$$k + 4 = \pm\sqrt{\frac{47}{3}}$$
$$-4 \qquad -4$$
$$k = -4 \pm \sqrt{\frac{47}{3}}$$

16. $$\frac{4}{4}(a^2 + 9a) = \frac{-2}{4}$$

$$a^2 + 9a + 20.25 = -\frac{1}{2} + 20.25$$

$$(a + 4.5)^2 = 19.75$$

$$\sqrt{(a+4.5)^2} = \pm\sqrt{19.75}$$
$$a + 4.5 = \pm\sqrt{19.75}$$
$$-4.5 \qquad -4.5$$
$$a = -4.5 \pm \sqrt{19.75}$$

Answer Key 10.4

a. $2^2 - 4(4)(-5) \Rightarrow 4 + 80 = 84 \qquad \therefore 2$ real solutions
b. $(-6)^2 - 4(9)(1) \Rightarrow 36 - 36 = 0 \qquad \therefore 1$ real solution
c. $(3)^2 - 4(2)(-5) \Rightarrow 9 + 40 = 49 \qquad \therefore 2$ real solutions
d. $3x^2 + 5x - 3 \Rightarrow (5)^2 - 4(3)(-3) \Rightarrow 25 + 36 = 61 \qquad \therefore 2$ real solutions
e. $3x^2 + 5x - 2 \Rightarrow (5)^2 - 4(3)(-2) \Rightarrow 25 + 24 = 49 \qquad \therefore 2$ real solutions
f. $(-8)^2 - 4(1)(16) \Rightarrow 64 - 64 = 0 \qquad \therefore 1$ real solution
g. $a^2+10a-56 \Rightarrow (10)^2-4(1)(-56) \Rightarrow 100+224 = 324 \qquad \therefore 2$ real solutions
h. $x^2 - 4x + 4 \Rightarrow (-4)^2 - 4(1)(4) \Rightarrow 16 - 16 = 0 \qquad \therefore 1$ real solution
i. $5x^2 - 10x + 26 \Rightarrow (-10)^2 - 4(5)(26) \Rightarrow 100 - 520 = -420$
$\therefore 2$ non-real solutions
j. $n^2 - 10n + 21 \Rightarrow (-10)^2 - 4(1)(21) \Rightarrow 100 - 84 = 16$
$\therefore 2$ real solutions

1.
$$\begin{aligned} a &= 4 \\ b &= 3 \\ c &= -6 \end{aligned}$$

$$\begin{aligned} a &= \frac{-3 \pm \sqrt{3^2 - 4(4)(-6)}}{2(4)} \\ a &= \frac{-3 \pm \sqrt{9 + 96}}{8} \\ a &= \frac{-3 \pm \sqrt{105}}{8} \end{aligned}$$

2. $$\begin{aligned} a &= 3 \\ b &= 2 \\ c &= -3 \end{aligned}$$

$$\begin{aligned} k &= \frac{-2 \pm \sqrt{2^2 - 4(3)(-3)}}{2(3)} \\ k &= \frac{-2 \pm \sqrt{4+36}}{6} \\ k &= \frac{-2 \pm \sqrt{40}}{6} \\ k &= \frac{-2 \pm 2\sqrt{10}}{6} \Rightarrow \frac{-1 \pm \sqrt{10}}{3} \end{aligned}$$

3. $$\begin{aligned} a &= 2 \\ b &= -8 \\ c &= -2 \end{aligned}$$

$$\begin{aligned} x &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(-2)}}{2(2)} \\ x &= \frac{8 \pm \sqrt{64+16}}{4} \\ x &= \frac{8 \pm \sqrt{80}}{4} \\ x &= \frac{8 \pm 4\sqrt{5}}{4} \Rightarrow 2 \pm \sqrt{5} \end{aligned}$$

4. $$\begin{aligned} a &= 6 \\ b &= 8 \\ c &= -1 \end{aligned}$$

$$\begin{aligned} n &= \frac{-8 \pm \sqrt{8^2 - 4(6)(-1)}}{2(6)} \\ n &= \frac{-8 \pm \sqrt{64 + 24}}{12} \\ n &= \frac{-8 \pm \sqrt{88}}{12} \\ n &= \frac{-8 \pm 2\sqrt{22}}{12} \Rightarrow \frac{-4 \pm \sqrt{22}}{6} \end{aligned}$$

5. $$\begin{aligned} a &= 2 \\ b &= -3 \\ c &= 6 \end{aligned}$$

$$\begin{aligned} m &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(6)}}{2(2)} \\ m &= \frac{3 \pm \sqrt{9 - 48}}{4} \\ m &= \frac{3 \pm \sqrt{-39}}{4} \end{aligned}$$

A negative square root means there are 2 non-real solutions or no real solution.

6. $$\begin{aligned} a &= 5 \\ b &= 2 \\ c &= 6 \end{aligned}$$

$$\begin{aligned} p &= \frac{-2 \pm \sqrt{2^2 - 4(5)(6)}}{2(5)} \\ p &= \frac{-2 \pm \sqrt{4 - 120}}{10} \\ p &= \frac{-2 \pm \sqrt{-116}}{10} \end{aligned}$$

A negative square root means there are 2 non-real solutions or no real solution.

7. $$\begin{aligned} a &= 3 \\ b &= -2 \\ c &= -1 \\ \\ r &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)} \\ \\ r &= \frac{2 \pm \sqrt{4+12}}{6} \\ \\ r &= \frac{2 \pm \sqrt{16}}{6} \\ \\ r &= \frac{2 \pm 4}{6} \Rightarrow 1, -\frac{1}{3} \end{aligned}$$

8. $$\begin{aligned} a &= 2 \\ b &= -\ 2 \\ c &= -15 \\ \\ x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-15)}}{2(2)} \\ \\ x &= \frac{2 \pm \sqrt{4+120}}{4} \\ \\ x &= \frac{2 \pm \sqrt{124}}{4} \\ \\ x &= \frac{2 \pm 2\sqrt{31}}{4} \Rightarrow \frac{1 \pm \sqrt{31}}{2} \end{aligned}$$

9. $$\begin{aligned} a &= 4 \\ b &= -3 \\ c &= 10 \\ \\ n &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(10)}}{2(4)} \\ \\ n &= \frac{3 \pm \sqrt{9-160}}{8} \\ \\ n &= \frac{3 \pm \sqrt{-151}}{8} \end{aligned}$$

$\therefore$ 2 non-real solutions

10.
$$\begin{aligned} a &= 1 \\ b &= 6 \\ c &= 9 \end{aligned}$$

$$b = \frac{-6 \pm \sqrt{6^2 - 4(1)(9)}}{2(1)}$$

$$b = \frac{-6 \pm 0\sqrt{36 - 36}}{2}$$

$$b = \frac{-6}{2} \Rightarrow -3$$

11.
$$\begin{array}{ccccccccccc} v^2 & - & 4v & - & 5 & = & -8 & & & & \\ & & & + & 8 & & +8 & & & & \\ & & & & 0 & = & v^2 & - & 4v & + & 3 \end{array}$$

$$\begin{aligned} a &= 1 \\ b &= -4 \\ c &= 3 \end{aligned}$$

$$v = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

$$v = \frac{4 \pm \sqrt{16 - 12}}{2}$$

$$v = \frac{4 \pm \sqrt{4}}{2}$$

$$v = \frac{4 \pm 2}{2} \Rightarrow 2 \pm 1$$

$$v = 3, 1$$

12.
$$\begin{array}{ccccccccccc} x^2 & + & 2x & + & 6 & = & 4 & & & & \\ & & & - & 4 & & -4 & & & & \\ & & & & 0 & = & x^2 & + & 2x & + & 2 \end{array}$$

$$\begin{aligned} a &= 1 \\ b &= 2 \\ c &= 2 \end{aligned}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 - 8}}{2}$$

$$x = \frac{-2 \pm \sqrt{-4}}{2}$$

$\therefore$ 2 non-real solutions

Answer Key 10.5

1. let $u = x^2$
   $\therefore u^2 - 5u + 4 = 0$
   factors to $(u-4)(u-1) = 0$
   replace $u : (x^2-4)(x^2-1) = 0$

   $(x-2)(x+2)(x-1)(x+1) = 0$
   $x = \pm 2, \pm 1$
2. let $u = y^2$
   $\therefore u^2 - 9y + 20 = 0$
   factors to $(u-5)(u-4) = 0$
   replace $u : (y^2-5)(y^2-4) = 0$

$y^2 - 5 = 0$	$(y-2)(y+2) = 0$
$y^2 = 5$	$y = \pm 2$
$y = \pm\sqrt{5}$	
3. $u = m^2$
   $\therefore u^2 - 7u - 8 = 0$
   $(u-8)(u+1) = 0$
   $(m^2-8)(m^2+1) = 0$

   $(m+\sqrt{8})(m-\sqrt{8})(m^2+1) = 0$
   $m = \pm\sqrt{8}$ or $\pm 2\sqrt{2}$
   $m^2 + 1$ has 2 non-real solutions
4. $u = y^2$
   $\therefore u^2 - 29y + 100 = 0$
   $(u-25)(u-4) = 0$
   $(y^2-25)(y^2-4) = 0$

   $(y-5)(y+5)(y-2)(y+2) = 0$
   $y = \pm 5, \pm 2$
5. let $u = a^2$
   $\therefore u^2 - 50u + 49 = 0$
   $(u-49)(u-1) = 0$
   $(a^2-49)(a^2-1) = 0$

   $(a-7)(a+7)(a-1)(a+1) = 0$
   $a = \pm 7, \pm 1$
6. let $u = b^2$
   $\therefore u^2 - 10u + 9 = 0$
   $(u-9)(u-1) = 0$
   $(b^2-9)(b^2-1) = 0$

$(b-3)(b+3)(b-1)(b+1)=0$
$b=\pm 3, \pm 1$

7. $x^4-20x^2+64=0$
let $u=x^2$
$\therefore u^2-20u+64=0$
$(u-16)(u-4)=0$
$(x^2-16)(x^2-4)=0$

$(x-4)(x+4)(x-2)(x+2)=0$
$x=\pm 4, \pm 2$

8. $6z^6-z^3-12=0$
let $u=z^3$
$\therefore 6u^2-u-12=0$
$(3u+4)(2u-3)=0$
$(3z^3+4)(2z^3-3)=0$

$3z^3+4=0 \qquad 2z^3-3=0$
$3z^3=-4 \qquad 2z^3=3$

$$z^3=-\frac{4}{3} \qquad z^3=\frac{3}{2}$$

$$z=\sqrt[3]{-\frac{4}{3}} \qquad z=\sqrt[3]{\frac{3}{2}}$$

9. $z^6-19z^3-216=0$
let $u=z^3$
$\therefore u^2-19u-216=0$
$(u-27)(u+8)=0$
$(z^3-27)(z^3+8)=0$

$(z-3)(z^2+3z+9)(z+2)(z^2-2z+4)=0$
$z=3,-2$
2 non-real solutions each for the 2nd and 4th factors

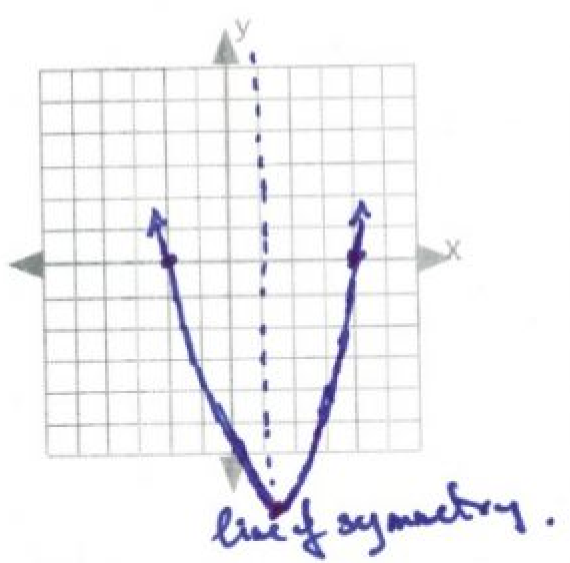
10. let $u=x^3$
$\therefore u^2-35u+216=0$
$(u-27)(u-8)=0$
$(x^3-27)(x^3-8)=0$
$(x-3)(x^2+3x+9)(x-2)(x^2+2x+4)$
$x=2,3$
2 non-real solutions each for the 2nd and 4th factors

Answer Key 10.6

1. intercepts:
$$\begin{aligned} y &= 0 \\ 0 &= x^2 - 2x - 8 \\ 0 &= (x-4)(x+2) \\ x &= 4, -2 \end{aligned}$$
vertex: $\left[\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right]$ line of symmetry:

$(1, -9)$

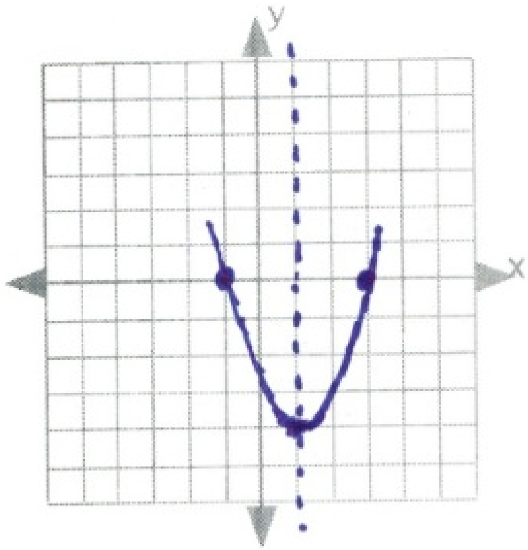
$$\begin{aligned} x &= \frac{-b}{2a} \\ x &= \frac{-(-2)}{2(1)} \Rightarrow \frac{2}{2} \text{ or } 1 \\ \therefore f(1) &= 1^2 - 2(1) - 8 \\ f(1) &= -9 \end{aligned}$$



2. intercepts:
$$\begin{aligned} 0 &= x^2 - 2x - 3 \\ 0 &= (x-3)(x+1) \\ x &= 3, -1 \end{aligned}$$
line of symmetry: $x = \frac{-(-2)}{2(1)} \Rightarrow \frac{2}{2}$ or 1 vertex:

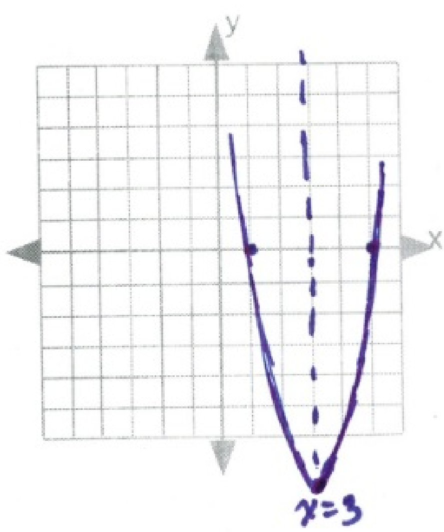
$$\begin{aligned} f(1) &= 1^2 - 2(1) - 3 \\ f(1) &= 1 - 2 - 3 \\ f(1) &= -4 \end{aligned}$$

$(1, -4)$



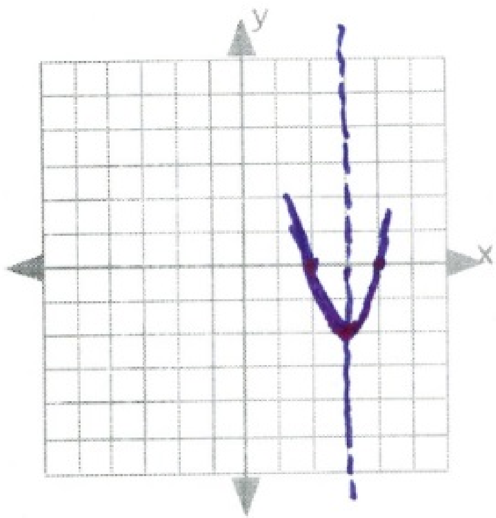
3. intercepts:
$$\begin{aligned} 0 &= 2x^2 - 12x + 10 \\ 0 &= 2(x^2 - 6x + 5) \\ 0 &= 2(x-5)(x-1) \\ x &= 5, 1 \end{aligned}$$
line of symmetry:
$$\begin{aligned} x &= \frac{-b}{2a} \\ x &= \frac{-6}{2(1)} \Rightarrow \frac{6}{2} \text{ or } 3 \end{aligned}$$
vertex:
$$\begin{aligned} f(3) &= 2(3)^2 - 12(3) + 10 \\ f(3) &= 18 - 36 + 10 \\ f(3) &= -8 \end{aligned}$$

$(3, -8)$



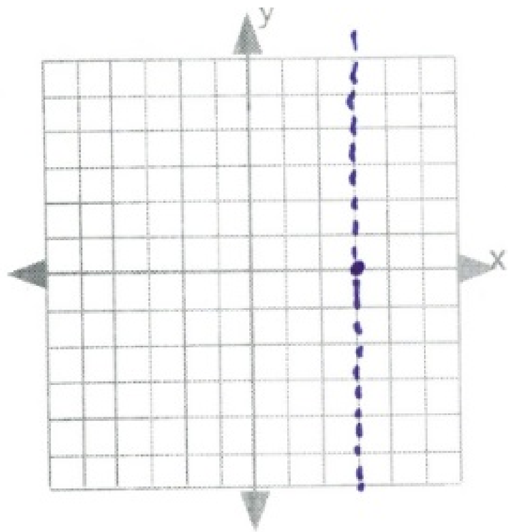
4. intercepts:
$$\begin{aligned} 0 &= 2x^2 - 12x + 16 \\ 0 &= 2(x^2 - 6x + 8) \\ 0 &= 2(x-4)(x-2) \\ x &= 4, 2 \end{aligned}$$
line of symmetry:
$$\begin{aligned} x &= \frac{-b}{2a} \\ x &= \frac{-(-12)}{2(2)} \Rightarrow \frac{12}{4} \text{ or } 3 \end{aligned}$$
$$\begin{aligned} f(3) &= 2(3)^2 - 12(3) + 16 \\ f(3) &= 18 - 36 + 16 \\ \text{vertex:} f(3) &= -2 \end{aligned}$$

$(3, -2)$



5. intercepts:
$$\begin{aligned} 0 &= -2x^2 + 12x - 18 \\ 0 &= -2(x^2 - 6x + 9) \\ 0 &= -2(x-3)(x-3) \\ x &= 3 \end{aligned}$$
line of symmetry:
$$\begin{aligned} x &= \frac{-b}{2a} \\ x &= \frac{-12}{2(-2)} \Rightarrow \frac{-12}{-4} \text{ or } 3 \end{aligned}$$
$$\begin{aligned} f(3) &= -2(3)^2 - 12(3) - 18 \\ f(3) &= -18 + 36 - 18 \\ \text{vertex:} f(3) &= 0 \end{aligned}$$

$(0, 3)$



6. intercepts:

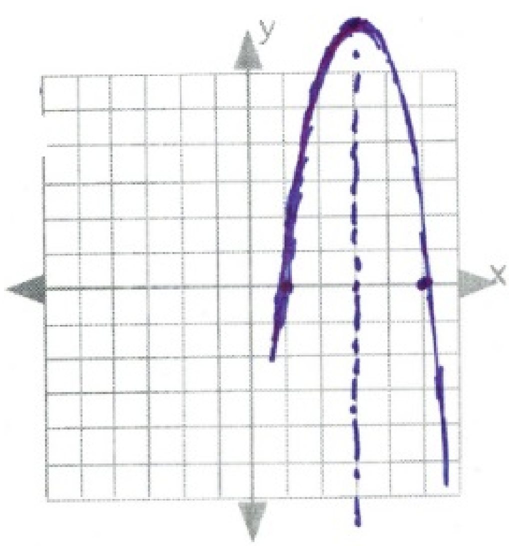
$$\begin{aligned} 0 &= -2x^2 + 12x - 10 \\ 0 &= -2(x^2 - 6x + 5) \\ 0 &= -2(x-5)(x-1) \\ x &= 5, 1 \end{aligned}$$

line of symmetry:

$$\begin{aligned} x &= \frac{-b}{2a} \\ x &= \frac{-12}{2(-2)} \Rightarrow \frac{-12}{-4} \text{ or } 3 \end{aligned}$$

$$\begin{aligned} f(3) &= -2(3)^2 - 12(3) - 10 \\ f(3) &= -18 + 36 - 10 \\ \text{vertex:} f(3) &= 8 \end{aligned}$$

$(3, 8)$



7. intercepts:

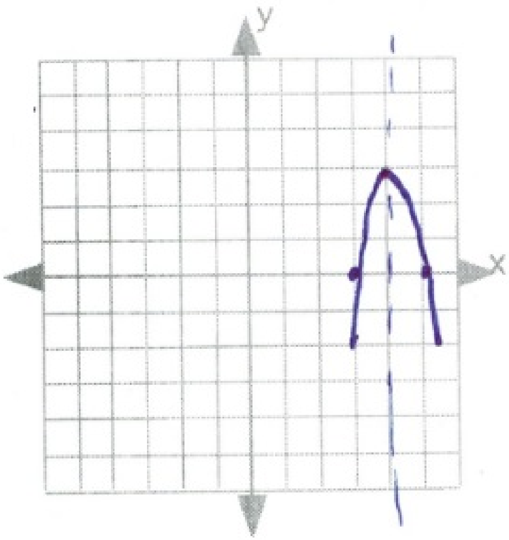
$$\begin{aligned} 0 &= -3x^2 + 24x - 45 \\ 0 &= -3(x^2 - 8x + 15) \\ 0 &= -3(x-3)(x-5) \\ x &= 3, 5 \end{aligned}$$

line of symmetry:

$$\begin{aligned} x &= \frac{-b}{2a} \\ x &= \frac{-24}{2(-3)} \Rightarrow \frac{-24}{-6} \text{ or } 4 \end{aligned}$$

$$\begin{aligned} f(4) &= -3(4)^2+24(4)-45 \\ f(4) &= -48+96-45 \\ \text{vertex:} f(4) &= 3 \end{aligned}$$

$$(4,3)$$



8. intercepts:

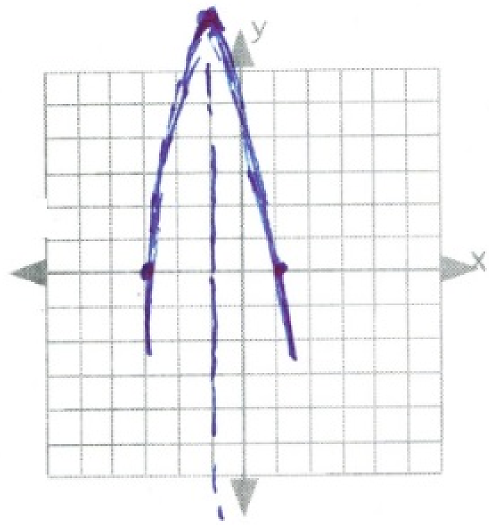
$$\begin{aligned} 0 &= -2(x^2+2x)+6 \\ 0 &= -2x^2-4x+6 \\ 0 &= -2(x^2+2x-3) \\ 0 &= -2(x+3)(x-1) \\ x &= -3,1 \end{aligned}$$

line of symmetry:

$$\begin{aligned} x &= \frac{-b}{2a} \\ x &= \frac{-(-4)}{2(-2)} \Rightarrow \frac{4}{-4} \text{ or } -1 \end{aligned}$$

$$\begin{aligned} f(-1) &= -2(-1)^2-4(-1)+6 \\ f(-1) &= -2+4+6 \\ \text{vertex:} f(-1) &= 8 \end{aligned}$$

$$(-1,8)$$

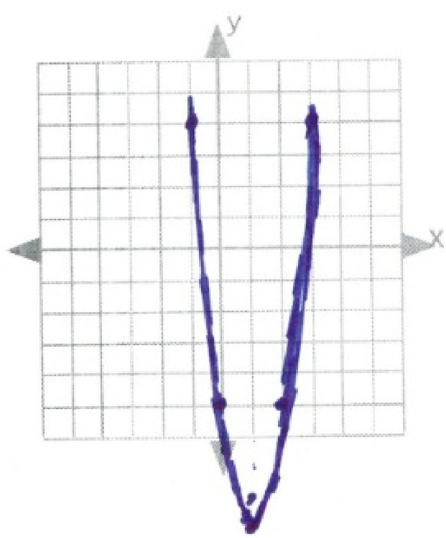


line of symmetry:

9.

$$x = \frac{-b}{2a} \Rightarrow \frac{-(-6)}{2(3)} \Rightarrow \frac{6}{6} \text{ or } 1$$

x	y
3	4
2	−5
1	−9
0	−5
−1	4

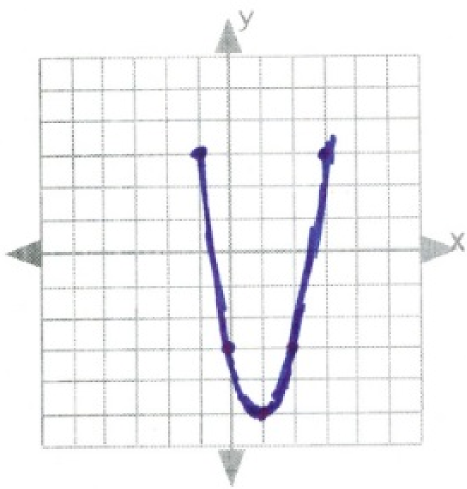


line of symmetry:

10.

$$x = \frac{-b}{2a} \Rightarrow \frac{-(-4)}{2(2)} \Rightarrow \frac{4}{4} \text{ or } 1$$

x	y
3	3
2	−3
1	−5
0	−3
−1	3

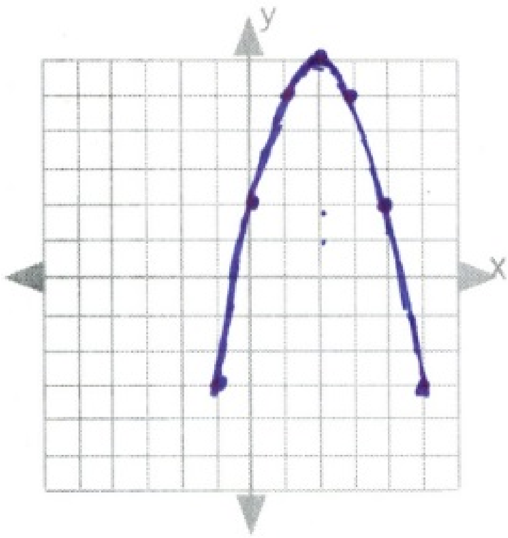


line of symmetry:

11.

$$x = \frac{-b}{2a} \Rightarrow \frac{-4}{2(-1)} \Rightarrow \frac{-4}{-2} \text{ or } 2$$

x	y
5	−3
4	2
3	5
2	6
1	5
0	2
−1	−3

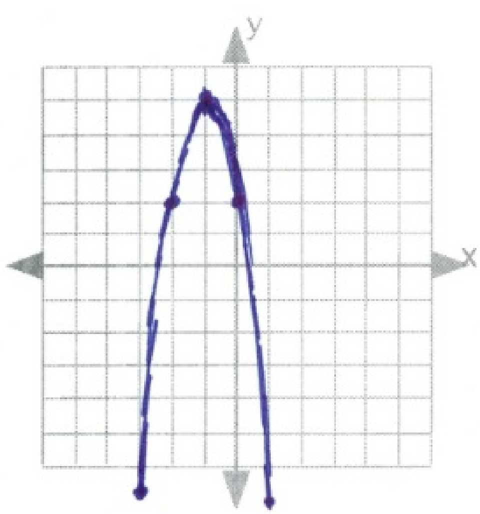


line of symmetry:

12.

$$x = \frac{-b}{2a} \Rightarrow \frac{-(-6)}{2(-3)} \Rightarrow \frac{6}{-6} \text{ or } -1$$

x	y
1	–7
0	2
–1	5
–2	2
–3	–7



Answer Key 10.7

1.

$$\begin{aligned} x + y &= 22 \Rightarrow x = 22 - y \\ x - y &= 120 \end{aligned}$$

$$\begin{aligned} (22 - y)y &= 120 \\ 22y - y^2 &= 120 \end{aligned}$$

$$\begin{aligned} 0 &= y^2 - 22y + 120 \\ 0 &= y^2 - 12y - 10y + 120 \\ 0 &= y(y - 12) - 10(y - 12) \\ 0 &= (y - 12)(y - 10) \end{aligned}$$

$$y = 12, \quad 10$$

$\therefore$ numbers are $10, 12$

2.

$$\begin{aligned} x - y &= 4 \Rightarrow x = y + 4 \\ x \cdot y &= 140 \end{aligned}$$

$$\begin{aligned} (y + 4)y &= 140 \\ y^2 + 4y &= 140 \end{aligned}$$

$$\begin{aligned} y^2 + 4y - 140 &= 0 \\ y^2 - 10y + 14y - 140 &= 0 \\ y(y - 10) + 14(y - 10) &= 0 \\ (y - 10)(y + 14) &= 0 \end{aligned}$$

$$y = 10, \quad -14$$

$$\begin{aligned} y &= 10, & x &= 10 + 4 = 14 \\ y &= -14, & x &= -14 + 4 = -10 \end{aligned}$$

$\therefore$ numbers are $10, 14$ and $-10, -14$

3.
$$\begin{array}{rcrcl} A - B &=& 8 &\Rightarrow& A = B + 8 \\ A^2 + B^2 &=& 320 & & \end{array}$$

$$\begin{array}{rcrcrcrcr} & & (B &+& 8)^2 &+& B^2 &=& 320 \\ B^2 &+& 16B &+& 64 &+& B^2 &=& 320 \\ & & &-& 320 & & & & -320 \\ & & 2B^2 &+& 16B &-& 256 &=& 0 \\ & & 2(B^2 &+& 8B &-& 128) &=& 0 \\ & & 2(B &+& 16)(B &-& 8) &=& 0 \end{array}$$

$$B = -16, \quad 8$$

$$\therefore A = B + 8$$
$$A = -8, \quad 16$$

$\therefore (-16, -8)$ and $(8, 16)$

4. $x, \quad x + 2$

$$\begin{array}{rcrcrcrcr} & & x^2 &+& (x &+& 2)^2 &=& 244 \\ x^2 &+& x^2 &+& 4x &+& 4 &=& 244 \\ & & & & & & -244 & & -244 \\ & & 2x^2 &+& 4x &-& 240 &=& 0 \\ & & 2(x^2 &+& 2x &-& 120) &=& 0 \\ & & 2(x &-& 10)(x &+& 12) &=& 0 \end{array}$$

$$x = 10, \quad -12$$

$$\begin{array}{rcrcrcl} x &=& 10, & & x + 2 &=& 12 \\ x &=& -12, & & x + 2 &=& -10 \end{array}$$

$\therefore$ numbers are $10, 12$ or $-12, -10$

5. $x, \quad x + 2$

$$\begin{array}{rcl} x^2 - (x+2)^2 &=& 60 \\ x^2 - (x^2 + 4x + 4) &=& 60 \\ x^2 - x^2 - 4x - 4 &=& 60 \\ +4 && +4 \\ \dfrac{-4x}{-4} &=& \dfrac{64}{-4} \\ x &=& -16 \\ x + 2 &\Rightarrow& -16 + 2 \\ &\Rightarrow& -14 \end{array}$$

$-16, -14$

6. $x, \quad x + 2$

$$\begin{array}{rcl} x^2 + (x+2)^2 &=& 452 \\ x^2 + x^2 + 4x + 4 &=& 452 \\ -452 && -452 \\ 2x^2 + 4x - 448 &=& 0 \\ 2(x^2 + 2x - 224) &=& 0 \\ 2(x - 14)(x + 16) &=& 0 \\ x &=& 14, -16 \\ x &=& 14 \\ x + 2 &=& 16 \\ x &=& -16 \\ x + 2 &=& -14 \end{array}$$

$14, 16 \text{ and } -16, -14$

7. $x, \quad x + 2, \quad x + 4$

$$\begin{array}{rcl} x(x+2) &=& 38 + x + 4 \\ x^2 + 2x &=& 42 + x \\ -x - 42 && -42 - x \\ x^2 + x - 42 &=& 0 \\ (x+7)(x-6) &=& 0 \\ x &=& -7, \quad 6 \end{array}$$

$\therefore$ numbers are $6, 8, 10$

8. $x, x+2, x+4$

$$\begin{array}{rrrrrrrrr}
& & (x)(x & + & 2) & = & 52 & + & x & + & 4 \\
\\
x^2 & + & 2x & & & = & 56 & + & x \\
& - & x & - & 56 & & -56 & - & x \\
x^2 & + & x & - & 56 & = & 0 \\
(x & + & 8)(x & - & 7) & = & 0 \\
\\
& & & & x & = & -8, 7
\end{array}$$

$\therefore$ numbers are $7, 9, 11$

9.

$$\begin{array}{rrrrrrrrrrrr}
& & A & = & T & + & 4 \\
\\
A & \cdot & T & = & 80 & + & (A & - & 4) & (T & - & 4) \\
(T & + & 4)T & = & 80 & + & (T & + & 4-4) & (T & - & 4) \\
\\
T^2 & + & 4T & = & 80 & + & T^2 & - & 4T \\
-T^2 & + & 4T & & & - & T^2 & + & 4T \\
& & \frac{8T}{8} & = & \frac{80}{8} \\
\\
& & T & = & 10 \\
\\
& & \therefore A & = & T & + & 4 \\
& & A & = & 10 & + & 4 & = & 14
\end{array}$$

10.

$$\begin{array}{rrrrrrrrrrrrr}
& & C & = & K & + & 3 \\
& & CK & = & (C & + & 5) & & (K & + & 5) & - & 130 \\
\\
(K & + & 3)K & = & (K & + & 3 & + & 5)(K & + & 5) & - & 130 \\
K^2 & + & 3K & = & K^2 & + & 13K & + & 40 & - & 130 \\
-K^2 & - & 13K & & -K^2 & - & 13K \\
& & \frac{-10K}{-10} & = & \frac{-90}{-10} \\
\\
& & K & = & 9 \\
\\
& & \therefore C & = & 9 & + & 3 & = & 12
\end{array}$$

11.

$$\begin{aligned}
J &= S + 1 \\
(J + 5)(S + 5) &= 230 + J \cdot S \\
(S + 1 + 5)(S + 5) &= 230 + (S + 1)S \\
(S + 6)(S + 5) &= 230 + S^2 + S \\
S^2 + 11S + 30 &= S^2 + S + 230 \\
-S^2 - S - 30 & \quad -S^2 - S - 30 \\
\frac{10S}{10} &= \frac{200}{10} \\
S &= 20 \\
J &= 21
\end{aligned}$$

12.

$$\begin{aligned}
J &= S + 2 \\
(S + 2)(J + 2) &= 48 + S \cdot J \\
(S + 2)(S + 2 + 2) &= 48 + S(S + 2) \\
(S + 2)(S + 4) &= 48 + S^2 + 25 \\
S^2 + 6S + 8 &= 48 + S^2 + 25 \\
-S^2 - 2S - 8 & \quad -8 - S^2 - 25 \\
\frac{4S}{4} &= \frac{40}{4} \\
S &= 10 \\
J &= 12
\end{aligned}$$

13.

$$\begin{aligned}
d &= r \cdot t \\
r \cdot t &= 240 \\
\therefore r &= \frac{240}{t} \\
(r + 20)(t - 1) &= 240 \\
(\frac{240}{t} + 20)(t - 1) &= 240 \\
240 + 20t - \frac{240}{t} - 20 &= 240 \\
(20t - \frac{240}{t} - 20 &= 0)(t) \\
(20t^2 - 240 - 20t &= 0)(\div 20)
\end{aligned}$$

$$\begin{aligned}
t^2 - 12 - t &= 0 \\
(t - 4)(t + 3) &= 0 \\
t &= 4, -3 \\
r &= \frac{240}{4} \text{ or } 60 \text{ km/h} \\
\text{faster} &= 80 \text{ km/h}
\end{aligned}$$

14.

$$\begin{aligned} d &= r \cdot t \\ r \cdot t &= 100 \\ r &= \frac{100}{t} \end{aligned}$$

$$\begin{aligned} (r + 20)(t - 0.5) &= 120 \\ \left(\frac{100}{t} + 20\right)(t - 0.5) &= 120 \\ 100 + 20t - \frac{50}{t} - 10 &= 120 \\ -120 & \quad -120 \\ 20t - 30 - \frac{50}{t} &= 0 \\ (20t - 30 - \frac{50}{t} &= 0)(t) \\ (20t^2 - 30t - 50 &= 0)(\div 10) \end{aligned}$$

$$\begin{aligned} 2t^2 - 3t - 5 &= 0 \\ t &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-5)}}{2(2)} \\ t &= \frac{3 \pm 7}{4} = \frac{10}{4} \text{ or } \frac{-4}{4} \\ t &= 2.5 \text{ h} \end{aligned}$$

$$\text{Answer: } \frac{100 \text{ km}}{2.5 \text{ h}} = \frac{40 \text{ km}}{\text{h}}, \frac{120 \text{ km}}{2 \text{ h}} = \frac{60 \text{ km}}{\text{h}}$$

15.

$$\begin{aligned} d &= r \cdot t \\ r \cdot t &= 150 \\ r &= \frac{150}{t} \end{aligned}$$

$$\begin{aligned} (r + 5)(t - 1.5) &= 150 \\ \left(\frac{150}{t} + 5\right)(t - 1.5) &= 150 \\ 150 + 5t - \frac{225}{t} - 7.5 &= 150 \\ (5t - \frac{225}{t} - 7.5 &= 0)(t) \\ (5t^2 - 7.5t - 225 &= 0)(2) \\ (10t^2 - 15t - 450 &= 0)(\div 5) \end{aligned}$$

$$\begin{aligned} 2t^2 - 3t - 90 &= 0 \\ (t + 6)(2t - 15) &= 0 \\ t &= -6, \frac{15}{2} \\ r &= \frac{150}{t} \\ r &= \frac{150}{\frac{15}{2}} \\ r &= \frac{150}{1} \cdot \frac{2}{15} \\ r &= 20 \text{ km/h} \end{aligned}$$

16.

$$
\begin{aligned}
d &= r\cdot t\\
r \cdot t &= 180 \Rightarrow r = \frac{180}{t}\\
(r + 15)(t - 1) &= 180\\
\left(\frac{180}{t} + 15\right)(t - 1) &= 180\\
180 + 15t - \frac{180}{t} - 15 &= 180\\
(15t - 15 - \frac{180}{t} &= 0)(t)\\
(15t^2 - 15t - 180 &= 0)(\div 15)\\
t^2 - t - 12 &= 0\\
(t - 4)(t + 3) &= 0\\
t &= 4, -3\\
r &= \frac{180}{4} = 45
\end{aligned}
$$

17.

$$
\begin{aligned}
r \cdot t &= 72 \Rightarrow r = \frac{72}{t}\\
(r + 12)(9 - t) &= 72\\
\left(\frac{72}{t} + 12\right)(9 - t) &= 72\\
\frac{648}{t} + 108 - 72 - 12t &= 72\\
-72 \qquad & \quad -72\\
(-12t - 36 + \frac{648}{t} &= 0)(t)\\
(-12t^2 - 36t + 648 &= 0)(\div - 12)\\
t^2 + 3t - 54 &= 0\\
(t + 9)(t - 6) &= 0\\
t &= -9, 6\\
r &= \frac{72}{6} = 12 \text{ (there)}\\
r &= 24 \text{ (return)}
\end{aligned}
$$

18.

$$\begin{aligned}
r \cdot t &= 120 \Rightarrow r = \frac{120}{t} \\
(r + 10)(7 - t) &= 120 \\
\left(\frac{120}{t} + 10\right)(7 - t) &= 120 \\
\frac{840}{t} + 70 - 120 - 10t &= 120 \\
-120 \qquad\qquad &\quad -120 \\
\left(-10t - 170 + \frac{840}{t}\right. &= 0)(t) \\
(-10t^2 - 170t + 840 &= 0)(\div -10) \\
t^2 + 17t - 84 &= 0 \\
(t + 21)(t - 4) &= 0 \\
t &= -21, 4 \\
r &= \frac{120}{4} \text{ or } 30 \text{ km/h} \\
r + 10 &= 40 \text{ km/h}
\end{aligned}$$

19.

$$\begin{aligned}
r \cdot t &= 240 \Rightarrow r = \frac{240}{t} \\
(r + 20)(t - 1) &= 240 \\
\left(\frac{240}{t} + 20\right)(t - 1) &= 240 \\
240 + 20t - \frac{240}{t} - 20 &= 240 \\
\left(20t - 20 - \frac{240}{t}\right. &= 0)(t) \\
(20t^2 - 20t - 240 &= 0)(\div 20) \\
t^2 - t - 12 &= 0 \\
(t - 4)(t + 3) &= 0 \\
t &= 4, -3 \\
r &= \frac{240}{4} \text{ or } 60 \text{ km/h}
\end{aligned}$$

20.

$$
\begin{aligned}
r \cdot t &= 600 \Rightarrow r = \frac{600}{t} \\
(r-50)(7-t) &= 600 \\
(\frac{600}{t} - 50)(7-t) &= 600 \\
\frac{4200}{t} - 350 - 600 + 50t &= 600 \\
-600 \qquad & \quad -600 \\
(50t - 1550 + \frac{4200}{t} &= 0)(t) \\
(50t^2 - 1550t + 4200 &= 0)(\div 50) \\
t^2 - 31t + 84 &= 0 \\
(t-3)(t-28) &= 0 \\
t &= 3, 28 \\
r &= \frac{600}{3} \text{ or } 200 \text{ km/h}
\end{aligned}
$$

21.

$$
\begin{aligned}
L &= 4 + W \\
\text{Area} &= L \cdot W \\
60 &= (4+W)W \\
60 &= 4W + W^2 \\
0 &= W^2 + 4W - 60 \\
0 &= W^2 + 10W - 6W - 60 \\
0 &= W(W+10) - 6(W+10) \\
0 &= (W+10)(W-6) \\
W &= -10, \quad 6 \\
L &= 6 + 4 = 10
\end{aligned}
$$

22. $$\begin{aligned} W &= L - 10 \\ \text{Area} &= L \cdot W \\ 200 &= L(L - 10) \\ 200 &= L^2 - 10L \\ 0 &= L^2 - 10L - 200 \\ 0 &= L^2 + 10L - 20L - 200 \\ 0 &= L(L + 10) - 20(L + 10) \\ 0 &= (L + 10)(L - 20) \\ L &= -10,\ 20 \\ W &= 20 - 10 = 10 \end{aligned}$$

23. $$\begin{aligned} \text{Area}_{\text{large}} - \text{Area}_{\text{small}} &= 2800\ \text{m}^2 \\ (150 + 2x)(120 + 2x) - (150)(120) &= 2800 \\ 18000 + 240x + 300x + 4x^2 - 18000 &= 2800 \\ -2800 &\quad -2800 \\ 4x^2 + 540x - 2800 &= 0 \\ x^2 + 135x - 700 &= 0 \\ (x - 5)(x + 140) &= 0 \\ x &= 5, -140 \end{aligned}$$

walkway $= 5$ m

24. $$\begin{aligned} \text{Area}_{\text{large}} - \text{Area}_{\text{small}} &= 74\ \text{m}^2 \\ (25 + 2x)(10 + 2x) - (25)(10) &= 74 \\ 250 + 20x + 50x + 4x^2 - 250 &= 74 \\ -74 &\quad -74 \\ 4x^2 + 70x - 74 &= 0 \\ 2x^2 + 35x - 37 &= 0 \\ (x - 1)(2x + 37) &= 0 \\ x &= 1, -\frac{37}{2} \end{aligned}$$

the overlap $= 1$ m

25.

$$\begin{aligned}
L &= W + 4 \\
L \cdot W &= 60 \\
\\
(W + 4)W &= 60 \\
W^2 + 4W &= 60 \\
-60 &\quad -60 \\
W^2 + 4W - 60 &= 0 \\
(W - 6)(W + 10) &= 0 \\
\\
W &= 6, \quad -10 \\
L &= 6 + 4 = 10
\end{aligned}$$

26.

$$\begin{aligned}
(x + 5)^2 &= 4(x)^2 \\
\\
x^2 + 10x + 25 &= 4x^2 \\
-x^2 - 10x - 25 &\quad -x^2 - 10x - 25 \\
0 &= 3x^2 - 10x - 25 \\
0 &= (x - 5)(3x + 5) \\
x &= 5, \quad -\frac{5}{3}
\end{aligned}$$

27.

$$\begin{aligned}
L &= 20 + W \\
L \cdot W &= 2400 \\
\\
(20 + W)W &= 2400 \\
W^2 + 20W &= 2400 \\
-2400 &\quad -2400 \\
W^2 + 20W - 2400 &= 0 \\
(W + 60)(W - 40) &= 0 \\
\\
W &= -60, \quad 40 \\
L &= 20 + 40 = 60
\end{aligned}$$

28.

$$\begin{aligned}
L &= W + 8 \\
(L + 2)(W + 2) &= L \cdot W + 60 \\
\\
(W + 8 + 2)(W + 2) &= (W + 8)W + 60 \\
W^2 + 12W + 20 &= W^2 + 8W + 60 \\
-W^2 - 8W - 20 &\quad -W^2 - 8W - 20 \\
\frac{4W}{4} &= \frac{40}{4} \\
\\
W &= 10 \\
L &= 10 + 8 = 18
\end{aligned}$$

Answer Key 10.8

1. $x = 5, x = 2$
   $x - 5 = 0, x - 2 = 0$

   $0 = (x-5)(x-2)$
   $0 = x^2 - 7x + 10$
2. $x = 3, x = 6$
   $x - 3 = 0, x - 6 = 0$

   $(x-3)(x-6) = 0$
   $0 = x^2 - 9x + 18$
3. $x = 20, x = 2$
   $x - 20 = 0, x - 2 = 0$

   $0 = (x-20)(x-2)$
   $0 = x^2 - 22x + 40$
4. $x = 13, x = 1$
   $x - 13 = 0, x - 1 = 0$

   $0 = (x-13)(x-1)$
   $0 = x^2 - 14x + 13$
5. $x = 4, x = 4$
   $x - 4 = 0, x - 4 = 0$

   $0 = (x-4)(x-4)$
   $0 = x^2 - 8x + 16$
6. $x = 0, x = 9$
   $x - 9 = 0, x$

   $x(x-9) = 0$
   $0 = x^2 - 9x$
7. $x = \frac{3}{4}, x = \frac{1}{4}$

   $$x - \frac{3}{4} = 0, x - \frac{1}{4} = 0$$

   $$0 = \left(x - \frac{3}{4}\right)\left(x - \frac{1}{4}\right)$$

   $$0 = x^2 - x + \frac{3}{16}$$
8. $x = \frac{5}{8}, x = \frac{5}{7}$

   $$x - \frac{5}{8} = 0, x - \frac{5}{7} = 0$$

$$0 = \left(x - \frac{5}{8}\right)\left(x - \frac{5}{7}\right)$$

$$0 = x^2 - \frac{75}{56}x + \frac{25}{56}$$

9. $x = \frac{1}{2}, x = \frac{1}{3}$

$$x - \frac{1}{2} = 0, x - \frac{1}{3} = 0$$

$$0 = \left(x - \frac{1}{2}\right)\left(x - \frac{1}{3}\right)$$

$$0 = x^2 - \frac{5}{6}x + \frac{1}{6}$$

10. $x = \frac{1}{2}, x = \frac{2}{3}$

$$x - \frac{1}{2} = 0, x - \frac{2}{3} = 0$$

$$0 = \left(x - \frac{1}{2}\right)\left(x - \frac{2}{3}\right)$$

$$0 = x^2 - \frac{7}{6}x + \frac{1}{3}$$

11. $x = 5, x = -5$

$$x - 5 = 0, x + 5 = 0$$

$$0 = (x - 5)(x + 5)$$
$$0 = x^2 - 25$$

12. $x = 1, x = -1$

$$x - 1 = 0, x + 1 = 0$$

$$0 = (x - 1)(x + 1)$$
$$0 = x^2 - 1$$

13. $x = \frac{1}{5}, x = -\frac{1}{5}$

$$x - \frac{1}{5} = 0, x + \frac{1}{5} = 0$$

$$0 = (x - \frac{1}{5})(x + \frac{1}{5})$$

$$0 = x^2 - \frac{1}{25}$$

14. $x = \sqrt{7}, x = -\sqrt{7}$

$$x - \sqrt{7} = 0, x + \sqrt{7} = 0$$

$0 = (x - \sqrt{7})(x + \sqrt{7})$
$0 = x^2 - 7$

15. $x = \sqrt{11}, x = -\sqrt{11}$
$x - \sqrt{11} = 0, x + \sqrt{11} = 0$

$0 = (x - \sqrt{11})(x + \sqrt{11})$
$0 = x^2 - 11$

16. $x = 2\sqrt{3}, x = -2\sqrt{3}$
$x - 2\sqrt{3} = 0, x + 2\sqrt{3} = 0$

$0 = (x - 2\sqrt{3})(x + 2\sqrt{3})$
$0 = x^2 - 12$

17. $x = 3, x = 5, x = 8$
$(x - 3) = 0, (x - 5) = 0, (x - 8) = 0$

$$\begin{array}{ccccccccc} (x & - & 3)(x & - & 5)(x & - & 8) & = & 0 \\ (x^2 & - & 8x & + & 15)(x & - & 8) & = & 0 \\ x^3 & - & 8x^2 & + & 15x & & & & \\ & - & 8x^2 & + & 64x & - & 120 & = & 0 \\ x^3 & - & 16x^2 & + & 79x & - & 120 & = & 0 \end{array}$$

18. $x = -4, x = 0, x = 4$
$x + 4 = 0, x, x - 4 = 0$

$x(x + 4)(x - 4) = 0$
$x(x^2 - 16) = 0$
$x^3 - 16x = 0$

19. $x = -9, x + 6 = 0, x = -2$
$x + 9 = 0, x + 6 = 0, x + 2 = 0$

$$\begin{array}{ccccccccc} (x & + & 9)(x & + & 6)(x & + & 2) & = & 0 \\ (x^2 & + & 15x & + & 54)(x & + & 2) & = & 0 \\ x^3 & + & 15x^2 & + & 54x & & & & \\ & + & 2x^2 & + & 30x & + & 108 & = & 0 \\ x^3 & + & 17x^2 & + & 84x & + & 108 & = & 0 \end{array}$$

20. $x = -1, x = 1, x = 5$
$x + 1 = 0, x - 1 = 0, x - 5 = 0$

$(x + 1)(x - 1)(x - 5) = 0$
$(x^2 - 1)(x - 5) = 0$
$x^3 - 5x^2 - x + 5 = 0$

21. $x = -2, x = 2, x = 5, x = -5$
$x + 2 = 0, x - 2 = 0, x - 5 = 0, x + 5 = 0$

$(x + 2)(x - 2)(x - 5)(x + 5) = 0$
$(x^2 - 4)(x^2 - 25) = 0$
$x^4 - 29x^2 + 100 = 0$

22. $x = 2\sqrt{3}, x = -2\sqrt{3}, x = \sqrt{5}, x = -\sqrt{5}$
$x - 2\sqrt{3} = 0, x + 2\sqrt{3} = 0, x - \sqrt{5} = 0, x + \sqrt{5} = 0$

$$(x-2\sqrt{3})(x+2\sqrt{3})(x-\sqrt{5})(x+\sqrt{5})=0$$
$$(x^2-12)(x^2-5)=0$$
$$x^4-17x^2+60=0$$

Midterm 3 Prep Answer Key

Midterm Three Review

1. $\dfrac{6(a-b)}{(a+b)(a^2-ab+b^2)} \cdot \dfrac{a^2-ab+b^2}{(a+b)(a-b)} \Rightarrow \dfrac{b}{(a+b)^2}$
2. $\dfrac{x}{(x+5)(x-5)} - \dfrac{2}{(x-5)(x-1)}$

   $\text{LCD} = (x+5)(x-5)(x-1)$

   $\therefore \dfrac{x(x-1)-2(x+5)}{(x+5)(x-5)(x-1)} \Rightarrow \dfrac{x^2-x-2x-10}{(x+5)(x-5)(x-1)} \Rightarrow \dfrac{x^2-3x-10}{(x+5)(x-5)(x-1)}$

   $\Rightarrow \dfrac{(x-5)(x+2)}{(x+5)(x-5)(x-1)} \Rightarrow \dfrac{x+2}{(x+5)(x-1)}$
3. $\dfrac{\left(1-\dfrac{6}{x}\right)x^2}{\left(\dfrac{4}{x}-\dfrac{24}{x^2}\right)x^2} \Rightarrow \dfrac{x^2-6x}{4x-24} \Rightarrow \dfrac{x(x-6)}{4(x-6)} \Rightarrow \dfrac{x}{4}$
4. $$\left(\frac{4}{x+4}-\frac{5}{x-2}=5\right)(x+4)(x-2)$$

$$\begin{array}{rcrcrcrcrcrcrcr}
4(x & - & 2) & - & 5(x & + & 4) & = & 5(x & + & 4)(x & - & 2) \\
4x & - & 8 & - & 5x & - & 20 & = & 5(x^2 & + & 2x & - & 8) \\
 & & & & -x & - & 28 & = & 5x^2 & + & 10x & - & 40 \\
 & & & & +x & + & 28 & & & + & x & + & 28 \\
 & & & & & & 0 & = & 5x^2 & + & 11x & - & 12 \\
 & & & & & & & & & & & & \\
 & & & & & & 0 & = & 5x^2 & + & 15x-4x & - & 12 \\
 & & & & & & 0 & = & 5x(x & + & 3)-4(x & + & 3) \\
 & & & & & & 0 & = & (x & + & 3)(5x & - & 4) \\
 & & & & & & & & & & & & \\
 & & & & & & x & = & -3, & & \dfrac{4}{5} & &
\end{array}$$

5. True
6. False

7. $4 \cdot 6 + 3\sqrt{36 \cdot 2} + 4$
$24 + 3 \cdot 6\sqrt{2} + 4$
$28 + 18\sqrt{2}$

8. $\dfrac{\sqrt{300100a^{54}b^2}}{\sqrt{3ab^2}} \Rightarrow \sqrt{100a^4} \Rightarrow 10a^2$

9. $\dfrac{(12)(3+\sqrt{6})}{(3-\sqrt{6})(3+\sqrt{6})} \Rightarrow \dfrac{36+12\sqrt{6}}{9-6} \Rightarrow \dfrac{3612+124\sqrt{6}}{31} \Rightarrow 12+4\sqrt{6}$

10. $\left(\dfrac{a^0 1b^3}{c^6 d^{-12}}\right)^{\frac{1}{3}} \Rightarrow \left(\dfrac{b^3 d^{12}}{c^6}\right)^{\frac{1}{3}} \Rightarrow \dfrac{bd^4}{c^2}$

11.
$$\begin{aligned}
(\sqrt{5x-6})^2 &= (x)^2 \\
5x-6 &= x^2 \\
0 &= x^2-5x+6 \\
0 &= (x-3)(x-2) \\
\\
x &= 3, 2
\end{aligned}$$

12.
$$\begin{array}{ccccccccc}
\sqrt{2x+9} & + & 3 & = & x & & & & \\
 & - & 3 & & & - & 3 & & \\
 & & \sqrt{2x+9} & = & x & - & 3 & & \\
 & & & & & & & & \\
 & & (\sqrt{2x+9})^2 & = & (x & - & 3)^2 & & \\
2x & + & 9 & = & x^2 & - & 6x & + & 9 \\
-2x & - & 9 & & & - & 2x & - & 9 \\
 & & 0 & = & x^2 & - & 8x & & \\
 & & 0 & = & x(x & - & 8) & & \\
 & & & & & & & & \\
 & & x & = & 0, & & 8 & &
\end{array}$$

13.
$(\sqrt{x-3})^2 = (\sqrt{2x-5})^2$

$$\begin{array}{cccccccc}
 & x & - & 3 & = & 2x & - & 5 \\
- & x & + & 5 & & -x & + & 5 \\
 & & & 2 & = & x & &
\end{array}$$

14.

a. $b^2 - 4ac$
$= (4)^2 - 4(2)(3)$
$= 16 - 24$
$= -8$
2 non-real solutions

b. $b^2 - 4ac$
$= (-2)^2 - 4(3)(-8)$
$= 4 + 96$
$= 100$
2 real solutions

15.

a.
$$\begin{aligned} \frac{3x^2}{3} &= \frac{27}{3} \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

b.
$$\begin{aligned} 2x^2 - 16x &= 0 \\ 2x(x-8) &= 0 \\ x &= 0, 8 \end{aligned}$$

16.

a. $(x-4)(x+3) \Rightarrow x = 4, -3$

b.
$$\begin{aligned} x^2 + 9x + 8 &= 0 \\ (x+8)(x+1) &= 0 \\ x &= -1, -8 \end{aligned}$$

17.

$$\left(\frac{x-3}{2} + \frac{6}{x+3} = 1\right)(2)(x+3)$$

$$\begin{array}{ccccccccccc} (x & - & 3)(x & + & 3) & + & 6(2) & = & 2(x & + & 3) \\ x^2 & & & - & 9 & + & 12 & = & 2x & + & 6 \\ & - & 2x & & & - & 6 & & -2x & - & 6 \\ & & x^2 & - & 2x & - & 3 & = & 0 & & \\ & & (x & - & 3)(x & + & 1) & = & 0 & & \\ & & & & & & & & & & \\ & & & & & & x & = & 3, & -1 & \end{array}$$

18.

$$\left(\frac{x-2}{x} = \frac{x}{x+4}\right)(x)(x+4)$$

$$\begin{array}{cccccccc} & (x & - & 2)(x & + & 4) & = & x^2 \\ & x^2 & + & 2x & - & 8 & = & x^2 \\ - & x^2 & & & + & 8 & & -x^2 + 8 \\ & & & & & \frac{2x}{2} & = & \frac{8}{2} \\ & & & & & & & \\ & & & & & x & = & 4 \end{array}$$

19.

width $= W$ length $= L = 3 + 2W$

$$\begin{aligned}
A &= L \cdot W \\
65 &= W(3+2W) \\
65 &= 3W + 2W^2 \\
\\
0 &= 2W^2 + 3W - 65 \\
0 &= 2W^2 - 10W + 13W - 65 \\
0 &= 2W(W-5) + 13(W-5) \\
0 &= (W-5)(2W+13) \\
\\
W &= 5, -\frac{13}{2} \\
\\
L &= 3 + 2W \\
L &= 13
\end{aligned}$$

20.

$x, x+2, x+4$

$$\begin{array}{ccccccccccc}
 & & x(x & + & 2) & = & 68 & + & x & + & 4 \\
x^2 & + & 2x & & & = & x & + & 72 & & \\
 & - & x & - & 72 & & -x & - & 72 & & \\
x^2 & + & x & - & 72 & = & 0 & & & & \\
\\
(x & + & 9)(x & - & 8) & = & 0 & & & & \\
 & & & & x & = & -9, & 8 & & &
\end{array}$$

$\therefore$ 8, 10, 12

21.

$d = r \cdot t$ and $d_{\text{up}} = d_{\text{down}}$

$$\begin{array}{cccccccc}
 & 8(r & - & 4) & = & 6(r & + & 4) \\
 & 8r & - & 32 & = & 6r & + & 24 \\
- & 6r & + & 32 & & -6r & + & 32 \\
 & & & 2r & = & 56 & & \\
 & & & r & = & 28 & \text{km/h} &
\end{array}$$

$$\begin{aligned}
22. \qquad A &= \frac{1}{2}bh \\
(330 &= \frac{1}{2}(h+8)h)(2) \\
660 &= h^2 + 8h \\
0 &= h^2 + 8h - 660 \\
0 &= h^2 + 30h - 22h - 660 \\
0 &= h(h+30) - 22(h+30) \\
0 &= (h+30)(h-22) \\
h &= -30, 22 \\
\therefore b &= h + 8 \\
&= 22 + 8 \\
&= 30
\end{aligned}$$

Midterm 3: Version A Answer Key

1. $\dfrac{15m^3}{4n^2} \cdot \dfrac{171m^3}{124n} \cdot \dfrac{31m^4}{342n^2} \Rightarrow \dfrac{15m^{10}}{32n^5}$
2. $\dfrac{8x-8y}{x^3+y^3} \cdot \dfrac{x^2-xy+y^2}{x^2-y^2}$

   $\Rightarrow \dfrac{8(x-y)}{(x+y)(x^2-xy+y^2)} \cdot \dfrac{x^2-xy+y^2}{(x+y)(x-y)} \Rightarrow \dfrac{8}{(x+y)^2}$
3. $\text{LCD} = 6(n-3)$

   $\dfrac{5(n-3) - 2 \cdot 6(n-3) - 5 \cdot 6}{6(n-3)}$

   $\dfrac{5n - 15 - 12n + 36 - 30}{6(n-3)}$

   $\dfrac{-7n-9}{6(n-3)}$
4. $\dfrac{\left(\dfrac{x^2}{y^2} - 4\right) y^3}{\left(\dfrac{x+2y}{y^3}\right) y^3} \Rightarrow \dfrac{x^2y - 4y^3}{x+2y} \Rightarrow \dfrac{y(x^2-4y^2)}{x+2y} \Rightarrow \dfrac{y(x-2y)(x+2y)}{(x+2y)}$

   $\Rightarrow y(x-2y)$
5. $3 \cdot 5 + 2\sqrt{36 \cdot 2} - 4$

   $15 + 2 \cdot 6\sqrt{2} - 4$

   $11 + 12\sqrt{2}$
6. $\dfrac{\sqrt{m^7n^{32}}}{\sqrt{2n}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} \Rightarrow \dfrac{\sqrt{m^6 \cdot m \cdot n^2 \cdot 2}}{\sqrt{4}} \Rightarrow \dfrac{m^3n\sqrt{2m}}{2}$
7. $\dfrac{2-x}{1-\sqrt{3}} \cdot \dfrac{1+\sqrt{3}}{1+\sqrt{3}} \Rightarrow \dfrac{2+2\sqrt{3}-x-x\sqrt{3}}{1-3}$

   $\Rightarrow \dfrac{2+2\sqrt{3}-x-x\sqrt{3}}{-2}$ or $\dfrac{x+x\sqrt{3}-2-2\sqrt{3}}{2}$
8. $$\begin{aligned} (\sqrt{7x+8})^2 &= (x)^2 \\ 7x+8 &= x^2 \\ 0 &= x^2-7x-8 \\ 0 &= (x-8)(x+1) \\ \\ x &= 8, -1 \end{aligned}$$

9.

a. $$\begin{aligned} \frac{4x^2}{4} &= \frac{64}{4} \\ x^2 &= 16 \\ x &= \pm 4 \end{aligned}$$

b. $$\begin{aligned} 3x^2 - 12x &= 0 \\ 3x(x-4) &= 0 \\ x &= 0, 4 \end{aligned}$$

10.

a. $$\begin{aligned} (x-5)(x-1) &= 0 \\ x &= 5, 1 \end{aligned}$$

b. $$\begin{aligned} x^2 + 10x + 9 &= 0 \\ (x+9)(x+1) &= 0 \\ x &= -9, -1 \end{aligned}$$

11.

$$\left(\frac{x+4}{-4} = \frac{8}{x}\right)(-4)(x)$$

$$\begin{aligned} x(x+4) &= -4(8) \\ x^2 + 4x &= -32 \\ 0 &= x^2 + 4x + 32 \qquad \text{Does not factor} \end{aligned}$$

12.

$$\begin{aligned} \text{Let } u &= x^2 \\ u^2 - 13u + 36 &= 0 \\ u^2 - 4u - 9u + 36 &= 0 \\ u(u-4) - 9(u-4) &= 0 \\ (u-4)(u-9) &= 0 \\ (x^2-4)(x^2-9) &= 0 \\ (x-2)(x+2)(x-3)(x+3) &= 0 \\ x &= \pm 2, \pm 3 \end{aligned}$$

13.
$$\begin{aligned}A &= \frac{1}{2}bh\\[1em] 300 &= \frac{1}{2}(h+10)h\\[1em] 600 &= h^2+10h\\ 0 &= h^2+10h-600\\ 0 &= (h-20)(h+30)\\[1em] h &= 20,-30\\[1em] \therefore b &= h+10=30\end{aligned}$$

14.
$$x, x+2, x+4$$

$$\begin{array}{rrrrrrrrrrr} & & x(x & + & 4) & = & 38 & + & x & + & 2\\ x^2 & + & 4x & & & = & x & + & 40 & & \\ & - & x & - & 40 & & -x & - & 40 & & \\ x^2 & + & 3x & - & 40 & = & 0 & & & & \\[1em] & & & & 0 & = & (x & + & 8)(x & - & 5)\\ & & & & x & = & -8, & & 5 & & \end{array}$$

$\therefore$ 5, 7, 9

15.
$$\begin{aligned}r_s t_s &= r_f t_f\\[1em] r(4.5\text{ h}) &= (r+150)(3.0\text{ h})\\ 4.5r &= \quad 3.0r+450\\ -3.0r & \quad\;\; -3.0r\\ 1.5r &= 450\\[1em] r &= \frac{450}{1.5}\text{ or 300 km/h}\\[1em] r_f &= 300+150\\ r_f &= 450\text{ km/h}\end{aligned}$$

Midterm 3: Version B Answer Key

1. $\dfrac{5m^3}{4n^2}\cdot\dfrac{13n^3}{3m^3}\cdot\dfrac{12m^4}{262n^2}\Rightarrow\dfrac{5m^4}{2n}$
2. $\dfrac{3x(x+3)}{3(x+3)}\cdot\dfrac{6x(x+3)}{(x+6)(x-3)}\Rightarrow\dfrac{6x^2(x+3)}{(x+6)(x-3)}$
3. $\left(\dfrac{5x}{x+3}-\dfrac{5x}{x-3}+\dfrac{90}{x^2-9}\right)(x-3)(x+3)$

   $\Rightarrow\dfrac{5x(x-3)-5x(x+3)+90}{(x+3)(x-3)}$

   $\Rightarrow\dfrac{5x^2-15x-5x^2-15x+90}{(x+3)(x-3)}$

   $\Rightarrow\dfrac{-30x+90}{(x+3)(x-3)}\Rightarrow\dfrac{-30(x-3)}{(x+3)(x-3)}\Rightarrow\dfrac{-30}{x+3}$
4. $\dfrac{\left(\dfrac{9a^2}{b^2}-25\right)(b^2)}{\left(\dfrac{3a}{b}+5\right)(b^2)}\Rightarrow\dfrac{9a^2-25b^2}{3ab+5b^2}\Rightarrow\dfrac{(3a-5b)(3a+5b)}{b(3a+5b)}\Rightarrow\dfrac{3a-5b}{b}$
5. $\sqrt{2\cdot 36\cdot d^2\cdot d}+4\sqrt{2\cdot 9\cdot d^2\cdot d}-2(7d^2)$
   $6d\sqrt{2d}+4\cdot 3d\sqrt{2d}-14d^2$
   $6d\sqrt{2d}+12d\sqrt{2d}-14d^2$
   $18d\sqrt{2d}-14d^2$
6. $\dfrac{\sqrt{a^{65}b^3}}{\sqrt{5a}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}}\Rightarrow\dfrac{\sqrt{5a^5b^3}}{\sqrt{25}}\Rightarrow\dfrac{\sqrt{5\cdot a^4\cdot a\cdot b^2\cdot b}}{5}\Rightarrow\dfrac{a^2b\sqrt{5ab}}{5}$
7. $\dfrac{\sqrt{5}}{3+\sqrt{5}}\cdot\dfrac{3-\sqrt{5}}{3-\sqrt{5}}\Rightarrow\dfrac{3\sqrt{5}-5}{9-5}\Rightarrow\dfrac{3\sqrt{5}-5}{4}$
8. $$\begin{aligned}(\sqrt{4x+12})^2&=(x)^2\\4x+12&=x^2\\0&=x^2-4x-12\\0&=(x-6)(x+2)\\ \\x&=6,-2\end{aligned}$$
9.

a. $$\begin{aligned}\frac{2x^2}{2} &= \frac{98}{2}\\ x^2 &= 49\\ x &= \pm 7\end{aligned}$$

b. $$\begin{aligned}4x^2-12x &= 0\\ 4x(x-3) &= 0\\ x &= 0,3\end{aligned}$$

10.

a. $$\begin{aligned}(x-5)(x+4) &= 0\\ x &= 5,-4\end{aligned}$$

b. $$\begin{aligned}x^2-2x-35 &= 0\\ (x-7)(x+5) &= 0\\ x &= 7,-5\end{aligned}$$

11.

$$\left(\frac{x-3}{x+2}+\frac{6}{x+3}=1\right)(x+2)(x+3)$$

$$\begin{array}{ccccccccccccccc} & (x-3) & \cdot & (x+3) & + & 6(x & + & 2) & = & (x & + & 2)(x & + & 3) \\ & x^2 & - & 9 & + & 6x & + & 12 & = & x^2 & + & 5x & + & 6 \\ - & x^2 & + & 9 & - & 6x & - & 12 & & -x^2 & - & 6x & - & 12 \\ & & & & & & & 0 & = & -x & + & 3 & & \\ & & & & & & + & x & & +x & & & & \\ & & & & & & & x & = & 3 & & & & \end{array}$$

$$\text{Let } u = x^2$$

12.

$$\begin{aligned}u^2-5u+4 &= 0\\ (u-4)(u-1) &= 0\\ \\ (x^2-4)(x^2-1) &= 0\\ (x-2)(x+2)(x-1)(x+1) &= 0\\ x &= \pm 2, \pm 1\end{aligned}$$

13.
$$\begin{array}{rcrcrcr} L & = & 3 & + & W & & \\ \\ P & = & 2L & + & 2W & & \\ 46 & = & 2(3 & + & W) & + & 2W \\ 46 & = & 6 & + & 2W & + & 2W \\ -6 & & -6 & & & & \\ 40 & = & 4W & & & & \\ \\ W & = & \dfrac{40}{4} & = & 10 & & \\ \\ \therefore L & = & W & + & 3 & & \\ L & = & 10 & + & 3 & = & 13 \end{array}$$

14.
$x, x+2, x+4$

$$\begin{array}{rcrcrcrcrcr} & & x(x & + & 2) & = & 16 & + & x & + & 4 \\ x^2 & + & 2x & & & = & 20 & + & x & & \\ & - & x & - & 20 & & -20 & - & x & & \\ x^2 & + & x & - & 20 & = & 0 & & & & \\ \\ & & & & 0 & = & (x & + & 5)(x & - & 4) \\ & & & & x & = & -5, & & 4 & & \end{array}$$

$\therefore$ 4, 6, 8

15.
$$\begin{array}{rcl} d & = & r \cdot t \\ d_{\text{up}} & = & d_{\text{return}} \\ \\ 4(r-5) & = & 2(r+5) \\ 4r-20 & = & 2r+10 \\ -2r+20 & & -2r+20 \\ \dfrac{2r}{2} & = & \dfrac{30}{2} \\ \\ r & = & 15 \end{array}$$

Midterm 3: Version C Answer Key

1. $\dfrac{153m^3}{4n^2} \cdot \dfrac{17n^3}{30102m^3} \cdot \dfrac{31m^4}{342n^2} \Rightarrow \dfrac{3m^4}{16n}$
2. $\dfrac{5v^2-25v}{5v+25} \cdot \dfrac{10v}{v^2-11v+30} \Rightarrow \dfrac{5v(v-5)}{5(v+5)} \cdot \dfrac{10v}{(v-5)(v-6)} \Rightarrow \dfrac{10v^2}{(v+5)(v-6)}$
3. $$\left(\frac{8}{2x} = \frac{2}{x} + 1\right)(2x)$$
$$\begin{array}{rcl} 8 & = & 2 \cdot 2 + 1(2x) \\ 8 & = & 4 + 2x \\ -4 & & -4 \\ \dfrac{4}{2} & = & \dfrac{2x}{2} \\ & & \\ x & = & 2 \end{array}$$
4. $\dfrac{\left(\dfrac{x^2}{y^2} - 16\right) y^3}{\left(\dfrac{x+4y}{y^3}\right) y^3} \Rightarrow \dfrac{x^2y - 16y^3}{x+4y} \Rightarrow \dfrac{y(x^2-16y^2)}{x+4y} \Rightarrow \dfrac{y(x-4y)(x+4y)}{x+4y}$

   $\Rightarrow y(x-4y)$
5. $5y + 2 \cdot 9y + 6y\sqrt{y}$

   $23y + 6y\sqrt{y}$
6. $\dfrac{(28)(7+3\sqrt{5})}{(7-3\sqrt{5})(7+3\sqrt{5})} \Rightarrow \dfrac{196+84\sqrt{5}}{49 - 9 \cdot 5} \Rightarrow \dfrac{196+84\sqrt{5}}{4} \Rightarrow 49 + 21\sqrt{5}$
7. $(27a^{-\frac{3}{8}})^{\frac{1}{3}}$

   $27^{\frac{1}{3}} a^{-\frac{3}{8} \cdot \frac{1}{3}}$

   $3a^{-\frac{1}{8}}$

   $\dfrac{3}{a^{\frac{1}{8}}} \Rightarrow \dfrac{3}{\sqrt[8]{a}}$
8. $$(\sqrt{3x-2})^2 = (\sqrt{5x+4})^2$$

$$\begin{array}{rrrcrrr} & 3x & - & 2 & = & 5x & + & 4 \\ - & 3x & - & 4 & & -3x & - & 4 \\ & & & -6 & = & 2x & & \\ & & & x & = & \dfrac{-6}{2} & = & -3 \end{array}$$

9.

a. $$\begin{aligned} \frac{2x^2}{2} &= \frac{72}{2} \\ x^2 &= 36 \\ x &= \pm 6 \end{aligned}$$

b. $$\begin{aligned} 2x^2 - 8x &= 0 \\ 2x(x-4) &= 0 \\ x &= 0, 4 \end{aligned}$$

10.

a. $$\begin{aligned} (x+5)(x+1) &= 0 \\ x &= -1, -5 \end{aligned}$$

b. Quadratic:

$$x^2 - 10x + 4 = 0$$

$$\frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(4)}}{2}$$

$$\frac{-10 \pm \sqrt{100-16}}{2}$$

$$\frac{10 \pm \sqrt{84}}{2}$$

$$\frac{10 \pm 2\sqrt{21}}{2} \Rightarrow 5 \pm \sqrt{21}$$

11.

$$\left(\frac{8}{4x} = \frac{2}{x} + 3\right)(4x)$$

$$\begin{array}{ccccc} 8 & = & 8 & + & 3(4x) \\ -8 & & -8 & & \\ \dfrac{0}{12} & = & \dfrac{12x}{12} & & \\ x & = & 0 & & \therefore \text{Undefined. No solution} \end{array}$$

12.

$$\begin{aligned} \text{Let } u &= x^2 \\ \\ u^2 - 17u + 16 &= 0 \\ (u-16)(u-1) &= 0 \\ \\ (x^2-16)(x^2-1) &= 0 \\ (x-4)(x+4)(x-1)(x+1) &= 0 \\ x &= \pm 1, \pm 4 \end{aligned}$$

13.

$$\begin{aligned} L &= W + 6 \\ \text{Area} &= 12 + \text{Perimeter} \\ \\ L \cdot W &= 12 + 2L + 2W \\ (W+6)W &= 12 + 2(W+6) + 2W \\ W^2 + 6W &= 12 + 2W + 12 + 2W \end{aligned}$$

$$\begin{array}{rcrcrcr} 0 & = & W^2 & + & 6W & & \\ & & & - & 4W & - & 24 \\ 0 & = & W^2 & + & 2W & - & 24 \\ 0 & = & (W & + & 6)(W & - & 4) \\ W & = & -6, & & 4 & & \\ \\ L & = & W & + & 6 & & \\ L & = & 4 & + & 6 & & \\ L & = & 10 & & & & \end{array}$$

14.

$x, x+2, x+4$

$$\begin{array}{rcrcrcrcrcrcr} & & x(x & + & 4) & = & 31 & + & x & + & 2 \\ x^2 & + & 4x & & & = & 33 & + & x & & \\ & - & x & - & 33 & & -33 & - & x & & \\ x^2 & + & 3x & - & 33 & = & 0 & & & & \end{array}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-3 \pm \sqrt{3^2 - 4(1)(-33)}}{2}$$

$$\frac{-3 \pm \sqrt{141}}{2}$$

15.

$$\begin{array}{rrrcrl} & d & = & r & \cdot & t \\ \text{To outpost:} & 60 & = & (B & - & C)5 \\ \text{Back:} & 60 & = & (B & + & C)3 \\ \\ & 60 & = & 5B & - & 5C \\ & 60 & = & 3B & + & 3C \\ \\ & 12 & = & B & - & C \\ + & 20 & = & B & + & C \\ & 32 & = & 2B & & \\ & \therefore B & = & 16 & \text{km/h} & \\ \\ & \therefore B & + & C & = & 20 \\ & 16 & + & C & = & 20 \\ - & 16 & & & & -16 \\ & & & C & = & 4\text{ km/h} \end{array}$$

Midterm 3: Version D Answer Key

1. $\dfrac{151m^3}{4n^2}\cdot\dfrac{131n^3}{453m^6}\cdot\dfrac{31m^4}{3931n^2}\Rightarrow\dfrac{m}{12n}$
2. $\dfrac{3x^2-9x}{3x+9}\cdot\dfrac{12x}{x^2+2x-15}\Rightarrow\dfrac{3x(x-3)}{3(x+3)}\cdot\dfrac{124x}{(x+5)(x-3)}\Rightarrow\dfrac{12x^2}{(x+3)(x+5)}$
3. $$\left(\frac{2}{x-4}-\frac{6}{x-3}=3\right)(x+4)(x-3)$$
$$\begin{array}{rrrrrrrrrrrr} 2(x & - & 3) & - & 6(x & + & 4) & = & 3(x & + & 4)(x & - & 3) \\ 2x & - & 6 & - & 6x & - & 24 & = & 3(x^2 & + & x & - & 12) \\ & & & & -4x & - & 30 & = & 3x^2 & + & 3x & - & 36 \\ & & & & +4x & + & 30 & & & + & 4x & + & 30 \\ & & & & & & 0 & = & 3x^2 & + & 7x & - & 6 \\ & & & & & & 0 & = & (x & + & 3)(3x & - & 2) \\ \\ & & & & & & x & = & -3, & \frac{2}{3} \end{array}$$
4. $\dfrac{\left(\dfrac{x^2}{y^2}-9\right)y^3}{\left(\dfrac{x+3y}{y^3}\right)y^3}\Rightarrow\dfrac{x^2y-9y^3}{x+3y}\Rightarrow\dfrac{y(x^2-9y^2)}{x+3y}\Rightarrow\dfrac{y(x-3y)(x+3y)}{(x+3y)}$
$$\Rightarrow y(x-3y)$$
5. $5y^2+2\cdot 7y+\sqrt{25y^2\cdot y}$
$5y^2+14y+5y\sqrt{y}$
6. $\dfrac{15}{3-\sqrt{5}}\cdot\dfrac{3+\sqrt{5}}{3+\sqrt{5}}\Rightarrow\dfrac{45+15\sqrt{5}}{9-5}\Rightarrow\dfrac{45+15\sqrt{5}}{4}$
7. $\left(\dfrac{a^0 1b^4}{c^8d^{-12}}\right)^{\frac{1}{4}}\Rightarrow\dfrac{b^{4\cdot\frac{1}{4}}}{c^{8\cdot\frac{1}{4}}d^{-12\cdot\frac{1}{4}}}\Rightarrow\dfrac{b}{c^2d^{-3}}\Rightarrow\dfrac{bd^3}{c^2}$
8. $$\begin{array}{rrrrl} \sqrt{2x+9} & - & 3 & = & x \\ & + & 3 & & +3 \\ (\sqrt{2x+9})^2 & & & = & (x+3)^2 \end{array}$$
$$\begin{array}{rrrrrrrrr} & 2x & + & 9 & = & x^2 & + & 6x & + & 9 \\ - & 2x & - & 9 & & & - & 2x & - & 9 \\ & & & 0 & = & x^2 & + & 4x \\ & & & 0 & = & x(x & + & 4) \\ \\ & & & x & = & 0, & -4 \end{array}$$
9.
   a. $$\begin{aligned} 8x^2-32x &= 0 \\ 8x(x-4) &= 0 \\ x &= 0, 4 \end{aligned}$$

b. $$\begin{aligned}\frac{3x^2}{2} &= \frac{48}{3}\\ \sqrt{x^2} &= \sqrt{16}\\ x &= \pm 4\end{aligned}$$

10.

a. $$\begin{aligned}x^2-5x+4 &= 0\\ (x-4)(x-1) &= 0\\ x &= 1,4\end{aligned}$$

b. $$\begin{aligned}(x-3)(x-1) &= 0\\ x &= 1,3\end{aligned}$$

11. $$\begin{aligned}2(x+4) &= x(x)\\ 2x+8 &= x^2\\ 0 &= x^2-2x-8\\ 0 &= x^2-4x+2x-8\\ 0 &= x(x-4)+2(x-4)\\ 0 &= (x-4)(x+2)\\ x &= -2,\ 4\end{aligned}$$

12. $$\begin{aligned}\text{Let } u &= x^2\\ u^2-48u-49 &= 0\\ (u-49)(u+1) &= 0\\ (x^2-49)(x^2+1) &= 0\\ (x-7)(x+7)(x^2+1) &= 0\\ x^2+1 &= \text{cannot be factored}\\ x &= \pm 7\end{aligned}$$

13. $$\begin{aligned}40 &= \frac{1}{2}(h-2)(h)\\ 80 &= h^2-2h\\ 0 &= h^2-2h-80\\ 0 &= (h-10)(h+8)\\ h &= 10,-8\\ b &= 10-2=8\end{aligned}$$

14.

$x, x+2, x+4$

$$\begin{array}{rrrrrrrrr}
x(x & + & 4) & = & 41 & + & 4(x & + & 2) \\
x^2 & + & 4x & = & 41 & + & 4x & + & 8 \\
 & - & 4x & & & - & 4x & & \\
 & & \sqrt{x^2} & = & \sqrt{49} & & & & \\
 & & x & = & \pm 7 & & & &
\end{array}$$

numbers are $7, 9, 11$ or $-7, -5, -3$

15. $$d_{\text{d}} = d_{\text{u}} + 4$$

$$\begin{array}{rrrrrrrrr}
 & 2(6 & + & r) & = & 3(6 & - & r) & + & 4 \\
 & 12 & + & 2r & = & 18 & - & 3r & + & 4 \\
- & 12 & + & 3r & & -12 & + & 3r & & \\
 & & & \dfrac{5r}{5} & = & \dfrac{10}{5} & & & & \\
 & & & r & = & 2 & \text{km/h} & & &
\end{array}$$

Midterm 3: Version E Answer Key

1. $\dfrac{121m^3}{51n^2} \cdot \dfrac{1531n^3}{3631m^6} \cdot \dfrac{84m^4}{63n^2} \Rightarrow \dfrac{4m}{3n}$
2. $\dfrac{x1(x+2)}{(x+2)(x+7)} \cdot \dfrac{21(x+7)}{21x^{32}} = \dfrac{1}{x^2}$
3. $$\left(\frac{x-3}{7} - \frac{x-15}{28} = \frac{3}{4}\right)(28)$$
$$\begin{array}{rrrrrrrl} 4(x & - & 3) & - & (x & - & 15) & = & 3(7) \\ 4x & - & 12 & - & x & + & 15 & = & 21 \\ & + & 12 & & & - & 15 & & -15+12 \\ & & & & & & 3x & = & 18 \\ & & & & & & x & = & 6 \end{array}$$
4. $\dfrac{\left(\dfrac{x^2}{y^2} - 36\right)y^3}{\left(\dfrac{x+6y}{y^3}\right)y^3} \Rightarrow \dfrac{x^2y - 36y^3}{x+6y} \Rightarrow \dfrac{y(x^2-36y^2)}{x+6y} \Rightarrow \dfrac{y(x-6y)(x+6y)}{(x+6y)}$
$$\Rightarrow y(x-6y)$$
5. $\sqrt{x^6 \cdot x \cdot y^4 \cdot y} + 2xy\sqrt{36 \cdot x \cdot y^4 \cdot y} - \sqrt{x \cdot y^2 \cdot y}$
$$x^3y^2\sqrt{xy} + 2xy \cdot 6y^2\sqrt{xy} - y\sqrt{xy}$$
$$x^3y^2\sqrt{xy} + 12xy^3\sqrt{xy} - y\sqrt{xy}$$
$$\sqrt{xy}(x^3y^2 + 12xy^3 - y)$$
6. $\dfrac{\sqrt{7}}{3-\sqrt{7}} \cdot \dfrac{3+\sqrt{7}}{3+\sqrt{7}} \Rightarrow \dfrac{3\sqrt{7}+7}{9-7} \Rightarrow \dfrac{3\sqrt{7}+7}{2}$
7. $\left(\dfrac{x^01y^4}{z^{-12}}\right)^{\frac{1}{4}} \Rightarrow \dfrac{y^{4\cdot\frac{1}{4}}}{z^{-12\cdot\frac{1}{4}}} \Rightarrow \dfrac{y^1}{z^{-3}} \Rightarrow yz^3$
8. $$(\sqrt{4x-5})^2 = (\sqrt{2x+3})^2$$
$$\begin{array}{rrrrrrrr} & 4x & - & 5 & = & 2x & + & 3 \\ - & 2x & + & 5 & & -2x & + & 5 \\ & & & 2x & = & 8 & & \\ & & & x & = & 4 & & \end{array}$$
9.

a. $\left(\frac{x^2}{3}=27\right)(3) \Rightarrow x^2=81 \Rightarrow x=\pm 9$

b.
$$\begin{aligned} 27x^2+3x &= 0 \\ 3x(9x+1) &= 0 \\ x &= 0, -\frac{1}{9} \end{aligned}$$

10.

a.
$$\begin{aligned} (x-12)(x+1) &= 0 \\ x &= 12, -1 \end{aligned}$$

b.
$$\begin{aligned} x^2+13x+12 &= 0 \\ (x+12)(x+1) &= 0 \\ x &= -1, -12 \end{aligned}$$

11.

$$\left(\frac{2}{x}=\frac{2x}{3x+8}\right)(x)(3x+8)$$

$$\begin{aligned} 2(3x+8) &= 2x^2 \\ 6x+16 &= 2x^2 \\ 0 &= 2x^2-6x-16 \\ 0 &= 2(x^2-3x-8) \end{aligned}$$

$$\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-8)}}{2(1)}$$

$$\frac{3\pm\sqrt{9+32}}{2}$$

$$\frac{3\pm\sqrt{41}}{2}$$

12.
$$\begin{aligned} (x^2-64)(x^2+1) &= 0 \\ (x-8)(x+8)(x^2+1) &= 0 \\ x &= \pm 8 \end{aligned}$$

13.
$$\begin{aligned} A &= 20 + P \\ L(L-5) &= 20 + 2(L-5) + 2L \\ L^2 - 5L &= 20 + 2L - 10 + 2L \\ -4L - 10 & \quad -10 - 4L \\ L^2 - 9L - 10 &= 0 \\ (L-10)(L+1) &= 0 \\ L &= 10, -1 \end{aligned}$$

$$\begin{aligned} W &= L - 5 \\ W &= 10 - 5 \\ W &= 5 \end{aligned}$$

14.

$x, x+2, x+4$

$$
\begin{array}{rrrrrrrrrrr}
 & & x(x & + & 4) & = & 35 & + & 10(x & + & 2) \\
x^2 & + & 4x & & & = & 35 & + & 10x & + & 20 \\
 & - & 10x & - & 55 & & -55 & - & 10x & & \\
x^2 & - & 6x & - & 55 & = & 0 & & & & \\
(x & - & 11)(x & + & 5) & = & 0 & & & & \\
 & & & & x & = & 11, & -5 & & &
\end{array}
$$

$$
\text{numbers are } 11, 13, 15 \text{ or } -5, -3, -1
$$

15. $$
\begin{array}{rrrr}
d_{\text{d}} & = & d_{\text{u}} & + & 9
\end{array}
$$

$$
\begin{array}{rrrrrrrrr}
 & 3(5 & + & r) & = & 4(5 & - & r) & + & 9 \\
 & 15 & + & 3r & = & 20 & - & 4r & + & 9 \\
- & 15 & + & 4r & & -15 & + & 4r & & \\
 & & & 7r & = & 14 & & & & \\
 & & & r & = & 2 & \text{km/h} & & &
\end{array}
$$

Answer Key 11.1

1.
   a. No
   b. Yes
   c. No
   d. Yes
   e. Yes
   f. No
   g. Yes
   h. $y^2 = 1 + x^2$
   $y = \pm\sqrt{1 + x^2}$
   No
   i. $\sqrt{y} = 2 - x$
   $y = (2 - x)^2$
   Yes
   j. $y^2 = 1 - x^2$
   $y = \pm\sqrt{1 - x^2}$
   No
2. All real numbers $-\infty, \infty$
3. $$\begin{array}{rcrcr} 5 & - & 4x & \geq & 0 \\ -5 & & & & -5 \\ & & \dfrac{-4x}{-4} & \geq & \dfrac{-5}{-4} \\ & & x & \leq & \dfrac{5}{4} \end{array}$$
   $\left(-\infty, \dfrac{5}{4}\right]$
4. $t^2 \neq 0$
   $t \neq \sqrt{0}$ or 0
5. All real or $(-\infty, \infty)$
6. $$\begin{array}{rcrcr} t^2 & + & 1 & \neq & 0 \\ & - & 1 & & -1 \\ & & t^2 & \neq & -1 \\ & & t & \neq & i \\ & & t & = & \mathbb{R} \end{array}$$
7. $$\begin{array}{rcrcr} x & - & 16 & \geq & 0 \\ & + & 16 & & +16 \\ & & x & \geq & 16 \end{array}$$
   $[16, \infty)$
8. $x^2 - 3x - 4 \neq 0$
   $(x - 4)(x + 1) \neq 0$

9. $x \neq 4, 1$

$$\begin{array}{rcrcr} 3x & - & 12 & \geq & 0 \\ & + & 12 & & +12 \\ & & \dfrac{3x}{3} & \geq & \dfrac{12}{3} \\ & & x & \geq & 4 \end{array}$$

$$\begin{aligned} x^2 - 25 &\neq 0 \\ (x-5)(x+5) &\neq 0 \\ x &\neq 5, -5 \\ \therefore x &\geq 4, \neq \pm 5 \end{aligned}$$

10. $$\begin{aligned} g(0) &= 4(0) - 4 \\ &= -4 \end{aligned}$$

11. $$\begin{aligned} g(2) &= -3 \cdot 5^{-2} \\ &= -\frac{3}{25} \end{aligned}$$

12. $$\begin{aligned} f(-9) &= (-9)^2 + 4 \\ &= 81 + 4 \\ &= 85 \end{aligned}$$

13. $$\begin{aligned} f(10) &= 10 - 3 \\ &= 7 \end{aligned}$$

14. $$\begin{aligned} f(-2) &= 3^{-2} - 2 \\ &= \frac{1}{9} - 2 \\ &= \frac{1}{9} - \frac{18}{9} \\ &= -\frac{17}{9} \end{aligned}$$

15. $$\begin{aligned} f(2) &= -3^{2-1} - 3 \\ &= -3^1 - 3 \\ &= -6 \end{aligned}$$

16. $$\begin{aligned} k(2) &= -2 \cdot 4^{2(2)-2} \\ &= -2 \cdot 4^{4-2} \\ &= -2 \cdot 4^2 \\ &= -32 \end{aligned}$$

17. $$\begin{aligned} p(-2) &= -2 \cdot 4^{2(-2)+1} + 1 \\ &= -2 \cdot 4^{-4+1} + 1 \\ &= -2 \cdot 4^{-3} + 1 \\ &= -\frac{2}{64} + 1 \\ &= -\frac{1}{32} + 1 \Rightarrow \frac{-31}{32} \end{aligned}$$

18. $$\begin{aligned} h(-4x) &= (-4x)^3 + 2 \\ &= -64x^3 + 2 \end{aligned}$$

19. $$\begin{aligned} h(n+2) &= 4(n+2)+2 \\ &= 4n+8+2 \\ &= 4n+10 \end{aligned}$$

20. $$\begin{aligned} h(-1+x) &= 3(-1+x)+2 \\ &= -3+3x+2 \\ &= 3x-1 \end{aligned}$$

21. $$\begin{aligned} h\left(\frac{1}{3}\right) &= -3\cdot 2^{\frac{1}{3}+3} \\ &= -2^3\cdot 3\sqrt[3]{2} \\ &= -8\cdot 3\sqrt[3]{2} \\ &= -24\sqrt[3]{2} \end{aligned}$$

22. $$\begin{aligned} h(x^4) &= (x^4)^2+1 \\ &= x^8+1 \end{aligned}$$

23. $$\begin{aligned} h(t^2) &= (t^2)^2+t \\ &= t^4+t \end{aligned}$$

24. $$\begin{aligned} f(0) &= |3(0)+1|+1 \\ &= 1+1 \text{ or } 2 \end{aligned}$$

25. $$\begin{aligned} f(-6) &= -2|-(-6)-2|+1 \\ &= -2|6-2|+1 \\ &= -2(4)+1 \\ &= -8+1 \text{ or } -7 \end{aligned}$$

26. $$\begin{aligned} f(10) &= |10+3| \\ &= 13 \end{aligned}$$

27. $$\begin{aligned} p(5) &= -|5|+1 \\ &= -5+1 \\ &= -4 \end{aligned}$$

Answer Key 11.2

1. $$\begin{aligned} g(3) &= (3)^3+5(3)^2 \\ &= 27+45 \\ &= 72 \end{aligned} \qquad \begin{aligned} f(3) &= 2(3)+4 \\ &= 6+4 \\ &= 10 \end{aligned}$$

   $g(3)+f(3)=72+10=82$

2. $$\begin{aligned} f(-4) &= -3(-4)^2+3(-4) \\ &= -3(16)-12 \\ &= -48-12 \\ &= -60 \end{aligned} \qquad \begin{aligned} g(-4) &= 2(-4)+5 \\ &= -8+5 \\ &= -3 \end{aligned}$$

   $$\frac{f(-4)}{g(-4)}=\frac{-60}{-3}=20$$

3. $$\begin{aligned} g(5) &= -4(5)+1 \\ &= -20+1 \\ &= -19 \end{aligned} \qquad \begin{aligned} h(5) &= -2(5)-1 \\ &= -10-1 \\ &= -11 \end{aligned}$$

   $g(5)+h(5)=-19-11=-30$

4. $$\begin{aligned} g(2) &= 3(2)+1 \\ &= 6+1 \\ &= 7 \end{aligned} \qquad \begin{aligned} f(2) &= (2)^3+3(2)^2 \\ &= 8+3\cdot 4 \\ &= 8+12 \\ &= 20 \end{aligned}$$

   $g(2)\cdot f(2)=7\cdot 20=140$

5. $$\begin{aligned} g(1) &= 1-3 \\ &= -2 \end{aligned} \qquad \begin{aligned} h(1) &= -3(1)^3+6(1) \\ &= -3+6 \\ &= 3 \end{aligned}$$

   $g(1)+h(1)=-2+3=1$

6. $$\begin{aligned} g(-6) &= (-6)^2-2 \\ &= 36-2 \\ &= 34 \end{aligned} \qquad \begin{aligned} h(-6) &= 2(-6)+5 \\ &= -12+5 \\ &= -7 \end{aligned}$$

   $g(-6)+h(-6)=34-7=27$

7. $$\begin{aligned} h(0) &= 2(0)-1 \\ &= -1 \end{aligned} \qquad \begin{aligned} g(0) &= 3(0)-5 \\ &= -5 \end{aligned}$$

   $$\frac{h(0)}{g(0)}=\frac{-1}{-5}=\frac{1}{5}$$

8. $$(g+h) = \begin{array}{rrcr} & 3a & - & 2 \\ + & 4a & - & 2 \\ \hline & 7a & - & 4 \end{array} \qquad \begin{aligned} (g+h)(10) &= 7(10)-4 \\ &= 70-4 \\ &= 66 \end{aligned}$$

9. $$(g+f) = \begin{array}{rrcr} & 3a & + & 3 \\ + & 2a & - & 2 \\ \hline & 5a & + & 1 \end{array} \qquad \begin{aligned} (g+f)(9) &= 5(9)+1 \\ &= 45+1 \\ &= 46 \end{aligned}$$

10. $(g-h) = \begin{array}{r} 4x+3 \\ -\ (x^3-2x^2) \\ \hline -x^3+2x^2+4x+3 \end{array}$ $\quad \begin{aligned}(g-h)(-1) &= -(-1)^3+2(-1)^2+4(-1)+3 \\ &= 1+2-4+3 \\ &= 2\end{aligned}$

11. $(g-f) = \begin{array}{rrrr} & x & + & 3 \\ - & (-x & + & 4) \\ \hline & 2x & - & 1 \end{array}$ $\quad \begin{aligned}(g-f)(3) &= 2(3)-1 \\ &= 6-1 \\ &= 5\end{aligned}$

12. $(g-f) = \begin{array}{rrrrrr} & x^2 & & & + & 2 \\ - & & & (2x & + & 5) \\ \hline & x^2 & - & 2x & - & 3 \end{array}$ $\quad \begin{aligned}(g-f)(0) &= (0)^2-2(0)-3 \\ &= -3\end{aligned}$

13. $(f+g) = \begin{array}{rrrr} & n & - & 5 \\ + & 4n & + & 2 \\ \hline & 5n & - & 3 \end{array}$ $\quad \begin{aligned}(f+g)(-8) &= 5(-8)-3 \\ &= -40-3 \\ &= -43\end{aligned}$

14. $(h\cdot g) = \begin{array}{rrrrrrr} & & & & t & + & 5 \\ \times & & & & 3t & - & 5 \\ \hline & 3t^2 & + & 15t & & & \\ & & - & 5t & - & 25 & \\ \hline & 3t^2 & + & 10t & - & 25 & \end{array}$ $\quad \begin{aligned}(h\cdot g)(5) &= 3(5)^2+10(5)-25 \\ &= 75+50-25 \\ &= 100\end{aligned}$

15. $(g\cdot h) = \begin{array}{rrrr} & t & - & 4 \\ \times & & & 2t \\ \hline 2t^2 & - & 8t & \end{array}$ $\quad \begin{aligned}(g\cdot h)(3t) &= 2(3t)^2-8(3t) \\ &= 2(9t^2)-24t \\ &= 18t^2-24t\end{aligned}$

16. $\dfrac{g(n)}{f(n)} = \dfrac{n^2+5}{2n+5}$ Does not reduce

17. $\dfrac{g}{f} = \dfrac{-2a+5}{3a+5}$ $\quad \left(\dfrac{g}{f}\right)(a^2) = \dfrac{-2a^2+5}{3a^2+5}$ Does not reduce

18. $h(n)+g(n) = \begin{array}{rrrrr} n^3 & + & 4n & & \\ + & & 4n & + & 5 \\ \hline n^3 & + & 8n & + & 5 \end{array}$

19. $\begin{aligned}g(n^2) &= (n^2)^2-4(n^2) \\ &= n^4-4n^2\end{aligned}$ $\quad h(n^2) = n^2-5$

$g(n^2)\cdot h(n^2) = \begin{array}{rrrrrr} & & & n^4 & - & 4n^2 \\ \times & & & n^2 & - & 5 \\ \hline n^6 & - & 4n^4 & & & \\ & - & 5n^4 & + & 20n^2 & \\ \hline n^6 & - & 9n^4 & + & 20n^2 & \end{array}$

20. $(g\cdot h) = \begin{array}{rrrrrr} & & & n & + & 5 \\ \times & & & 2n & - & 5 \\ \hline 2n^2 & + & 10n & & & \\ & - & 5n & - & 25 & \\ \hline 2n^2 & + & 5n & - & 25 & \end{array}$ $\quad \begin{aligned}(g\cdot h)(-3n) &= 2(-3n)^2+5(-3n)-25 \\ &= 2(9n^2)-15n-25 \\ &= 18n^2-15n-25\end{aligned}$

21. $\begin{aligned}(f\circ g) &= -4(4x+3)+1 \\ &= -16x-12+1 \\ &= -16x-11\end{aligned}$ $\quad \begin{aligned}(f\circ g)(9) &= -16(9)-11 \\ &= -144-11 \\ &= -155\end{aligned}$

22. $$\begin{aligned}(h\circ g) &= 3(a+1)+3 \\ &= 3a+3+3 \\ &= 3a+6\end{aligned} \qquad \begin{aligned}(h\circ g)(5) &= 3(5)+6 \\ &= 15+6 \\ &= 21\end{aligned}$$

23. $$\begin{aligned}(g\circ h) &= (x^2-1)+4 \\ &= x^2-1+4 \\ &= x^2+3\end{aligned} \qquad \begin{aligned}(g\circ h)(10) &= (10)^2+3 \\ &= 100+3 \\ &= 103\end{aligned}$$

24. $$\begin{aligned}(f\circ g) &= -4(n+4)+2 \\ &= -4n-16+2 \\ &= -4n-14\end{aligned} \qquad \begin{aligned}(f\circ g)(9) &= -4(9)-14 \\ &= -36-14 \\ &= -50\end{aligned}$$

25. $$\begin{aligned}(g\circ h) &= 2(2x^3+4x^2)-4 \\ &= 4x^3+8x^2-4\end{aligned} \qquad \begin{aligned}(g\circ h)(3) &= 4(3)^3+8(3)^2-4 \\ &= 108+72-4 \\ &= 176\end{aligned}$$

26. $$\begin{aligned}(g\circ h) &= (4x+4)^2-5(4x+4) \\ &= 16x^2+32x+16-20x-20 \\ &= 16x^2+12x-4\end{aligned}$$

27. $$\begin{aligned}(f\circ g) &= -2(4a)+2 \\ &= -8a+2\end{aligned}$$

28. $$\begin{aligned}(g\circ f) &= 4(x^3-1)+4 \\ &= 4x^3-4+4 \\ &= 4x^3\end{aligned}$$

29. $$\begin{aligned}(g\circ f) &= -(2x-3)+5 \\ &= -2x+6+5 \\ &= -2x+11\end{aligned}$$

30. $$\begin{aligned}(f\circ g) &= 4(-4t-2)+3 \\ &= -16t-8+3 \\ &= -16t-5\end{aligned}$$

Answer Key 11.3

1. $$\begin{aligned}(g\circ f)(x) &= -(\sqrt[5]{-x-3})^5-3\\ &= -(-x-3)-3\\ &= x+3-3\\ &= x \qquad \text{Inverse}\end{aligned}$$

2. $$\begin{aligned}(g\circ f)(x) &= 4-\left(\frac{4}{x}\right)\\ &= 4-\frac{4}{x} \qquad \text{Not inverse}\end{aligned}$$

3. $$\begin{aligned}(g\circ f)(x) &= -10\left(\frac{x-5}{10}\right)+5\\ &= -x+5+5\\ &= -x+10 \qquad \text{Not inverse}\end{aligned}$$

4. $$\begin{aligned}(f\circ g)(x) &= \frac{(10x+5)-5}{10}\\ &= \frac{10x+5-5}{10}\\ &= \frac{10x}{10}\\ &= x \qquad \text{Inverse}\end{aligned}$$

5. $$\begin{aligned}(f\circ g)(x) &= \frac{-2}{\frac{3x+2}{x+2}+3}\\ &= \frac{-2(x+2)}{3x+2+3(x+2)}\\ &= \frac{-2x-4}{3x+2+3x+6}\\ &= \frac{-2x-4}{6x+8}\\ &= \frac{-x-2}{3x+4} \qquad \text{Not inverse}\end{aligned}$$

6. $$(f \circ g) = \frac{-\left(\dfrac{-2x+1}{-x-1}\right)-1}{\dfrac{-2x+1}{-x-1}-2}$$

$$= \frac{-(-2x+1)-1(-x-1)}{-2x+1-2(-x-1)}$$

$$= \frac{2x-1+x+1}{-2x+1+2x+2}$$

$$= \frac{3x}{3}$$

$$= x$$

Inverse

7.
$$\begin{array}{rcll} y & = & (x-2)^5 & + \quad 3 \\ x & = & (y-2)^5 & + \quad 3 \\ -3 & & & - \quad 3 \\ x-3 & = & (y-2)^5 & \\ \sqrt[5]{x-3} & = & y-2 & \\ & & & \\ y & = & \sqrt[5]{x-3} & + \quad 2 \end{array}$$

8.
$$\begin{array}{rcll} y & = & \sqrt[3]{x+1} & + \quad 2 \\ x & = & \sqrt[3]{y+1} & + \quad 2 \\ -2 & & & - \quad 2 \\ x-2 & = & \sqrt[3]{y+1} & \\ (x-2)^3 & = & y+1 & \\ & & & \\ y & = & (x-2)^3 & - \quad 1 \end{array}$$

9.
$$\begin{array}{rcl} y & = & \dfrac{4}{x+2} \\ & & \\ x & = & \dfrac{4}{y+2} \\ & & \\ y+2 & = & \dfrac{4}{x} \\ & & \\ y & = & \dfrac{4}{x}-2 \end{array}$$

10.
$$\begin{aligned} y &= \frac{3}{x-3} \\ x &= \frac{3}{y-3} \\ y-3 &= \frac{3}{x} \\ y &= \frac{3}{x}+3 \end{aligned}$$

11.
$$\begin{aligned} y &= \frac{-2x-2}{x+2} \\ x &= \frac{-2y-2}{y+2} \\ x(y+2) &= -2y-2 \\ xy+2x &= -2y-2 \\ -xy+2 & \quad -xy+2 \\ 2x+2 &= -2y-xy \\ 2x+2 &= y(-2-x) \\ y &= \frac{2x+2}{-2-x} \\ y &= -\frac{2x+2}{2+x} \end{aligned}$$

12.
$$\begin{aligned} y &= \frac{9+x}{3} \\ x &= \frac{9+y}{3} \\ 3x &= 9+y \\ y &= 3x-9 \end{aligned}$$

13.
$$\begin{aligned} y &= \frac{10-x}{5} \\ x &= \frac{10-y}{5} \\ 5x &= 10-y \\ y &= 10-5x \end{aligned}$$

14.
$$\begin{aligned} y &= \frac{5x-15}{2} \\ x &= \frac{5y-15}{2} \\ 5y-15 &= 2x \\ 5y &= 2x+15 \\ y &= \frac{2x+15}{5} \end{aligned}$$

15.
$$\begin{aligned} y &= -(x-1)^3 \\ x &= -(y-1)^3 \\ \sqrt[3]{x} &= -(y-1) \\ \sqrt[3]{x} &= -y+1 \\ -y &= \sqrt[3]{x}-1 \\ y &= 1-\sqrt[3]{x} \end{aligned}$$

16.
$$\begin{aligned} y &= \frac{12-3x}{4} \\ x &= \frac{12-3y}{4} \\ 4x &= 12-3y \\ 3y &= 12-4x \\ y &= \frac{12-4x}{3} \\ y &= 4-\frac{4}{3}x \end{aligned}$$

17.
$$\begin{aligned} y &= (x-3)^3 \\ x &= (y-3)^3 \\ \sqrt[3]{x} &= y-3 \\ y &= \sqrt[3]{x}+3 \end{aligned}$$

18.
$$\begin{aligned} y &= \sqrt[5]{-x} + 2 \\ x &= \sqrt[5]{-y} + 2 \\ x - 2 &= \sqrt[5]{-y} \\ (x-2)^5 &= -y \\ \\ y &= -(x-2)^5 \end{aligned}$$

19.
$$\begin{aligned} y &= \frac{x}{x-1} \\ x &= \frac{y}{y-1} \\ x(y-1) &= y \\ xy - x &= y \\ y - xy &= -x \\ y(1-x) &= -x \\ y &= \frac{-x}{1-x} \\ y &= \frac{x}{x-1} \end{aligned}$$

20.
$$\begin{aligned} y &= \frac{-3-2x}{x+3} \\ x &= \frac{-3-2y}{y+3} \\ x(y+3) &= -3-2y \\ xy + 3x &= -3-2y \\ +2y - 3x & \quad -3x + 2y \\ xy + 2y &= -3-3x \\ y(x+2) &= -3-3x \\ y &= \frac{-3-3x}{x+2} \\ y &= -\frac{3x+3}{x+2} \end{aligned}$$

21.

$$
\begin{aligned}
y &= \frac{x-1}{x+1} \\
x &= \frac{y-1}{y+1} \\
x(y+1) &= y-1 \\
xy+x &= y-1 \\
xy-y &= -x-1 \\
y(x-1) &= -x-1 \\
y &= \frac{-x-1}{x-1} \\
y &= -\frac{x+1}{x-1}
\end{aligned}
$$

22.

$$
\begin{aligned}
y &= \frac{x}{x+2} \\
x &= \frac{y}{y+2} \\
x(y+2) &= y \\
xy+2x &= y \\
2x &= y-xy \\
2x &= y(1-x) \\
y &= \frac{2x}{1-x}
\end{aligned}
$$

Answer Key 11.4

1. $$\begin{array}{rrrrrrr} & 1 & - & 2n & = & 1 & - & 3n \\ - & 1 & + & 3n & & -1 & + & 3n \\ & & & n & = & 0 & & \end{array}$$

2. $$\begin{aligned} 4^{2x} &= 4^{-2} \\ \frac{2x}{2} &= \frac{-2}{2} \\ x &= -1 \end{aligned}$$

3. $$\begin{aligned} 4^{2a} &= 4^{0} \\ 2a &= 0 \\ a &= 0 \end{aligned}$$

4. $$\begin{aligned} (4^2)^{-3p} &= (4^3)^{3p} \\ 4^{-6p} &= 4^{9p} \\ -6p &= 9p \\ +6p & \quad +6p \\ 0 &= 15p \\ p &= 0 \end{aligned}$$

5. $$\begin{aligned} (5^{-2})^{-k} &= (5^3)^{-2k-2} \\ 5^{2k} &= 5^{-6k-6} \\ 2k &= -6k-6 \\ +6k & \quad +6k \\ 8k &= -6 \\ k &= -\frac{6}{8} \Rightarrow -\frac{3}{4} \end{aligned}$$

6. $$\begin{aligned} (5^4)^{-n-2} &= 5^{-3} \\ 5^{-4n+8} &= 5^{3} \\ -4n+8 &= 3 \\ -8 & \quad -8 \\ -4n &= -5 \\ n &= \frac{5}{4} \end{aligned}$$

7. $$\begin{aligned} 6^{2m+1} &= 6^{-2} \\ 2m+1 &= -2 \\ -1 & \quad\; -1 \\ 2m &= -3 \\ m &= -\frac{3}{2} \end{aligned}$$

8. $$\begin{aligned} 2r-3 &= r-3 \\ -r+3 & \quad -r+3 \\ r &= 0 \end{aligned}$$

9. $$\begin{aligned} 6^{-3x} &= 6^{2} \\ \therefore -3x &= 2 \\ x &= -\frac{2}{3} \end{aligned}$$

10. $$\begin{aligned} 2n &= -n \\ +n & \quad +n \\ 3n &= 0 \\ n &= 0 \end{aligned}$$

11. $$\begin{aligned} (2^{6})^{b} &= 2^{5} \\ 6b &= 5 \\ b &= \frac{5}{6} \end{aligned}$$

12. $$\begin{aligned} (6^{3})^{-3v} &= (6^{2})^{3v} \\ 6^{-9v} &= 6^{6v} \\ -9v &= 6v \\ +9v & \quad +9v \\ 0 &= 15v \\ v &= 0 \end{aligned}$$

13. $$\begin{aligned} (4^{-1})^{x} &= 4^{2} \\ 4^{-x} &= 4^{2} \\ \therefore -x &= 2 \\ x &= -2 \end{aligned}$$

14. $$\begin{aligned} (3^{3})^{-2n-1} &= 3^{2} \\ 3^{-6n-3} &= 3^{2} \\ -6n-3 &= 2 \\ +3 &= +3 \\ -6n &= 5 \\ n &= -\frac{5}{6} \end{aligned}$$

15. $$\begin{aligned} \therefore 3a &= 3 \\ a &= 1 \end{aligned}$$

16. $$\begin{aligned} 4^{-3v} &= 4^3 \\ \therefore -3v &= 3 \\ v &= -1 \end{aligned}$$

17. $$\begin{aligned} (6^2)^{3x} &= (6^3)^{2x+1} \\ 6^{6x} &= 6^{6x+3} \\ \therefore 6x &= 6x+3 \\ -6x & \quad\ -6x \\ 0 &= 3 \Rightarrow \text{no solution} \end{aligned}$$

18. $$\begin{aligned} (4^3)^{x+2} &= 4^2 \\ 4^{3x+6} &= 4^2 \\ \therefore 3x+6 &= 2 \\ -6 & \quad -6 \\ 3x &= -4 \\ x &= -\frac{4}{3} \end{aligned}$$

19. $$\begin{aligned} (3^2)^{2n+3} &= 3^5 \\ 3^{4n+6} &= 3^5 \\ \therefore 4n+6 &= 5 \\ -6 & \quad -6 \\ 4n &= -1 \\ n &= -\frac{1}{4} \end{aligned}$$

20. $$\begin{aligned} (4^2)^{2k} &= 4^{-3} \\ 4^{4k} &= 4^{-3} \\ \therefore 4k &= -3 \\ k &= -\frac{3}{4} \end{aligned}$$

21. $$\begin{aligned} 3x-2 &= 3x+1 \\ -3x+2 & \quad -3x+2 \\ 0 &= 3 \Rightarrow \text{no solution} \end{aligned}$$

22. $$\begin{aligned} (2^5)^p &= (3^2)^{-3p} \\ \therefore 5p &= -6p \\ +6p & \quad +6p \\ 11p &= 0 \\ p &= 0 \end{aligned}$$

23. $$\begin{aligned} -2x &= 3 \\ x &= -\frac{3}{2} \end{aligned}$$

24. $$\begin{aligned} 2n &= 2 - 3n \\ +3n & \quad\ \ + 3n \\ 5n &= 2 \\ \\ n &= \frac{2}{5} \end{aligned}$$

25. $$\begin{aligned} m + 2 &= -m \\ +m - 2 &= +m - 2 \\ 2m &= -2 \\ \\ m &= -1 \end{aligned}$$

26. $$\begin{aligned} (5^4)^{2x} &= 5^2 \\ 5^{8x} &= 5^2 \\ \therefore 8x &= 2 \\ \\ x &= \frac{1}{4} \end{aligned}$$

27. $$\begin{aligned} (6^{-2})^{b-1} &= 6^3 \\ 6^{-2b+2} &= 6^3 \\ \therefore -2b + 2 &= 3 \\ -2 & \quad\ \ -2 \\ -2b &= 1 \\ \\ b &= -\frac{1}{2} \end{aligned}$$

28. $$\begin{aligned} (6^3)^{2n} &= 6^2 \\ 6^{6n} &= 6^2 \\ \therefore 6n &= 2 \\ \\ n &= \frac{1}{3} \end{aligned}$$

29. $$\begin{aligned} 2 - 2x &= 2 \\ -2 &= -2 \\ -2x &= 0 \\ \\ x &= 0 \end{aligned}$$

30. $$\begin{aligned} (2^{-2})^{3v-2} &= (2^6)^{1-v} \\ 2^{-6v+4} &= 2^{6-6v} \\ \therefore -6v+4 &= 6-6v \\ +6v-4 & \quad -4+6v \\ 0 &= 2 \Rightarrow \text{No solution} \end{aligned}$$

Answer Key 11.5

1. $9^2 = 81$
2. $b^{-16} = a$
3. $\left(\frac{1}{49}\right)^{-2} = 7$
4. $16^2 = 256$
5. $13^2 = 169$
6. $11^0 = 1$
7. $\log_8 1 = 0$
8. $\log_{17} \frac{1}{289} = -2$
9. $\log_{15} 225 = 2$
10. $\log_{144} 12 = \frac{1}{2}$
11. $\log_{64} 2 = \frac{1}{6}$
12. $\log_{19} 361 = 2$
13. $$\begin{aligned} \log_{125} 5 &= x \\ 125^x &= 5 \\ 5^{3x} &= 5 \\ 3x &= 1 \\ x &= \frac{1}{3} \end{aligned}$$
14. $$\begin{aligned} \log_5 125 &= x \\ 5^x &= 125 \\ 5^x &= 5^3 \\ x &= 3 \end{aligned}$$
15. $$\begin{aligned} \log_{343} \frac{1}{7} &= x \\ 343^x &= \frac{1}{7} \\ 7^{3x} &= 7^{-1} \\ 3x &= -1 \\ x &= -\frac{1}{3} \end{aligned}$$

16. $$\begin{aligned} \log_7 1 &= x \\ 7^x &= 1 \\ 7^x &= 7^0 \\ x &= 0 \end{aligned}$$

17. $$\begin{aligned} \log_4 16 &= x \\ 4^x &= 16 \\ 4^x &= 4^2 \\ x &= 2 \end{aligned}$$

18. $$\begin{aligned} \log_4 \frac{1}{64} &= x \\ 4^x &= \frac{1}{64} \\ 4^x &= 4^{-3} \\ x &= -3 \end{aligned}$$

19. $$\begin{aligned} \log_6 36 &= x \\ 6^x &= 36 \\ 6^x &= 6^2 \\ x &= 2 \end{aligned}$$

20. $$\begin{aligned} \log_{36} 6 &= x \\ 36^x &= 6 \\ 6^{2x} &= 6^1 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

21. $$\begin{aligned} \log_2 64 &= x \\ 2^x &= 64 \\ 2^x &= 2^6 \\ x &= 6 \end{aligned}$$

22. $$\begin{aligned} \log_3 243 &= x \\ 3^x &= 243 \\ 3^x &= 3^5 \\ x &= 5 \end{aligned}$$

23. $$\begin{aligned} 5^1 &= x \\ x &= 5 \end{aligned}$$

24. $$\begin{aligned} 8^3 &= k \\ k &= 512 \end{aligned}$$

25. $$\begin{aligned} 2^{-2} &= x \\ x &= \frac{1}{4} \end{aligned}$$

26. $$\begin{aligned} 10^3 &= \\ n &= 1000 \end{aligned}$$

27. $$\begin{aligned} 11^2 &= k \\ k &= 121 \end{aligned}$$

28. $$\begin{aligned} 4^4 &= p \\ p &= 256 \end{aligned}$$

29. $$\begin{array}{rcrcr} 9^4 & = & n & + & 9 \\ -9 & & & - & 9 \\ n & = & 9^4 & - & 9 \\ n & = & 6561 & - & 9 \\ n & = & 6552 & & \end{array}$$

30. $$\begin{array}{rcrcr} 11^{-1} & = & x & - & 4 \\ +4 & & & + & 4 \\ x & = & 4 & + & \frac{1}{11} \\ x & = & 4\frac{1}{11} & & \end{array}$$

31. $$\begin{aligned} 5^3 &= -3m \\ m &= \frac{5^3}{-3} \\ m &= -\frac{125}{3} \end{aligned}$$

32. $$\begin{aligned} 2^1 &= -8r \\ r &= \frac{2}{-8} \Rightarrow -\frac{1}{4} \end{aligned}$$

33. $$\begin{array}{rcrcr} 11^{-1} & = & x & + & 5 \\ -5 & & & - & 5 \\ x & = & -5 & + & \frac{1}{11} \\ x & = & -4\frac{10}{11} & & \end{array}$$

34. $$\begin{aligned} 7^4 &= -3n \\ n &= \frac{7^4}{-3} \\ n &= -\frac{2401}{3} \end{aligned}$$

35.
$$\begin{array}{rcrcr} 4^0 & = & 6b & + & 4 \\ -4 & & & - & 4 \\ 6b & = & -4 & + & 1 \\ 6b & = & -3 & & \\ b & = & -\frac{1}{2} & & \end{array}$$

36.
$$\begin{array}{rcrcr} 11^{-1} & = & 10v & + & 1 \\ -1 & & & - & 1 \\ 10v & = & -1 & + & \frac{1}{11} \\ 10v & = & -\frac{10}{11} & & \\ v & = & -\frac{1}{11} & & \end{array}$$

37.
$$\begin{array}{rcrcr} 5^4 & = & -10x & + & 4 \\ 625 & = & -10x & + & 4 \\ -4 & & & - & 4 \\ \frac{621}{-10} & = & \frac{-10x}{-10} & & \\ x & = & -\frac{621}{10} & & \end{array}$$

38.
$$\begin{array}{rcrcr} 9^{-2} & = & 7 & - & 6x \\ -7 & & -7 & & \\ -6x & = & -7 & + & \frac{1}{81} \\ -6x & = & -\frac{566}{81} & & \\ x & = & \frac{566}{81 \cdot 6} & & \\ x & = & \frac{566}{486} & & \\ x & = & \frac{283}{243} & & \end{array}$$

39.
$$\begin{array}{rcrcr} 2^3 & = & 10 & - & 5a \\ -10 & & -10 & & \\ -5a & = & 8 & - & 10 \\ -5a & = & -2 & & \\ a & = & \dfrac{2}{5} & & \end{array}$$

40.
$$\begin{array}{rclcl} 8 & = & 3k & - & 1 \\ +1 & & & + & 1 \\ 9 & = & 3k & & \\ k & = & 3 & & \end{array}$$

Answer Key 11.6

1. a.
$$\begin{aligned} A &= \text{find} \qquad P = \$500 \qquad r = 0.04 \qquad n = 1 \qquad t = 10 \\ A &= 500\left(1+\frac{0.04}{1}\right)^{10} \\ A &= 500(1.04)^{10} \\ A &= \$740.12 \end{aligned}$$

b.
$$\begin{aligned} A &= \text{find} \qquad P = \$600 \qquad r = 0.06 \qquad n = 1 \qquad t = 6 \\ A &= 600\left(1+\frac{0.06}{1}\right)^{6} \\ A &= 600(1.06)^{6} \\ A &= \$851.11 \end{aligned}$$

c.
$$\begin{aligned} A &= \text{find} \qquad P = \$750 \qquad r = 0.03 \qquad n = 1 \qquad t = 8 \\ A &= 750\left(1+\frac{0.03}{1}\right)^{8} \\ A &= 750(1.03)^{8} \\ A &= \$950.08 \end{aligned}$$

d.
$$\begin{aligned} A &= \text{find} \qquad P = \$1500 \qquad r = 0.04 \qquad n = 2 \qquad t = 7 \\ A &= 1500\left(1+\frac{0.04}{2}\right)^{14} \\ A &= 1500(1.02)^{14} \\ A &= \$1979.22 \end{aligned}$$

e. $$\begin{aligned} A &= \text{find} \qquad P = \$900 \qquad r = 0.06 \qquad n = 2 \qquad t = 5 \\ A &= 900\left(1 + \frac{0.06}{2}\right)^{10} \\ A &= 900(1.03)^{10} \\ A &= \$1209.52 \end{aligned}$$

f. $$\begin{aligned} A &= \text{find} \qquad P = \$950 \qquad r = 0.04 \qquad n = 2 \qquad t = 12 \\ A &= 950\left(1 + \frac{0.04}{2}\right)^{24} \\ A &= 950(1.02)^{24} \\ A &= \$1528.02 \end{aligned}$$

g. $$\begin{aligned} A &= \text{find} \qquad P = \$2000 \qquad r = 0.05 \qquad n = 4 \qquad t = 6 \\ A &= 2000\left(1 + \frac{0.05}{4}\right)^{24} \\ A &= 2000(1.0125)^{24} \\ A &= \$2694.70 \end{aligned}$$

h. $$\begin{aligned} A &= \text{find} \qquad P = \$2250 \qquad r = 0.04 \qquad n = 4 \qquad t = 9 \\ A &= 2250\left(1 + \frac{0.04}{4}\right)^{36} \\ A &= 2250(1.01)^{36} \\ A &= \$3219.23 \end{aligned}$$

i. $$\begin{aligned} A &= \text{find} \qquad P = \$3500 \qquad r = 0.06 \qquad n = 4 \qquad t = 12 \\ A &= 3500\left(1 + \frac{0.06}{4}\right)^{48} \\ A &= 3500(1.015)^{48} \\ A &= \$7152.17 \end{aligned}$$

2. $$\begin{aligned} A &= \$10,000\left(1+\frac{0.04}{4}\right)^{40} \\ A &= \$10,000(1.01)^{40} \\ A &= \$14,888.64 \end{aligned}$$

3. $$\begin{aligned} A &= \$27,500\left(1+\frac{0.06}{12}\right)^{12(9)} \\ A &= \$27,500(1.005)^{108} \\ A &= \$47,126.74 \end{aligned}$$

4. $$\begin{aligned} A &= \$55,000\left(1+\frac{0.10}{12}\right)^{18} \\ A &= \$55,000(1.008\bar{3})^{18} \\ A &= \$63,861.18 \end{aligned}$$

5. $$\begin{aligned} \$20,000 &= P\left(1+\frac{0.06}{2}\right)^{10} \\ P &= \frac{\$20,000}{(1.03)^{10}} \\ P &= \$14,881.88 \end{aligned}$$

6. $$\begin{aligned} \$4200 &= P\left(1+\frac{0.04}{4}\right)^{4(5)} \\ P &= \frac{\$4200}{(1.01)^{20}} \\ P &= \$3442.09 \end{aligned}$$

Answer Key 11.7

1. 0.743145
2. 0.484810
3. 0.906308
4. 0.484810
5. 0.194380
6. 1.53986
7. 0.190810
8. 0.544639
9. 29°
10. 39°
11. 50°
12. 52°
13. 33.3°
14. 8.9°
15. 41°
16. 81°

1.
$$\begin{aligned} 20^2 + 10^2 &= z^2 \\ z &= \sqrt{500} \\ z &= 22.36\ldots \end{aligned}$$

$$\begin{aligned} \tan \text{Ø} &= \frac{\text{opp}}{\text{adj}} \\ \tan \text{Ø} &= \frac{10}{20} \\ \text{Ø} &= \tan^{-1} 0.5 \\ \text{Ø} &= 26.6^\circ \end{aligned}$$

2.
$$\begin{aligned} 20^2 + y^2 &= 28^2 \\ y &= \sqrt{28^2 - 20^2} \\ y &= \sqrt{384} \\ y &= 19.6 \end{aligned}$$

$$\begin{aligned} \cos \text{Ø} &= \frac{A}{H} \\ \cos \text{Ø} &= \frac{20}{28} \\ \text{Ø} &= \cos^{-1}\left(\frac{20}{28}\right) \\ \text{Ø} &= 44.4^\circ \end{aligned}$$

3. $$\begin{aligned} \cos Ø &= \frac{A}{H} \\ \cos Ø &= \frac{12}{20} \\ Ø &= \cos^{-1}\left(\frac{12}{20}\right) \\ Ø &= 53.1° \end{aligned}$$

$$\begin{aligned} 12^2 + x^2 &= 20^2 \\ x &= \sqrt{20^2 - 12^2} \\ x &= 16 \end{aligned}$$

4. $$\begin{aligned} \cos 32 &= \frac{x}{25} \\ x &= 25 \cos 32 \\ x &= 21.2 \end{aligned}$$

$$\begin{aligned} \sin 32° &= \frac{y}{25} \\ y &= 25 \sin 32 \\ y &= 13.2 \end{aligned}$$

5. $$\begin{aligned} \cos 42° &= \frac{x}{1200N} \\ x &= 1200N \cos 42° \\ x &= 891.8N \end{aligned}$$

$$\begin{aligned} \sin 42° &= \frac{y}{1200N} \\ y &= 1200N \sin 42° \\ y &= 803N \end{aligned}$$

6. $$\begin{aligned} \tan Ø &= \frac{100N}{220N} \\ Ø &= \tan^{-1}\left(\frac{100}{220}\right) \\ Ø &= 24.4° \end{aligned}$$

$$\begin{aligned} z^2 &= 100^2 + 220^2 \\ z &= \sqrt{58400} \\ z &= 241.7 \end{aligned}$$

7. $$\begin{aligned} \cos 55° &= \frac{y}{12} \\ y &= 12 \cos 55° \\ y &= 6.9 \end{aligned}$$

$$\begin{aligned} \sin 55° &= \frac{x}{12} \\ x &= 12 \sin 55° \\ x &= 9.8 \end{aligned}$$

8. $$\begin{aligned} \tan 28 &= \frac{20}{x} \\ x &= \frac{20}{\tan 28} \\ x &= 37.6 \end{aligned}$$

$$\begin{aligned} \sin 28° &= \frac{20}{z} \\ z &= \frac{20}{\sin 28} \\ z &= 42.6 \end{aligned}$$

9. $$\begin{aligned} \tan \emptyset &= \frac{20}{15} \\ \emptyset &= \tan^{-1}\left(\frac{20}{15}\right) \\ \emptyset &= 53.1^\circ \end{aligned}$$

$$\begin{aligned} 15^2+20^2 &= z^2 \\ z &= \sqrt{625} \\ z &= 25 \end{aligned}$$

10. $$\begin{aligned} y^2+100^2 &= 125^2 \\ y &= \sqrt{125^2-100^2} \\ y &= 75 \end{aligned}$$

$$\begin{aligned} \cos \emptyset &= \frac{100}{125} \\ \emptyset &= \cos^{-1}\left(\frac{100}{125}\right) \\ \emptyset &= 36.9^\circ \end{aligned}$$

11. $$\begin{aligned} \cos \emptyset &= \frac{3}{5} \\ \emptyset &= \cos\ ^{-1}\left(\frac{3}{5}\right) \\ \emptyset &= 53.1 \end{aligned}$$

$$\begin{aligned} 3^2+y^2 &= 5^2 \\ y &= \sqrt{5^2-3^2} \\ y &= 4 \end{aligned}$$

12. $$\begin{aligned} \cos\ 24^\circ &= \frac{25}{z} \\ z &= \frac{25}{\cos\ 24^\circ} \\ z &= 27.4 \end{aligned}$$

$$\begin{aligned} \tan\ 24^\circ &= \frac{y}{25} \\ y &= 25\ \tan\ 24^\circ \\ y &= 11.1 \end{aligned}$$

13. $$\begin{aligned} \sin \emptyset &= \frac{28}{40} \\ \emptyset &= \sin\ ^{-1}\left(\frac{28}{40}\right) \\ \emptyset &= 44.4^\circ \end{aligned}$$

$$\begin{aligned} z^2+28^2 &= 40^2 \\ z &= \sqrt{40^2-28^2} \\ z &= 28.6 \end{aligned}$$

14. $$\begin{aligned} \cos \emptyset &= \frac{20}{28} \\ \emptyset &= \cos\ ^{-1}\left(\frac{20}{28}\right) \\ \emptyset &= 44.4^\circ \end{aligned}$$

$$\begin{aligned} 20^2+y^2 &= 28^2 \\ y &= \sqrt{28^2-20^2} \\ y &= 19.6 \end{aligned}$$

15.
$$\begin{aligned} \sin \varnothing &= \frac{8}{12} \\ \varnothing &= \sin^{-1}\left(\frac{8}{12}\right) \\ \varnothing &= 41.8^\circ \end{aligned}$$

$$\begin{aligned} y^2 + 8^2 &= 12^2 \\ y &= \sqrt{12^2 - 8^2} \\ y &= 8.9 \end{aligned}$$

16.
$$\begin{aligned} \tan 35^\circ &= \frac{x}{50} \\ x &= 50 \tan 35^\circ \\ x &= 35 \end{aligned}$$

$$\begin{aligned} \cos 35^\circ &= \frac{50}{y} \\ y &= \frac{50}{\cos 35^\circ} \\ y &= 61 \end{aligned}$$

Answer Key 11.8

1. $$\begin{aligned} z^2 &= 10^2 + 20^2 - 2(10)(20)\ \cos\ 40° \\ z^2 &= 100 + 400 - 306.4 \\ z^2 &= 193.6 \\ \\ z &= 13.9\ \text{cm} \end{aligned}$$

2. $$\begin{aligned} 20^2 &= 28^2 + 28^2 - 2(28)(28)\ \cos\ Ø \\ 400 &= 784 + 784 - 1568\ \cos\ Ø \\ \cos\ Ø &= \frac{-1168}{-1568} \\ Ø &= \cos^{-1} 0.7449 \\ Ø &= 41.8° \end{aligned}$$

3. $$\begin{aligned} 140^2 &= 200^2 + 130^2 - 2(200)(130)\ \cos\ Ø \\ 19600 &= 400000 + 169000 - 52000\ \cos\ Ø \\ \cos\ Ø &= \frac{-37300}{-52000} \\ Ø &= \cos^{-1} 0.71730 \\ Ø &= 44.2° \end{aligned}$$

4. $$\begin{aligned} x^2 &= 125^2 + 120^2 - 2(125)(120)\ \cos\ 32° \\ x^2 &= 15625 + 14400 - 25441 \\ x^2 &= 4583.6 \\ x &= 67.7 \end{aligned}$$

5.
$$\begin{aligned}
18^2 &= 3^2 + 20^2 - 2(3)(20)\cos Ø \\
324 &= 9 + 400 - 120\cos Ø \\
\cos Ø &= \frac{-85}{-120} \\
Ø &= \cos^{-1}\left(\frac{-85}{-120}\right) \\
Ø &= 44.9^\circ
\end{aligned}$$

6.
$$\begin{aligned}
\frac{y}{\sin 35^\circ} &= \frac{40}{\sin 65^\circ} \\
y &= \frac{40 \sin 35^\circ}{\sin 65^\circ} \\
y &= 25.3
\end{aligned}$$

7.
$$\begin{aligned}
\frac{y}{\sin 28^\circ} &= \frac{12 \text{ m}}{\sin 25^\circ} \\
y &= \frac{12 \sin 28^\circ}{\sin 25^\circ} \\
y &= 13.3 \text{ m}
\end{aligned}$$

8.
$$\begin{aligned}
\frac{x}{\sin 25^\circ} &= \frac{10 \text{ m}}{\sin 15^\circ} \\
x &= \frac{10 \text{ m} \sin 25^\circ}{\sin 15^\circ} \\
x &= 16.3 \text{ m}
\end{aligned}$$

9.
$$\begin{aligned}
\frac{z}{\sin 10^\circ} &= \frac{8 \text{ cm}}{\sin 70^\circ} \\
z &= \frac{8 \text{ cm} \sin 10^\circ}{\sin 70^\circ} \\
z &= 1.48 \text{ cm}
\end{aligned}$$

10.
$$\begin{aligned}
y^2 &= 20^2 + 28^2 - 2(20)(28)\cos 130^\circ \\
y^2 &= 400 + 784 + 720 \\
y^2 &= 1904 \\
y &= 43.6 \text{ cm}
\end{aligned}$$

11.
$$\begin{aligned} 20^2 &= 15^2 + 30^2 - 2(15)(30)\ \cos\ \varnothing \\ 400 &= 225 + 900 - 900\ \cos\ \varnothing \\ \cos\ \varnothing &= \frac{-725}{-900} \\ \varnothing &= \cos^{-1}\left(\frac{-725}{-900}\right) \\ \varnothing &= 36.3^\circ \end{aligned}$$

12.
$$\begin{aligned} \frac{x}{\sin\ 95^\circ} &= \frac{8\ \text{m}}{\sin\ 20^\circ} \\ x &= \frac{8\ \text{m}\ \sin\ 95^\circ}{\sin\ 20^\circ} \\ x &= 23.3 \end{aligned}$$

13.
$$\begin{aligned} 16^2 &= 10^2 + 8^2 - 2(8)(10)\ \cos\varnothing \\ 256 &= 100 + 64 - 160\ \cos\ \varnothing \\ \cos\ \varnothing &= \frac{92}{-160} \\ \varnothing &= \cos\ ^{-1} - 0.575 \\ \varnothing &= 125^\circ \end{aligned}$$

14.
$$\begin{aligned} y^2 &= 20^2 + 24^2 - 2(20)(24)\ \cos\ 15^\circ \\ y^2 &= 400 + 576 - 960\ \cos\ 15^\circ \\ y^2 &= 976 - 927.3 \\ y &= \sqrt{48.7} \\ y &= 6.98\ \text{cm} \end{aligned}$$

15.
$$\begin{aligned}
20^2 &= 10^2 + 22^2 - 2(10)(22)\cos \varnothing \\
400 &= 100 + 484 - 440\cos \varnothing \\
\cos \varnothing &= \frac{-184}{-440} \\
\varnothing &= \cos^{-1}\left(\frac{-184}{-440}\right) \\
\varnothing &= 65.3^\circ
\end{aligned}$$

16.
$$\begin{aligned}
\frac{y}{\sin 25^\circ} &= \frac{20\text{ m}}{\sin 28^\circ} \\
y &= \frac{20\text{ m}\sin 25^\circ}{\sin 28^\circ} \\
y &= 18\text{ m}
\end{aligned}$$

Answer Key 11.9

1.

x	y
10	0.1
8	0.125
5	0.2
4	0.25
2	0.5
1	1
0.5	2
0.25	4
0.2	5
0.125	8
0.1	10

2.

x	y
3	0
2.5	±1.7
2	±2.4
1.5	±2.6
1	±2.8
0.5	±2.9
0	±3
−0.5	±2.9
−1	±2.8
−1.5	±2.6
−2	±2.4
−2.5	±1.7
−3	0

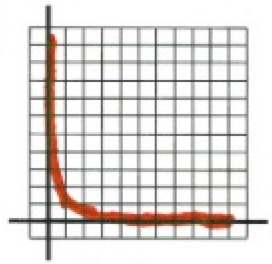
3.

x	y
5	10
4	8
3	6
2	4
1	2
0	0
−1	2
−2	4
−3	6
−4	8
−5	10

4.

x	y
0	180°
1.5	150°
2.6	120°
3	90°
2.6	60°
1.5	30°
0	0°
1.5	−30°
2.6	−60°
3	−90°
2.6	−120°
1.5	−150°
0	−180°

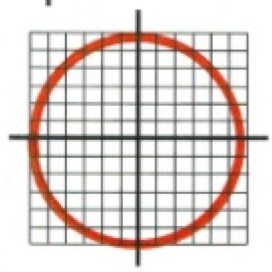
1.



Scaling:

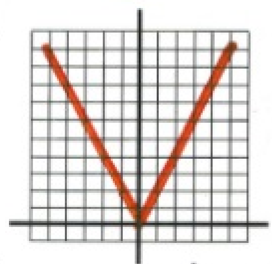
Graph 1.
x axis ... 1 square = 1
y axis ... 1 square = 1

2.



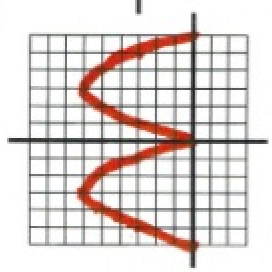
Graph 2.
x axis ... 2 squares = 1
y axis ... 2 squares = 1

3.



Graph 3.
x axis ... 1 square = 1
y axis ... 1 square = 1

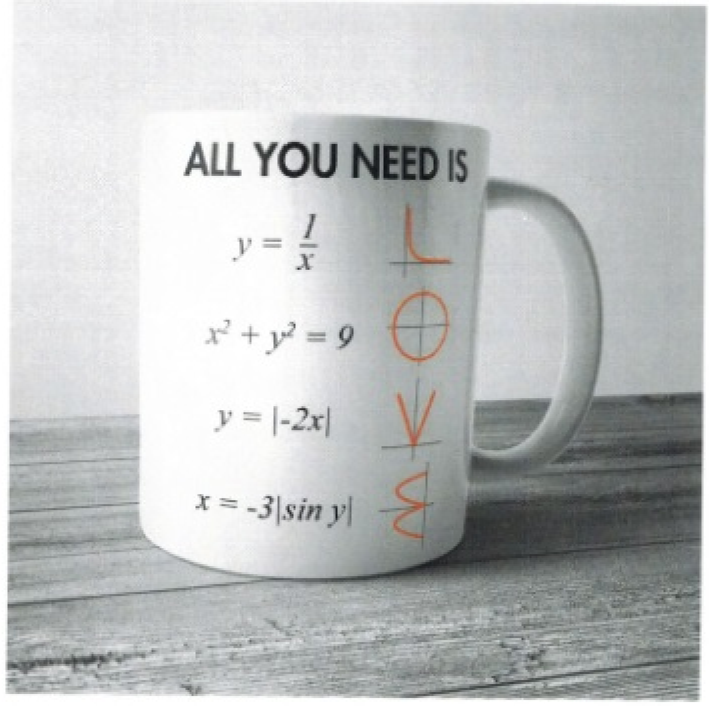
4.



Graph 4.
x axis ... 2 squares = 1
y axis ... 1 squares = 30°

Berg, Terrance (2017)

Or you could just find and buy this mug



Final Exam: Version A Answer Key

Questions from Chapters 1 to 3

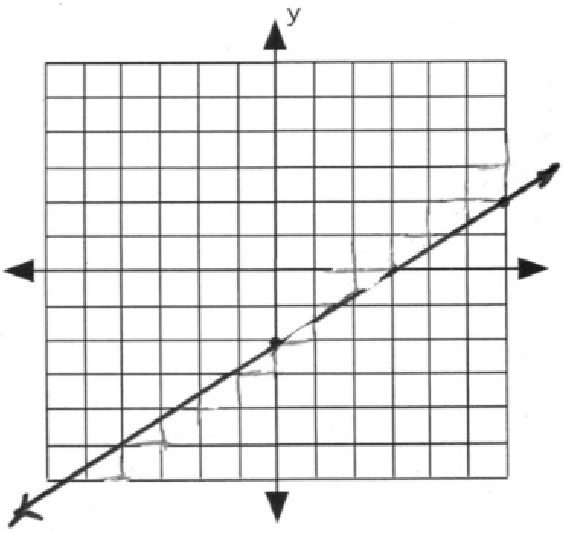
1. $-(6)-\sqrt{6^2-4(4)(2)}$

   $-6-\sqrt{36-32}$

   $-6-\sqrt{4}$

   $-6-2=-8$
2. $$\begin{array}{rrrcrrrrrrr} 6x & + & 24 & = & 35 & - & 5x & - & 8 & + & 12x \\ 6x & + & 24 & = & 27 & + & 7x \\ -7x & - & 24 & & -24 & - & 7x \\ & & -x & = & 3 \\ & & \therefore x & = & -3 \end{array}$$
3. $$\left(\frac{x+4}{2}-\frac{1}{2}=\frac{x+2}{4}\right)(4)$$
   $$\begin{array}{rrrrrcrrr} 2(x & + & 4) & - & 1(2) & = & x & + & 2 \\ 2x & + & 8 & - & 2 & = & x & + & 2 \\ -x & - & 8 & + & 2 & & -x & - & 8+2 \\ & & & & x & = & -4 \end{array}$$
4. $x=-2$
5. $$\begin{array}{rcl} d^2 & = & \Delta x^2+\Delta y^2 \\ & = & (2--4)^2+(6--2)^2 \\ & = & 6^2+8^2 \\ & = & 36+64 \\ & = & 100 \\ \\ \therefore d & = & \sqrt{100}=10 \end{array}$$
6. $2x-3y=6$

x	y
0	−2
3	0
6	2



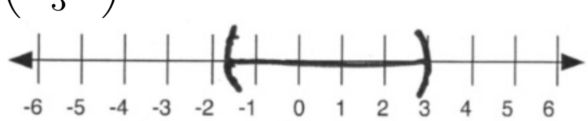
7.
$$\begin{array}{rrrrrrrr} x & - & 2x & + & 10 & \leq & 18 & + & 3x \\ & & -x & + & 10 & \leq & 18 & + & 3x \\ + & & -3x & - & 10 & & -10 & - & 3x \\ & & & & \dfrac{-4x}{-4} & \leq & \dfrac{8}{-4} & & \\ & & & & x & \geq & -2 & & \end{array}$$

$[-2, \infty)$

$$\left(-1 < \frac{3x-2}{7} < 1\right)(7)$$

$$\begin{array}{rrrrrrr} -7 & < & 3x & - & 2 & < & 7 \\ +2 & & & + & 2 & & +2 \\ \dfrac{-5}{3} & < & \dfrac{3x}{3} & & & < & \dfrac{9}{3} \\ -\dfrac{5}{3} & < & x & & & < & 3 \end{array}$$

$$\left(-\frac{5}{3}, 3\right)$$



8. $t = \dfrac{k}{r}$

$$\begin{array}{rclcrcl} & & \text{1st data} & \quad & & & \text{2nd data} \\ t & = & 45 \text{ min} & & t & = & \text{find} \\ k & = & \text{find 1st} & & k & = & 27000 \\ r & = & 600 \text{ kL/min} & & r & = & 1000 \text{ kL/min} \\ t & = & \dfrac{k}{r} & & t & = & \dfrac{k}{r} \\ 45 & = & \dfrac{k}{600} & & t & = & \dfrac{27000}{1000} \\ k & = & 45(600) & & t & = & 27 \text{ min} \\ k & = & 27000 \text{ kL} & & & & \end{array}$$

9.

$x, x+2$

$$\begin{array}{rrrrrcrrr} x & + & x & + & 2 & = & 4(x) & - & 12 \\ & & 2x & + & 2 & = & 4x & - & 12 \\ & - & 2x & + & 12 & & -2x & + & 12 \\ & & & & \dfrac{14}{2} & = & \dfrac{2x}{2} & & \\ & & & & x & = & 7 & & \end{array}$$

numbers are $7, 9$

Questions from Chapters 4 to 6

1.

$$\begin{array}{rrrrcr} & 2x & + & 5y & = & -18 \\ + & -2x & + & y & = & 6 \\ & & & \dfrac{6y}{6} & = & \dfrac{-12}{6} \\ & & & y & = & -2 \\ & \therefore y & - & 6 & = & 2x \\ & -2 & - & 6 & = & 2x \\ & & & 2x & = & -8 \\ & & & x & = & -4 \end{array}$$

Answer: $(-4,-2)$

2.
$$\begin{array}{rcrcr} (8x & + & 7y & = & 51)(-2) \\ (5x & + & 2y & = & 20)(7) \\ & & & & \\ -16x & - & 14y & = & -102 \\ +\quad 35x & + & 14y & = & 140 \\ & & \dfrac{19x}{19} & = & \dfrac{38}{19} \\ & & x & = & 2 \end{array}$$

$$\begin{array}{rcrcr} \therefore 5x & + & 2y & = & 20 \\ 5(2) & + & 2y & = & 20 \\ 10 & + & 2y & = & 20 \\ -10 & & & & -10 \\ & & 2y & = & 10 \\ & & y & = & 5 \end{array}$$

Answer: $(2,5)$

3.
$$\begin{array}{rcrcrcr} -2x & - & 2y & - & 12z & = & -10 \\ +\quad 2x & & & - & 3z & = & 4 \\ & & (-2y & - & 15z & = & -6)(3) \\ & & (3y & + & 4z & = & 9)(2) \\ & & & & & & \\ & & -6y & - & 45z & = & -18 \\ & + & 6y & + & 8z & = & 18 \\ & & & & -37z & = & 0 \\ & & & & z & = & 0 \end{array}$$

$$\begin{array}{rcrcl} 2x & - & 3z0 & = & 4 \\ & & x & = & \dfrac{4}{2} \text{ or } 2 \\ & & & & \\ 3y & + & 4z0 & = & 9 \\ & & y & = & \dfrac{9}{3} \text{ or } 3 \end{array}$$

Answer $(2,3,0)$

4. $24+\{-3x-[6x-3(5-2x)]^0 1\}+3x$

$24-3x-1+3x$

23

5. $2ab^3(a^2-16) \Rightarrow 2a^3b^3-32ab^3$

6. $(x^{1--2}y^{-3-4})^{-1}$

$(x^3y^{-7})^{-1}$

$x^{-3}y^7$

$\dfrac{y^7}{x^3}$

7. $3x^2+3x+8x+8$

$3x(x+1)+8(x+1)$

$(x+1)(3x+8)$

8. $(4x)^3-y^3 \Rightarrow (4x-y)(16x^2+4xy+y^2)$

9.
$$\begin{array}{rrrrl} & (A & + & B & = & 50)(-370) \\ & (3.95A & + & 3.70B & = & 191.25)(100) \\ \\ & -370A & - & 370B & = & -18500 \\ + & 395A & + & 370B & = & 19125 \\ & & & 25A & = & 625 \\ \\ & & & A & = & \dfrac{625}{25} \text{ or } 25 \\ \\ & A & + & B & = & 50 \\ & 25 & + & B & = & 50 \\ & & & B & = & 25 \end{array}$$

10.
$$\begin{array}{rrrrl} & (d & + & q & = & 16)(-10) \\ & 10d & + & 25q & = & 235 \\ \\ & -10d & - & 10q & = & -160 \\ + & 10d & + & 25q & = & 235 \\ & & & \dfrac{15q}{15} & = & \dfrac{75}{15} \\ \\ & & & q & = & 5 \\ & & & \therefore d & = & 16-5=11 \end{array}$$

Questions from Chapters 7 to 10

1. $\dfrac{153s^{32}}{3t^21} \cdot \dfrac{171s^3}{51t} \cdot \dfrac{3t^3}{342s^4} \Rightarrow \dfrac{3s^2}{2}$
2. $\text{LCD} = (x+2)(x-2)$

$$\frac{2x(x-2)-4x(x+2)+20}{(x+2)(x-2)}$$

$$\frac{2x^2-4x-4x^2-8x+20}{(x+2)(x-2)}$$

$$\frac{-2x^2-12x+20}{(x+2)(x-2)}$$

$$\frac{-2(x^2+6x-10)}{(x+2)(x-2)}$$

3. $$\frac{\left(\frac{x^2}{y^2}-9\right)y^3}{\left(\frac{x+3y}{y^3}\right)y^3} \Rightarrow \frac{x^2y-9y^3}{x+3y} \Rightarrow \frac{y(x^2-9y^2)}{x+3y} \Rightarrow \frac{y(x-3y)(x+3y)}{(x+3y)}$$

$$\Rightarrow y(x-3y)$$

4. $$3\cdot 5\sqrt{x}-2\sqrt{36\cdot 2x}-\sqrt{16\cdot x^2\cdot x}$$

$$15\sqrt{x}-2\cdot 6\sqrt{2x}-4x\sqrt{x}$$

$$15\sqrt{x}-12\sqrt{2x}-4x\sqrt{x}$$

5. $$\frac{\sqrt{m^6n}}{\sqrt{3n}} \Rightarrow \frac{m^3}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} \Rightarrow \frac{m^3\sqrt{3}}{3}$$

6. $$\left(\frac{a^0 1 b^4}{c^8 d^{-12}}\right)^{\frac{1}{4}} \Rightarrow \frac{b^{4\cdot\frac{1}{4}}}{c^{8\cdot\frac{1}{4}}d^{-12\cdot\frac{1}{4}}} \Rightarrow \frac{b}{c^2d^{-3}} \Rightarrow \frac{bd^3}{c^2}$$

7. $$(x-5)(x+1)=0$$
$$x=5,-1$$

8. $$\begin{array}{rrrrrrcr} & & (x & - & 3)^2 & = & (x)^2 \\ & x^2 & - & 6x & + & 9 & = & x^2 \\ - & x^2 & & & & & & -x^2 \\ & & & -6x & + & 9 & = & 0 \\ & & & & & \frac{-6x}{-6} & = & \frac{-9}{-6} \\ & & & & & x & = & \frac{3}{2} \end{array}$$

9.
$$\begin{aligned} A &= \frac{1}{2}bh \\ 20 &= \frac{1}{2}(h+6)h \\ 40 &= h^2+6h \\ 0 &= h^2+6h-40 \\ 0 &= h^2+10h-4h-40 \\ 0 &= h(h+10)-4(h+10) \\ 0 &= (h-4)(h+10) \\ h &= 4,-10 \\ b &= 4+6=10 \end{aligned}$$

10.
$$x, x+2, x+4$$

$$\begin{aligned} x(x+2) &= 8+6(x+4) \\ x^2+2x &= 8+6x+24 \\ -6x-8-24 & \quad -8-6x-24 \\ x^2-4x-32 &= 0 \\ x^2+4x-8x-32 &= 0 \\ x(x+4)-8(x+4) &= 0 \\ (x+4)(x-8) &= 0 \\ x &= -4, 8 \end{aligned}$$

$$\therefore \text{ numbers are } -4,-2,0 \text{ or } 8,10,12$$

Final Exam: Version B Answer Key

Questions from Chapters 1 to 3

1. $-2(-3) - \sqrt{(-3)^2 - 4(4)(-1)}$

   $6 - \sqrt{9 + 16}$

   $6 - 5$

   1

2. $$\begin{array}{rrrrcrrrr} & 18x & - & 30 & = & 3[4 & - & 4x & - & 7] \\ & 18x & - & 30 & = & 3[-4x & - & 3] \\ & 18x & - & 30 & = & -12x & - & 9 \\ + & 12x & + & 30 & & +12x & + & 30 \\ & & & 30x & = & 21 \\ \\ & & & x & = & \dfrac{21}{30} & = & \dfrac{7}{10} \end{array}$$

3. $$\left(\frac{x+4}{2} - \frac{1}{3} = \frac{x+2}{6}\right)(6)$$

   $$\begin{array}{rrrrrcrrr} (x & + & 4)(3) & - & 1(2) & = & x & + & 2 \\ 3x & + & 12 & - & 2 & = & x & + & 2 \\ -x & - & 10 & & & & -x & - & 10 \\ & & & & \dfrac{2x}{2} & = & \dfrac{-8}{2} \\ \\ & & & & x & = & -4 \end{array}$$

4.

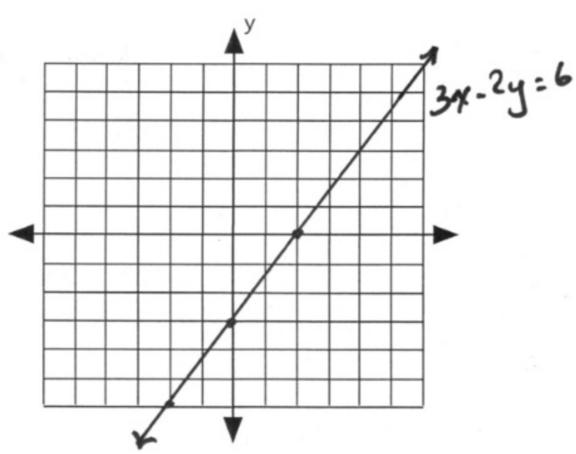
$$
\begin{aligned}
m &= \frac{\Delta y}{\Delta x} \\
\frac{2}{3} &= \frac{y-4}{x-1} \\
2(x-1) &= 3(y-4) \\
2x-2 &= 3y-12 \\
-3y+12 & \quad -3y+12 \\
2x-3y+10 &= 0 \\
y &= \frac{2}{3}x+\frac{10}{3}
\end{aligned}
$$

5.

$$
\begin{aligned}
d^2 &= \Delta x^2+\Delta y^2 \\
&= (4--4)^2+(4--2)^2 \\
&= 8^2+6^2 \\
&= 64+36 \\
&= 100 \\
d &= 10
\end{aligned}
$$

6. $3x-2y=6$

x	y
2	0
0	−3
−2	−6



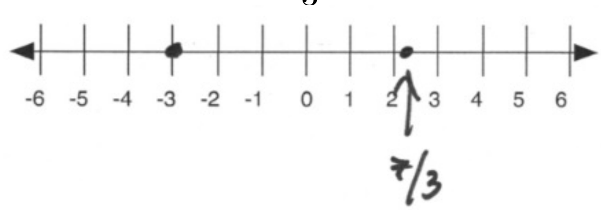
7.

$$\begin{array}{rcrcrcr} 3 & \leq & 6x & + & 3 & < & 9 \\ -3 & & & - & 3 & & -3 \\ \dfrac{0}{6} & \leq & & \dfrac{6x}{6} & & < & \dfrac{6}{6} \\ \\ 0 & \leq & & x & & < & 1 \end{array}$$

$(0, 1)$

8. $$\left(\frac{3x+1}{4} = 2\right)^4 \qquad \left(\frac{3x+1}{4} = -2\right)^4$$

$$\begin{array}{rcrcr} 3x & + & 1 & = & 8 \\ & - & 1 & & -1 \\ & & 3x & = & 7 \\ \\ & & x & = & \dfrac{7}{3} \end{array} \qquad \begin{array}{rcrcr} 3x & + & 1 & = & -8 \\ & - & 1 & & -1 \\ & & 3x & = & -9 \\ & & x & = & -3 \end{array}$$



9.

$$w_{\mathrm{m}} = kw_e$$

1st data

$$\begin{aligned} w_{\mathrm{m}} &= 38 \text{ lb} \\ k &= \text{find 1st} \\ w_{\mathrm{e}} &= 95 \text{ lb} \\ \\ w_{\mathrm{m}} &= kw_{\mathrm{e}} \\ 38 &= k(95) \\ k &= \frac{38}{95} \\ k &= 0.4 \end{aligned}$$

2nd data

$$\begin{aligned} w_{\mathrm{m}} &= \text{find} \\ k &= 0.4 \\ w_{\mathrm{e}} &= 240 \text{ lb} \\ \\ w_{\mathrm{m}} &= kw_{\mathrm{e}} \\ w_{\mathrm{m}} &= (0.4)(240) \\ w_{\mathrm{m}} &= 96 \text{ lb} \end{aligned}$$

10.

$x, x+2$

$$\begin{array}{ccccccccccc} x & + & x & + & 2 & = & x & + & 2 & - & 20 \\ & & 2x & + & 2 & = & x & - & 18 & & \\ & & -x & - & 2 & & -x & - & 2 & & \\ & & & & x & = & -20 & & & & \end{array}$$

$$\text{numbers are } -20, -18$$

Questions from Chapters 4 to 6

1.

$$\begin{array}{cccccc} & (4x & - & 3y & = & 13)(5) \\ & (6x & + & 5y & = & -9)(3) \\ \\ & 20x & - & 15y & = & 65 \\ + & 18x & + & 15y & = & -27 \\ & & & 38x & = & 38 \\ & & & x & = & 1 \\ \\ & 4(1) & - & 3y & = & 13 \\ & -4 & & & & -4 \\ & & & -3y & = & 9 \\ & & & y & = & -3 \end{array}$$

$(1,-3)$

2.
$$\begin{array}{rrrrrrl} & & & & x & = & -1-y \\ \\ \therefore 3(-1 & - & y) & - & 4y & = & -5 \\ -3 & - & 3y & - & 4y & = & -5 \\ +3 & & & & & & +3 \\ & & & & -7y & = & -2 \\ & & & & y & = & \dfrac{2}{7} \\ \\ & & x & + & y & = & -1 \\ & & x & + & \dfrac{2}{7} & = & -1 \\ \\ & & & - & \dfrac{2}{7} & & -\dfrac{2}{7} \\ & & & & x & = & -\dfrac{9}{7} \end{array}$$

3.
$$\begin{array}{rrrrrrl} & & & (x & - & 4z & = & 0)(-1) \\ \\ & x & + & 2y & & & = & 0 \\ + & -x & & & + & 4z & = & 0 \\ & & & (2y & + & 4z & = & 0)(\div 2) \\ & & & y & + & 2z & = & 0 \\ \\ & & & y & + & 2z & = & 0 \\ & & + & y & - & 2z & = & 0 \\ & & & & & 2y & = & 0 \\ & & & & & y & = & 0 \\ \\ & & & y0 & - & 2z & = & 0 \\ & & & & & z & = & 0 \\ \\ & & & x & + & 2y0 & = & 0 \\ & & & & & x & = & 0 \end{array}$$

$(0,0,0)$

4. $28-\{5x^01-[6x-3(5-2x)]^01\}+5x^01$
$28-\{5-1\}+5$
$28-4+5$
29

5.
$$\begin{array}{rrrrrrrr} & x^2 & - & 3x & + & 8 & & \\ \times & & & x & - & 4 & & \\ & x^3 & - & 3x^2 & + & 8x & & \\ + & & - & 4x^2 & + & 12x & - & 32 \\ & x^3 & - & 7x^2 & + & 20x & - & 32 \end{array}$$

6. $(x^{3n-6-3n})^{-1}$
$(x^{-6})^{-1}$
x^6

7. $5y(5y^2-3y+1)$

8. $x^3+(2y)^3$
$(x+2y)(x^2-2xy+4y^2)$

9.

Solution	Amount	Strength	Equation
Soda	x	0	0
Juice	2	35	2 (35)
Diluted	$x+2$	8	$(x+2)8$

$$\begin{array}{rrrrrr} & 2(35) & = & 8(x & + & 2) \\ & 70 & = & 8x & + & 16 \\ - & 16 & & & - & 16 \\ & 54 & = & 8x & & \\ & & & & & \\ & x & = & \dfrac{54}{8} & \text{or} & 6\dfrac{3}{4}\text{ litres} \end{array}$$

10.
$$\begin{array}{rrrrrr} & (d & + & q & = & 14)(-10) \\ & 10d & + & 25q & = & 185 \\ & & & & & \\ & -10d & - & 10q & = & -140 \\ + & 10d & + & 25q & = & 185 \\ & & & \dfrac{15q}{15} & = & \dfrac{45}{15} \\ & & & & & \\ & & & q & = & 3 \\ & & & & & \\ & \therefore d & + & 3 & = & 14 \\ & & & d & = & 11 \end{array}$$

Questions from Chapters 7 to 10

1. $\dfrac{9s^2}{7t^3}\cdot\dfrac{15t}{13s^2}\cdot\dfrac{262s}{9t}\Rightarrow\dfrac{15\cdot 2\cdot 5}{7t^3}\Rightarrow\dfrac{30s}{7t^3}$

2. $\dfrac{(a-1)2a}{(a-1)(a-6)(a+6)} - \dfrac{5(a+6)}{(a-6)(a-1)(a+6)} \Rightarrow \dfrac{2a^2-2a-5a-30}{(a-1)(a-6)(a+6)}$

$\Rightarrow \dfrac{2a^2-7a-30}{(a-1)(a-6)(a+6)} \Rightarrow \dfrac{2a^2-12a+5a-30}{(a-1)(a-6)(a+6)}$

$\Rightarrow \dfrac{2a(a-6)+5(a-6)}{(a-1)(a-6)(a+6)} \Rightarrow \dfrac{(a-6)(2a+5)}{(a-1)(a-6)(a+6)} \Rightarrow \dfrac{2a+5}{(a-1)(a+6)}$

3. $\dfrac{\left(1-\dfrac{8}{x}\right)x^2}{\left(\dfrac{3}{x}-\dfrac{24}{x^2}\right)x^2} \Rightarrow \dfrac{x^2-8x}{3x-24} \Rightarrow \dfrac{x(x-8)}{3(x-8)} \Rightarrow \dfrac{x}{3}$

4. $\sqrt{x^4\cdot x\cdot y^6\cdot y}+2xy\sqrt{16\cdot x\cdot y^2\cdot y}-\sqrt{x\cdot y^2\cdot y}$

$x^2y^3\sqrt{xy}+2xy\cdot 4y\sqrt{xy}-y\sqrt{xy}$

$(x^2y^3+8xy^2-y)\sqrt{xy}$

5. $\dfrac{2+x}{1-\sqrt{7}}\cdot\dfrac{1+\sqrt{7}}{1+\sqrt{7}} \Rightarrow \dfrac{2+2\sqrt{7}+x+x\sqrt{7}}{1-7} \Rightarrow \dfrac{2+x+2\sqrt{7}+x\sqrt{7}}{-6}$

6. $\left(\dfrac{a^6b^3}{c^0 1 d^{-9}}\right)^{\frac{2}{3}} \Rightarrow \dfrac{a^{6\cdot\frac{2}{3}}b^{3\cdot\frac{2}{3}}}{d^{-9\cdot\frac{2}{3}}} \Rightarrow \dfrac{a^4b^2}{d^{-6}} \Rightarrow a^4b^2d^6$

7. $$\begin{aligned}(x-5)(x+3) &= 0\\ x &= 5, -3\end{aligned}$$

8.

$$\left(\frac{2x-1}{3x}=\frac{x-3}{x}\right)(3x)$$

$$\begin{array}{rrrrcrrr} & 2x & - & 1 & = & (x & - & 3)(3)\\ \\ & 2x & - & 1 & = & 3x & - & 9\\ + & -3x & + & 1 & & -3x & + & 1\\ & & & -x & = & -8 & & \\ & & & x & = & 8 & & \end{array}$$

9.
$$\begin{aligned} A &= L \cdot W \\ L &= 5 + 2W \\ \\ 75 &= W(5+2W) \\ 75 &= 5W + 2W^2 \\ \\ 0 &= 2W^2 + 5W - 75 \\ 0 &= 2W^2 - 10W + 15W - 75 \\ 0 &= 2W(W-5) + 15(W-5) \\ 0 &= (W-5)(2W+15) \\ W &= 5, -\frac{15}{2} \\ \\ L &= 5 + 2(5) \\ L &= 15 \end{aligned}$$

10.
$$x, x+2, x+4$$

$$\begin{array}{ccccccccccc} & & x(x & + & 2) & = & 8(x & + & 4) & - & 25 \\ x^2 & + & 2x & & & = & 8x & + & 32 & - & 25 \\ & - & 8x & - & 32 & & -8x & - & 32 & + & 25 \\ & & & + & 25 & & & & & & \\ x^2 & - & 6x & - & 7 & = & 0 & & & & \\ (x & - & 7)(x & + & 1) & = & 0 & & & & \\ & & & & x & = & 7, & -1 & & & \end{array}$$

$$\text{numbers are } 7, 9, 11 \text{ or } -1, 1, 3$$